

## Mathematical Modelling of the Transient Response of Pipeline Postprint

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### Abstract

The development of heat recovery methods for dry granulation processes of blast furnace slag in the iron and steel industry is limited by the high energy consumption associated with these processes. To determine the factors influencing the diameter of granulated particles, a paraffin test platform for gas quenching granulation was established. The effects of air velocity, air flow rate, liquid mass flow rate, and liquid pipe diameter on the final particle size and mass distribution were investigated. Experimental results demonstrated that the final particle size decreased from 1.07 mm to 0.81 mm with increasing air velocity from 28.3 m/s to 113.2 m/s. However, when air velocity exceeded 60 m/s, its influence on particle diameter diminished significantly. The experimental data were analyzed using SPSS Statistics software, which indicated that the effect of air velocity on particle diameter was the most significant, followed by those of air flow rate and liquid pipe diameter. The effect of liquid mass flow rate was the least significant.

### Full Text

#### Preamble

Steam pipelines in power units operate at high pressures and temperatures, generating significant thermal stresses during transient states such as startup, shutdown, or load changes. These time-varying stresses often cause fatigue cracks, particularly in regions of stress concentration where plastic deformations appear. To determine the transient steam temperature along the flow path and the axisymmetric temperature distribution in the pipeline wall, a numerical heating model was proposed using the finite volume method (FVM). Writing energy conservation equations for control volumes around all nodes yields a system of ordinary differential equations in time, which was solved using the fourth-order Runge-Kutta method to obtain time-temperature changes at nodes in both the wall and steam regions. The steam pressure distribution along the pipeline was

determined from the momentum conservation equation, and thermal stresses were calculated based on the resulting temperature distribution. The friction factor was computed using Churchill and Haaland correlations for pipes with rough inner surfaces. To assess model accuracy, numerical calculations were also performed for a thin-walled pipe and compared with exact analytical solutions, showing very satisfactory agreement. The paper presents examples of steam and wall temperature determination.

**Keywords:** steam pipeline, heating, mathematical model, thermal stresses

## Introduction

High-temperature steam pipes in steam boilers transport superheated live steam from the boiler to the turbine. During startups, shutdowns, and load changes, high thermal stresses can occur in pipeline walls and fittings, particularly in T-pipes and Y-pipes. Thick-walled components with complex geometries such as gate valves and valve housings are also exposed to large thermal loads, and high, time-variable thermal stresses may lead to premature crack damage. Knowledge of stress ranges in critical components enables boiler startup procedures that ensure safe, long-term operation of both boiler and turbine.

Issues related to direct and inverse calculation and monitoring of thermal stresses in cylindrical components are subjects of current research [?]. This paper analyzes transient temperature and thermal stress distributions in a pipeline connecting the boiler and turbine. First, fluid and pipeline wall temperatures are determined analytically using the superposition method, assuming temperature-independent physical properties for fluid and metal and neglecting temperature differences across the wall thickness (i.e., modeling the wall as a lumped capacitance element). Next, the finite volume method (FVM) calculates transient steam and pipeline wall temperatures, from which thermal stresses are determined. Analytical and numerical results for fluid and metal temperature changes are compared, and pipeline heating with superheated steam is presented.

## Nomenclature

Symbol	Description	Units
$a$	thermal diffusivity, $a = \lambda/(c\rho)$	$\text{m}^2/\text{s}$
$A$	cross-section area	$\text{m}^2$
$c_p$	specific heat capacity at constant pressure	$\text{J}/(\text{kg} \cdot \text{K})$
$c$	specific heat capacity of material	$\text{J}/(\text{kg} \cdot \text{K})$
$d_i$	inner diameter	$\text{m}$
$I_n(x)$	modified Bessel functions of order $n$	-
$L$	length of the pipeline	$\text{m}$

Symbol	Description	Units
$m$	mass	kg
$\dot{m}$	fluid mass flow rate	kg/s
$n$	number of nodes in the radial direction	-
$N$	number of nodes in finite difference grid	-
$NTU$	number of transfer units	-
$Nu$	Nusselt number	-
$p$	absolute pressure	Pa
$Pr$	Prandtl number	-
$r$	radius	m
$r_i$	inner radius	m
$r_o$	outer radius	m
$Re$	Reynolds number	-
$s$	wall thickness	m
$T$	temperature	°C or K
$T_I, T_{wI}$	fluid and tube wall temperature for linear increase of fluid temperature at pipeline inlet	°C or K
$T_{II}, T_{wII}$	fluid and tube wall temperature for linear decrease of fluid temperature at pipeline inlet	°C or K
$T_{III}, T_{wIII}$	fluid and tube wall temperature for step change in fluid temperature at pipeline inlet	°C or K
$T_m(t)$	mean temperature of the wall	°C or K
$T_n$	nominal (design) temperature	°C or K
$t$	time	s
$t_{cn}$	time after which steam temperature reaches nominal temperature	s
$t_T$	transit time of fluid particle	s
$x, y, z$	Cartesian coordinates	m
$w$	fluid velocity	m/s

### Greek symbols

Symbol	Description	Units
$\alpha$	steam heat transfer coefficient	W/(m <sup>2</sup> · K)
$\Delta r$	radial step	m
$\Delta t$	time step size	s
$\lambda$	thermal conductivity	W/(m · K)

Symbol	Description	Units
$\delta$	relative roughness	m
$\eta$	dynamic viscosity	kg/(m · s)
$\nu$	kinematic viscosity	m <sup>2</sup> /s
$v_T$	rate of change of temperature	K/s
$\rho$	density	kg/m <sup>3</sup>
$\tau$	time constant	s
$\tau_w$	time constant for wall	s
$\tau_c$	time constant for steam	s

### Subscripts

- $i$ : inner surface
- $o$ : outer surface
- $w$ : wall surface

### Simplified Analytical Model

The simplified mathematical model for pipeline heating or cooling employs the following assumptions: - Fluid temperature in pipeline cross-sections is constant and depends only on time and axial coordinate - Temperature drop across wall thickness is negligible - Axial heat conduction in the pipeline wall is omitted - Outer pipeline surface is perfectly thermally insulated - Physical properties of fluid and wall material are temperature- and location-independent

The governing equations for steam and pipeline wall are:

#### Boundary conditions:

[Equations would appear here]

#### Initial condition:

[Equations would appear here]

The superposition method was applied to solve these equations. Formulas for transient liquid and wall temperatures for abrupt and linear fluid temperature increases at the pipeline inlet are given in [?].

The solution of the boundary-initial value problem is:

#### Steam temperature:

$$T = T(t), \quad 0 \leq T \leq T(t)$$

#### Pipeline wall temperature: where:

[Equation definitions]

The following designations from [?] were adopted in formulas (1-14):

$$z = NTU \cdot z, \quad NTU = \tau_w \cdot A \cdot c$$

[Figure 1: see original paper] shows time changes of fluid temperature at the pipeline inlet–boundary conditions for solution components I, II, and III in the superposition method.

The analytical solution given by these equations will be used for comparison with the numerical solution proposed in this paper.

## Numerical Model

A schematic of the pipeline connecting the boiler with a turbine in a 120 MW power unit is depicted in [Figure 2: see original paper].

The governing equations for steam along the flow path are:

Neglecting thermal expansion of steam ( $\beta = 0$ ), heat generation due to friction, and axial thermal conduction in steam, the energy conservation equation simplifies to:

The heat conduction equation in cylindrical coordinates is:

The system of partial differential equations (22)-(23) is subject to the following initial and boundary conditions:

In addition to conditions (24)-(30), steam pressure and mass flow rate are known at the pipeline inlet.

The initial-boundary value problem (24)-(30) was solved using the finite volume method (FVM) [?].

First, the computational domain (wall and steam regions) was divided into finite volumes ([Figure 3: see original paper]). The transient heat conduction equation for the pipeline wall is:

Taking into account small pressure changes along the pipeline length, small steam density changes can be assumed, implying  $\partial/\partial x = 0$  in Eq. (15). With constant mass flow rate  $\dot{m}$  over the tube length, steam velocity in each cross-section can be calculated from:

Considering this, the momentum conservation equation reduces to:

[Figure 3: see original paper] shows the division of the computational domain into finite volumes.

Using Eq. (21), steam pressure can be determined. For each control volume in both wall and steam regions, energy balance equations were formulated. For example, the energy balance equation for node  $i$  is set for a control volume located in the wall region ([Figure 4: see original paper]):

The energy balance equation for node  $i$  is:

Transformation of Eq. (31) gives:

Similarly, the heat balance equation for the  $i$ -th control volume in the steam region is:

The heat transfer coefficient  $\alpha$  on the inner pipeline surface was determined using Gnielinski's correlation [?]:

$$Nu = \frac{(\xi/8)(Re - 1000)Pr}{1 + 12.7\sqrt{\xi/8}(Pr^{2/3} - 1)}$$

where the friction factor  $\xi$  is given by the Colebrook correlation:

$$\frac{1}{\sqrt{\xi}} = -1.8 \log \left[ \frac{6.9}{Re} + \left( \frac{\delta}{3.7} \right)^{1.11} \right]$$

The Reynolds, Prandtl, and Nusselt numbers are defined as:

After formulating all balance equations for wall and steam, a system of ordinary differential equations for node temperatures is obtained. The number of equations for wall nodes is  $(m + 1)(n + 1)$ , while for steam nodes it is  $(m + 1)$ .

The system of ordinary differential equations was solved by the fourth-order Runge-Kutta method. To ensure stability in determining wall and steam temperatures, the Fourier stability condition for the wall and Courant-Friedrichs-Levy condition for steam must be satisfied. The smallest allowable time step  $\Delta t_{max}$  results from the Courant-Friedrichs-Levy condition:

Knowing the transient temperature distribution in the wall, thermal stresses can be determined. Considering that the axial temperature gradient  $\partial T/\partial z$  is very small, only radial temperature drop in the pipeline wall is taken into account. Assuming pipeline ends are free to expand, thermal stress components are given by [?]:

where the mean wall temperature is:

Radial stresses  $\sigma_r$  equal zero on inner and outer pipeline surfaces. Circumferential  $\sigma_\phi$  and axial stresses  $\sigma_z$  are equal on those surfaces.

## Results

This paper presents calculations for the pipeline connecting boiler OP-380 with a steam turbine. The pipeline is made of low-alloy steel 13HMF (C-0.18%, Mn-0.40%, Si-0.35%, Pmax-0.040%, Smax-0.040%, Cumax-0.25%, Cr-0.60%, Nimax-0.30%, Mo-0.65%, Almax-0.020%). Main dimensions are: outer diameter  $d_{out} = 0.324$  m, wall thickness  $s = 0.04$  m, and length  $L = 45$  m.

First, numerical results are compared with analytical solutions using constant physical properties for steam and steel (as assumed in the analytical solution). The following constant properties were used:  $\lambda_w = 44.68 \text{ W/(m} \cdot \text{K)}$ ,  $\rho_w = 7766 \text{ kg/m}^3$ ,  $c_w = 545.3 \text{ J/(kg} \cdot \text{K)}$ ,  $a = 0.0133 \text{ m}^2/\text{s}$ ,  $c_{pc} = 2672.3 \text{ J/(kg} \cdot \text{K)}$ ,  $\rho_c = 39.8 \text{ kg/m}^3$ ,  $\lambda_c = 0.0837 \text{ W/(m} \cdot \text{K)}$ ,  $\nu = 0.7755 \times 10^{-6} \text{ m}^2/\text{s}$ .

Initial fluid and pipeline wall temperature was  $T_0 = 20^\circ\text{C}$ . Initial temperature jump was  $\Delta T = 100 \text{ K}$ . Steam temperature increased at constant rate  $v_T = 10 \text{ K/min} = 1/6 \text{ K/s}$  from 0 to  $t_{cn} = 2519.9 \text{ s}$ . Steam velocity was constant:  $w_c = 56.57 \text{ m/s}$  ( $Re = 1.779894 \times 10^7$ ). Heat transfer coefficient was calculated from the Dittus-Boelter formula:

$$\alpha = 0.023 \frac{\lambda_c}{d_i} Re^{0.8} Pr^{0.4}$$

Given  $Re = 1.779894 \times 10^7$  and  $Pr = 0.985$ , Eq. (43) yields  $\alpha = 4959.2 \text{ W/(m}^2 \cdot \text{K)}$ . The number of transfer units is  $NTU = 0.6$ , with time constants  $\tau_c = 1.31 \text{ s}$  and  $\tau_w = 39.76 \text{ s}$  for fluid and pipeline wall, respectively.

[Figure 5: see original paper] compares fluid and pipeline wall temperatures from analytical formulas and the numerical method. Analysis shows that in the initial heating stage, mean wall temperatures from analytical and finite volume methods differ significantly. In later heating phases, the average wall temperature from analytical formulas becomes similar to the internal surface temperature from the numerical method. Differences between fluid temperatures from analytical and numerical methods are slight.

The numerical method must be used to calculate thermal stresses in the pipeline wall. Test calculations were performed accounting for temperature-dependent physical properties of fluid and pipeline material. Thermo-physical properties of steel are temperature-dependent.

[Figure 9: see original paper] shows pipeline wall temperature versus time at two cross-sections, displayed at five uniformly spaced nodes. Nodes 11 and 101 are located on the inner surface, while nodes 15 and 105 lie on the outer surface.

The thermal conductivity  $\lambda$  is in  $\text{W/(m} \cdot \text{K)}$ , thermal diffusivity  $a$  in  $\text{m}^2/\text{s}$ , and temperature  $T$  in  $^\circ\text{C}$ .

Transient fluid and pipeline wall temperatures were calculated for different numbers of finite volumes across the pipe wall thickness. [Figure 6: see original paper] shows that even with four finite volumes (five nodes evenly distributed), satisfactory accuracy is obtained. Results are almost identical for divisions into nine (10 nodes) or nineteen finite volumes (20 nodes) as for four finite volumes (5 nodes).

Wall temperature in the radial direction is computed at five evenly spaced nodes ( $n = 4$ ). The number of nodes in the axial direction is  $m + 1 = 21$ .

Pressure is  $p = 13.9$  MPa. Steam mass flow rate is  $\dot{m} = 100$  kg/s. Initial pipeline and steam temperature is  $T_{w0} = 20^\circ\text{C}$ . For  $t > 0$ , steam temperature at the pipeline inlet increases abruptly to constant temperature  $T_0 = 540^\circ\text{C}$ .

Selected modeling results appear in Figures 7-11. [Figure 7: see original paper] displays steam temperature versus time at nodes No. 2 ( $z = 2.25$  m), No. 10 ( $z = 20.25$  m), and No. 20 ( $z = 42.75$  m).

Analysis of [Figure 7: see original paper] shows steam temperature reaches steady state after approximately 600 s. [Figure 8: see original paper] illustrates steam temperature changes along the pipeline length at times 10 s, 60 s, 240 s, and 600 s.

At the heating process beginning, temperature differences across the wall thickness are large but rapidly decrease over time ([Figure 9: see original paper]).

[Figure 10: see original paper] presents circumferential stresses on inner and outer pipeline surfaces versus time. Analysis shows the inner surface experiences high compressive stresses while substantially lower tensile stresses occur on the outer surface. High thermal stresses result from the 520 K steam temperature jump at  $t = 0$ .

[Figure 11: see original paper] shows circumferential thermal stress distribution over wall thickness. Stresses are compressive near the inner surface and tensile near the outer surface, with maximum absolute values at heating onset that tend to zero over time.

## Summary

The mathematical model of steam pipeline heating developed in this paper determines steam and pipeline wall temperatures as functions of position and time. Comparison of transient steam and wall temperatures from analytical formulas with numerical model results indicates that analytical formulas are suitable only for steam temperature calculation. The developed model can determine transient thermal stresses caused by temperature differences across wall thickness. Examples of steam temperature, wall temperature, and circumferential thermal stress calculations on inner and outer pipeline surfaces have been presented. Calculation tests demonstrate that the developed mathematical model can simulate actual pipeline heating or cooling in power plants.

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