Multi-Focus Microlens Array Light Field Imaging for Three-Dimensional Flame Temperature Field Measurement (Postprint)

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Date: 2017-11-07T00:00:00+00:00

Abstract

Multi-focus microlens arrays can improve the depth resolution of focused light field cameras. To investigate the influence of multi-focus microlens arrays on three-dimensional temperature field measurement of flames via light field imaging, this study analyzes the flame radiation light field imaging characteristics of both single-focus and multi-focus microlens arrays based on a flame radiation light field imaging model, computes flame radiation images under the two different microlens arrays, and reconstructs the three-dimensional temperature field of the flame from the flame light field images. Experimental research on three-dimensional temperature field reconstruction of flames using a multi-focus microlens array focused light field camera was conducted, and both numerical calculation and experimental results were analyzed.

Full Text

Preamble

Three-dimensional Temperature Measurement of the Flame Using the Light Field Camera with a Multiple Focus Microlens Array

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Abstract: Multiple focus microlens array (MFMA) can improve the depth resolution of the focused light field camera. This paper investigates the effect of MFMA on the three-dimensional (3-D) temperature measurement of the flame. On the basis of the light field imaging model of the flame, the light field imaging characteristics of flame radiation were analyzed for both MFMA and single focus microlens array (SFMA). For the two different types of microlens arrays, the light field images of the flame were numerically calculated. According to the light field images, the 3-D temperature field of the flame was also reconstructed. Experiments were carried out on the 3-D temperature measurement of the flame using the light field camera with an MFMA. The simulation and experimental results were analyzed in detail.

Key words: focused light field camera; multiple focus microlens array; radiative transfer equation; inverse; temperature

Combustion flames are widely present in industrial processes such as gas turbines, power station boilers, and aero engines [1, 2]. Combustion is a complex, intense multiphase reaction process accompanied by continuous heat and mass transfer, generating multiple combustion products and releasing substantial heat. Since temperature is a crucial parameter characterizing combustion status, effective measurement of flame temperature fields is essential for improving combustion efficiency and stability while controlling combustion products, particularly pollutant gases.

Existing flame temperature field measurement techniques can be divided into two categories: laser spectroscopy-based methods [3-4] and flame image-based methods [5-7]. Laser spectroscopy-based techniques can measure not only flame temperature fields but also the distribution of intermediate species during combustion. For instance, Ma et al. used a hyperspectral detector to receive absorption spectra after laser transmission through flames and then employed tomographic inversion algorithms to reconstruct flame temperature fields and absorptive species concentrations [4]. However, laser-based temperature measurement techniques suffer from complex systems, high costs, and demanding operational requirements, making them primarily suitable for laboratory flame diagnostics. Flame image-based temperature measurement methods, which do not require external laser sources and only use conventional imaging detectors (cameras) to capture flame self-radiation images combined with inversion algorithms for temperature field reconstruction [5-8], offer reduced system complexity and easier installation and operation. These methods hold advantages for industrial flame temperature measurement where laser techniques cannot be applied due to measurement condition constraints. Hossain et al. used conventional cameras to capture flame images and applied tomographic reconstruction techniques such as the Filtered Back-Projection (FBP) algorithm to obtain cross-sectional images of flame interior layers, then calculated temperature distributions for each layer using the two-color method [6], with existing applications in industrial flame monitoring. Li et al. similarly established a radiative transfer model correlating flame radiation with image grayscale based on the flame radiation imaging process of conventional cameras, then reconstructed flame temperature fields through inversion algorithms such as Tikhonov regularization, with applications in measuring large-scale industrial flames such as power station boiler combustion [7]. However, since conventional cameras cannot distinguish flame radiation directions, image-based temperature measurement techniques require multiple conventional cameras to acquire multi-view radiation information for non-axisymmetric flame 3-D temperature field reconstruction, demanding high camera coupling and synchronization.

Light field cameras, equipped with a microlens array between the imaging detector and main lens, can record not only flame radiation intensity information with higher accuracy under single exposure conditions compared to conventional cameras but also resolve flame radiation directions. Combined with inversion algorithms such as the Least Squares based on QR decomposition (LSQR), light field cameras enable 3-D flame temperature field reconstruction using a single device [9, 10], significantly reducing measurement system complexity. Building upon conventional light field cameras [9], Lumsdaine proposed the focused light field camera design, where the imaging detector is positioned on the defocused plane of the microlens array. This configuration reduces directional sampling while increasing spatial sampling, effectively improving the resolution of refocused images [11]. Based on this, Perwass et al. employed a triple-focus microlens array with three different focal lengths uniformly distributed, enabling spatial points at different depths to be refocused onto the same image simultaneously and further improving the depth resolution of light field cameras [10]. However, no research has yet been conducted on flame temperature field reconstruction using triple-focus microlens array focused light field cameras.

To investigate whether adopting a triple-focus microlens array in focused light field cameras can improve measurement accuracy for 3-D flame temperature based on light field imaging, this paper conducts research on flame 3-D temperature field measurement methods using light field imaging technology with focused light field cameras as flame radiation detection devices. The distribution of captured radiation rays was analyzed for both single-focus and triple-focus microlens array focused light field cameras, and the influence of the two microlens arrays on flame 3-D temperature field reconstruction results was studied. Finally, experimental research on 3-D temperature field reconstruction of ethylene diffusion flames was conducted using a triple-focus microlens array focused light field camera.

1. Flame Radiation Light Field Imaging Model

Light field cameras install a microlens array between the imaging detector and main lens. In conventional light field cameras, the detector plane is located at the focal plane of the microlens array, whereas in focused light field cameras, the detector plane is offset from the microlens array’ s focal plane, reducing directional dimension sampling and increasing spatial dimension sampling, thereby overcoming the redundancy in directional sampling of conventional light field cameras. This paper adopts a focused light field camera as the flame temperature field measurement device, as shown in Figure 1: see original paper. The object-side conjugate plane of the camera detector plane with respect to the imaging system is defined as the virtual object plane, where the detector plane and virtual image plane are conjugate about the microlens, and the virtual image plane and virtual object plane are conjugate about the main lens. Points on the virtual object plane are called virtual source points, while points on the virtual image plane are called virtual image points.

When the microlens array contains three focal lengths, there exist three corresponding virtual image planes and three virtual object planes, as shown in Figure 1: see original paper. The flame radiation received by each pixel on the detector plane can be considered as originating from a bundle of radiation imaging rays from a virtual source point. As illustrated in Figure 1: see original paper, for the case where the distance between the microlens array and detector plane is less than the microlens focal length, the virtual image plane and detector plane lie on the same side of the microlens array. The flame radiation imaging beam from the same virtual source point is split into several bundles by the microlenses before being converged by the main lens to the virtual image point. Each bundle is projected onto different pixels on the detector plane through the converging action of the corresponding microlens. The direction of the beam corresponding to each pixel can be determined by the pixel position and its associated microlens location, simultaneously recording both the intensity and direction of flame radiation rays. For each pixel on the detector plane, since the aperture angle of the received radiation imaging ray bundle is small [10, 13], a pinhole imaging model is employed using the principal ray passing through the microlens center to represent this ray bundle. The intensity and direction of the principal ray represent the intensity and direction of flame radiation detected by that pixel. This principal ray is referred to as the sampling ray of the pixel. The spatial distribution of sampling rays from all pixels on the detector plane characterizes the flame radiation sampling pattern of the light field camera. Therefore, by studying the distribution of sampling rays for both single-focus and triple-focus microlens array focused light field cameras, the flame radiation sampling characteristics of the two configurations can be analyzed and compared.

For a single-focus microlens array focused light field camera, all microlenses on the array have identical focal lengths. The distribution of sampling rays for pixels on the detector plane is shown in Figure 1: see original paper. For a triple-focus microlens array focused light field camera, the microlenses have three different focal lengths arranged uniformly such that no two microlenses with the same focal length are adjacent. Microlenses with the same focal length can focus and image object points at a certain depth in object space, while three focal lengths facilitate simultaneous focusing of object points at different depths onto the detector plane, improving the camera’ s depth resolution [12]. The sampling ray distribution for pixels on the detector plane of a triple-focus microlens array focused light field camera is shown in Figure 1: see original paper.

In Figure 1: see original paper and (c), microlenses with three different focal lengths are represented by red, green, and blue colors, respectively. Comparing the two figures reveals that compared with the triple-focus microlens array, the single-focus microlens array exhibits more uniform virtual source point distribution and more uniform sampling ray distribution within the flame interior space, with higher independence among sampling rays. In contrast, the flame radiation sampling rays of the multiple-focus microlens array show non-uniform distribution within the flame interior space with higher overlap. With a comparable number of sampling rays, the single-focus microlens array demonstrates lower overlap of sampling rays within the flame interior space, acquiring more effective flame radiation information and thus being more favorable for flame 3-D temperature field reconstruction.

According to the thin lens formulas (1) and (2) in geometric optics [14], ray tracing is performed for the flame radiation sampling rays of each pixel on the detector plane to determine their positions and directions in object (flame) space.

$$\frac{1}{s'} + \frac{1}{s} = \frac{1}{F} \quad (1)$$

$$\frac{x'}{x} = -\frac{s'}{s} \quad (2)$$

where s' and s are the image distance and object distance of the lens, respectively. When rays pass through a microlens, s' is the distance between the virtual image plane and microlens array, s is the distance between the detector plane and microlens array, x' is the coordinate of the virtual image point relative to the microlens center, x is the coordinate of the pixel relative to the microlens center, and F is the focal length of the microlens. When rays pass through the main lens, s' is the distance between the virtual object plane and main lens, s is the distance between the virtual image plane and main lens, x' is the coordinate of the virtual source point relative to the main lens center, x is the coordinate of the virtual image point relative to the main lens center, and F is the focal length of the main lens.

Soot particles in flames are absorptive with small diameters. According to Mie theory, the scattering capability of soot particles is far weaker than their absorption capability [15]. This paper considers only absorption to simplify the flame radiation transfer process. Using the apparent ray method [16], the intensity of flame radiation sampling rays for each pixel on the camera detector plane is calculated through equation (3) to obtain flame radiation images.

$$I_n = \sum_{j=1}^{n} \left[I_{b,j} \cdot (1 - \exp(-\tau_j)) \cdot \exp\left(-\sum_{i=j+1}^{n} \tau_i \right) \right] \quad (3)$$

where I_n is the ray intensity detected by the detector plane, $I_{b,i}$ and τ_i are the blackbody radiation intensity and optical path of the i-th control volume along the ray path, respectively, and $I_{b,n}$ and τ_n are the blackbody radiation intensity and optical path of the last control volume along the ray path, respectively.

To reconstruct the flame 3-D temperature field, a linear equation system regarding the intensities of all sampling rays on the detector plane is established based on equation (3):

$$\mathbf{I}_{ccd} = \mathbf{A} \cdot \mathbf{I}_B \quad (4)$$

where $\mathbf{I}_{ccd}$ is the vector composed of flame radiation intensities detected by pixels on the camera imaging detector plane, $\mathbf{I}_B$ is the vector composed of blackbody radiation intensities of all flame control volumes, and $\mathbf{A}$ is the corresponding coefficient matrix. The coefficient matrix $\mathbf{A}$ is a large sparse matrix. The Least Squares method based on QR decomposition (LSQR) can solve least squares problems involving large sparse matrices. Similar to the conjugate gradient method, this algorithm exhibits higher stability when $\mathbf{A}$ is ill-conditioned [17]. Solving equation (4) involves solving the least squares problem shown in equation (5):

$$\min \|\mathbf{A}x - b\| \quad (5)$$

where x equals $\mathbf{I}_B$ and b equals $\mathbf{I}_{ccd}$. Assuming k bidiagonalization processes have been performed, the $m \times (k+1)$ orthogonal matrix $\mathbf{U}_{k+1}$, the $n \times k$ orthogonal matrix $\mathbf{V}_k$, and the $(k+1) \times k$ bidiagonal matrix $\mathbf{B}_k$ are obtained [18]:

$$\mathbf{A}\mathbf{V}_k = \mathbf{U}_{k+1}\mathbf{B}_k \quad (6)$$

where $\alpha_1, \alpha_2, \cdots, \alpha_k \in \mathbb{R}$ and $\beta_1, \beta_2, \cdots, \beta_{k+1} \in \mathbb{R}$. The bidiagonalization process yields:

$$\mathbf{U}_{k+1}^T b = \beta_1 e_1 \quad (7)$$

Since $\mathbf{U}_{k+1}$ is an orthogonal matrix and orthogonal transformations do not change matrix norms, the original problem becomes:

$$\min \left\| \begin{pmatrix} \mathbf{B}_k \\ \mathbf{0} \end{pmatrix} y - \begin{pmatrix} \beta_1 e_1 \\ 0 \end{pmatrix} \right\| \quad (10)$$

Thus, the complex least squares problem is transformed into a simpler one. The standard QR decomposition method is used to solve this least squares problem (11). After obtaining the blackbody radiation intensity $I_{b,i}$ of the i-th flame control volume, the flame temperature of each control volume is further calculated using the Stefan-Boltzmann law (12):

$$I_{b,i} = \frac{\sigma T_i^4}{\pi} \quad (12)$$

where the Stefan-Boltzmann constant σ is $5.670373 \times 10^{-8}\ \mathrm{W} \cdot \mathrm{m}^{-2} \cdot \mathrm{K}^{-4}$.

3.1 Numerical Simulation

To compare the accuracy of flame 3-D temperature field reconstruction between single-focus and triple-focus microlens array focused light field cameras, numerical simulations were conducted. Based on the parameters of the commercial focused light field camera Raytrix R29 [10, 12], the computational parameters were set as shown in .

Table 1 The parameters of the light field camera in the numerical simulations

Parameter	Value
d_p (mm)	0.006
f_m (mm)	A: 0.65, B: 0.60, C: 0.55
d_m (mm)	0.140
f (mm)	12.5
L_{ix} (mm)	A: -1.36, B: -1.65, C: -2.20
L_{ux} (mm)	2.5
L_{xp} (mm)	0.44
L_{ou} (mm)	A: 505, B: 482, C: 444

where d_m and d_p are the microlens diameter and pixel side length, respectively; N_m and N_s are the number of pixels covered by the diameter of each sub-image (a group of pixels covered by a microlens) and the number of microlenses in the vertical (or horizontal) direction on the microlens array, respectively; f_m and f are the focal lengths of the microlens and main lens, respectively; L_{ix}, L_{ux}, and L_{xp} are the distances between the microlens array and virtual image plane, main lens plane, and camera detector plane, respectively; and L_{ou} is the distance between the main lens plane and virtual object plane.

For both microlens arrays, the parameters d_m, d_p, N_m, N_s, f, L_{ux}, and L_{xp} are identical. The microlens focal length f_m for the single-focus microlens array is 600 m. For the triple-focus microlens array, the three types of microlenses (A, B, and C) have focal lengths f_m of 650 m, 600 m, and 550 m, respectively, all greater than the distance L_{xp} (440 mm) between the microlens array and detector plane. The three focal length microlenses are arranged alternately in the horizontal (or vertical) direction such that no two microlenses with the same focal length are adjacent. L_{ix} and L_{ou} are calculated based on the conjugate relationships described in Section 1.

The flame is configured as a cylinder with radius L of 80 mm and height H of 300 mm. The flame absorption coefficient is 8 m^{-1}. The flame is divided into $10\times10\times1$ control volumes along the axial, radial, and circumferential directions. The flame temperature is set to be symmetrically distributed about the cylinder central axis according to the distribution function $T(r,z)$:

$$T(r,z) = 1800\sin\left(\frac{\pi}{2}\cdot\frac{r}{L}\right)\cdot\left(1-\frac{z}{H}\right)+273.15 \quad (13)$$

where r and z are the radial and axial coordinates (in mm) of the control volume center position, respectively, with the origin at the center of the flame bottom surface. For the single-focus microlens array, the flame central plane is located on the virtual object plane. For the triple-focus microlens array, the flame central plane is located on the virtual object plane corresponding to microlens (B). Since the flame radius is 80 mm, the distances L_{ou} between the virtual object planes and main lens plane for the three focal length microlenses (A, B, and C) are 505 mm, 482 mm, and 444 mm, respectively. All three virtual object planes are positioned within the flame interior, ensuring that the flame is not in a defocused position for any of the three microlens focal lengths.

Based on equations (8) and (9), the flame radiation images were calculated as shown in [Figure 2: see original paper]. Figure 2(a) shows the flame image from the single-focus microlens array focused light field camera, while Figure 2(b) shows the flame image from the triple-focus microlens array focused light field camera. Macroscopically, the two images are essentially consistent, with the flame images conforming to the temperature distribution characteristics of equation (13), where brighter image regions correspond to higher temperatures and darker regions correspond to lower temperatures.

Comparing the flame sub-images reveals that the single-focus microlens array focused light field camera produces sub-images of uniform size, whereas the triple-focus microlens array focused light field camera generates sub-images of varying diameters despite identical microlens diameters d_m. Specifically, larger microlens focal lengths ($C \to B \to A$) correspond to smaller sub-image diameters, as shown in Figure 2(b). This occurs because the distance L_{xp} between the imaging detector plane and microlens array remains constant; larger microlens focal lengths f_m result in greater defocusing of the pupil’ s image on the detector plane about the microlens, producing blurrier imaging contours and smaller sub-image diameters.

Gaussian noise of varying levels was superimposed on the flame images to simulate measurement errors, with signal-to-noise ratios of 30 dB and 20 dB. The 3-D flame temperature field was reconstructed using the method described in Section 2, and the relative temperature reconstruction errors for each control volume were calculated. The average relative error across all control volumes ($10\times10\times1$) is presented in [Figure 3: see original paper]. The results show that as noise increases, the relative temperature reconstruction errors for both single-focus and triple-focus microlens arrays increase accordingly. However, at either noise level (30 dB or 20 dB), the relative temperature reconstruction error for the single-focus microlens array is smaller than that for the triple-focus microlens array. This is because, compared with the triple-focus microlens array focused light field camera, the single-focus microlens array focused light field camera exhibits higher uniformity in the distribution of flame radiation sampling rays within the flame interior, thereby achieving higher accuracy in flame temperature measurement.

3.2 Experimental Study

An experimental study was conducted on ethylene diffusion flame temperature measurement using a triple-focus microlens array focused light field camera (Raytrix R29). The captured flame light field image is shown in Figure 4: see original paper. The fuel flow rate was 3 mL/s, with flame radius and height of 10 mm and 90 mm, respectively. Using this image, the 3-D temperature field of the ethylene diffusion flame was reconstructed. Six cross-sections were uniformly selected along the flame height direction from $Z = 5$ mm to $Z = 90$ mm, as shown in Figure 4(a). The reconstructed temperature distributions for each cross-section are presented in Figure 4: see original paper. The flame temperature ranges from 700 K to 1400 K. In diffusion flames, fuel (ethylene) diffuses from inside outward while air diffuses from outside inward. When the fuel-air mixture ratio reaches the stoichiometric ratio for combustion, complete combustion occurs, releasing substantial heat and forming high-temperature regions where flame temperatures exceed other regions [20]. Consequently, the temperature distribution along the radial direction for each cross-section shows a trend of initially increasing then decreasing from inner to outer layers. For the highest flame cross-section (90 mm), fuel and air can mix sufficiently at the center, resulting in a gradual temperature decrease from inner to outer layers along the radial direction. The temperature distribution shown in Figure 4(b) aligns well with this pattern.

Thermocouples (Type R) were used to measure temperature values at different radial positions ($R = 0$ mm, $R = 5$ mm, $R = 10$ mm) on cross-sections at different flame heights ($Z = 30$ mm, $Z = 60$ mm). Since thermocouples exchange radiative heat with the cooler surrounding environment during flame temperature measurement, causing radiative heat loss, the thermocouple measurement results were corrected according to reference [19]. The corrected temperature values were compared with the temperature reconstruction results from the triple-focus focused light field camera, as shown in [Figure 5: see original paper]. The temperature distribution trends at both heights are essentially consistent, with flame temperature initially increasing then decreasing as radius increases. At height $Z = 60$ mm and radius $R = 5$ mm, the temperature difference between the two methods reaches a maximum of 182 K, while differences at other locations are all less than 85 K, with the minimum difference being 8 K. Relative to the flame temperature reaching 1400 K, the reconstruction results demonstrate considerable reliability.

To further improve the accuracy of flame 3-D temperature field reconstruction, based on the numerical simulation results, a focused light field camera with a single-focus microlens array can be designed and assembled for flame 3-D temperature field reconstruction.

Conclusion

This paper analyzed the light field imaging characteristics of single-focus and multiple (triple)-focus microlens arrays, comparing the distribution properties of flame radiation sampling rays for the two microlens arrays. The apparent ray method was used to calculate flame radiation images for both microlens arrays, with different noise levels superimposed as measurement errors. Based on these images, the 3-D flame temperature field was reconstructed. Finally, experimental research on 3-D temperature measurement of ethylene diffusion flames was conducted using a multiple-focus microlens array focused light field camera. The results demonstrate that for the triple-focus focused light field camera, sub-image contours in flame light field images decrease with increasing microlens focal length. The single-focus microlens array exhibits more uniform distribution of flame radiation sampling rays within the flame interior space with higher ray independence, resulting in lower flame temperature field reconstruction errors compared to the triple-focus microlens array. The reconstructed temperature field of the ethylene diffusion flame shows consistent distribution trends and good numerical agreement with thermocouple measurement results. Future work will focus on designing and implementing a single-focus microlens array focused light field camera for flame 3-D temperature field reconstruction to improve temperature reconstruction accuracy.

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