

Baryonium, Tetra-quark State and Glue-ball in Large Nc QCD Postprint

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Abstract

From the large-Nc QCD point of view, baryonia, tetra-quark states, hybrids, and glueballs are studied. The existence of these states is argued for. They are constructed from baryons. In $N_f = 1$, large Nc QCD, a baryonium is always identical to a glueball with Nc valence gluons. The groundstate 0^{-+} glueball has a mass about 2450 MeV. $f_0(1710)$ is identified as the lowest 0^{++} glueball. The lowest four-quark nonet should be $f_0(1370)$, $a_0(1450)$, $K^*_0(1430)$ and $f_0(1500)$. Combining with the heavy quark effective theory, spectra of heavy baryonia and heavy tetra-quark states are predicted. $1/N_c$ corrections are discussed.

Full Text

Baryonium, Tetra-quark State and Glue-ball in Large Nc QCD

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Abstract

From the perspective of large-Nc QCD, we investigate baryonia, tetra-quark states, hybrids, and glueballs, arguing for their existence and constructing them from baryons. In $N_f = 1$ large Nc QCD, a baryonium is always identical to a glueball with Nc valence gluons. The ground state 0^{-+} glueball has a mass of approximately 2450 MeV, while $f_0(1710)$ is identified as the lowest 0^{++} glueball. The lowest four-quark nonet should consist of $f_0(1370)$, $a_0(1450)$, $K^*_0(1430)$, and $f_0(1500)$. Combining with heavy quark effective theory, we predict spectra for heavy baryonia and heavy tetra-quark states, and discuss $1/N_c$ corrections.

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Introduction

The large- N_c limit represents one of the most important non-perturbative methods in QCD [1, 2, 3]. While meson properties can be understood through planar diagram analysis, baryons are described by a Hartree-Fock picture. Although mesons are non-interacting, baryon-baryon interactions scale as $O(N_c)$, suggesting a possible duality between the meson and baryon sectors [3]. This implication arises from the similarity between large- N_c baryons and Polyakov-'t Hooft monopoles when $1/N_c$ is treated as a coupling constant. Such duality was partially realized in the Skyrme model [5], where a baryon emerges as a soliton of a meson theory [4].

In this work, we describe hadron spectra starting from baryons, including not only conventional mesons and baryons but also certain multiquark systems. While color-singlet multiquark-gluon systems might be expected naively due to their color neutrality, we provide deeper reasoning for their existence and description from the large- N_c QCD viewpoint. In principle, dual theories to meson theories exist for large- N_c QCD, though these theories are strongly interacting and lack a perturbative description.

The approach of taking baryons as fundamental building blocks dates back to Fermi and Yang [6] long before QCD's establishment, with strange baryons later included by Sakita [7]; a recent version appears in Ref. [8]. As an alternative to using baryons as basic constituents, we employ Witten's Hartree-Fock picture of large- N_c baryons [3] for semi-quantitative analyses.

Baryons themselves have been studied extensively [3, 9, 10]. A baryon contains N_c valence quarks, and baryon-baryon interactions are strong. Consequently, molecular states of baryons can exist—these are precisely nuclei [3].

II. Baryonia

Let us consider baryon-antibaryon systems, which were mentioned in Ref. [3]. We study their properties under the assumption that they form bound states. The interaction between a baryon and an antibaryon can be as strong as baryon-baryon interactions, leading us to expect that molecular states of a baryon and an antibaryon also exist. Due to baryon-antibaryon annihilation, the binding in baryonia is stronger than in nuclei.

The relevant interactions fall into two categories. The first involves glueball exchanges. The antibaryon within a baryonium need not be the antiparticle of the baryon; a possible example is a baryonium composed of $\Delta^{++}(uuu)$ and $^{-}\Delta^{-}(\bar{d}\bar{d}\bar{d})$, with valence quark contents as indicated. In this case, the interaction at short distances is described by Fig. 1 [Figure 1: see original paper].

The N_c -dependence can be understood as follows: each gluon-quark vertex contributes a factor of $1/\sqrt{N_c}$. There are N_c^2 possible ways to create the first gluon, but only N_c ways for the second, since the two gluons must form a color singlet. Consequently, the interaction energy remains proportional to N_c . Generally, a baryonium mass is approximately $2N_c\Lambda_{\text{QCD}}$. Although the binding energy also scales as N_c , it is expected to be smaller than the baryon mass $N_c\Lambda_{\text{QCD}}$ because baryons are color singlets, consistent with the molecular picture of baryonia. In hadronic language, the interaction is mediated by glueballs with glueball-baryon coupling $\sqrt{N_c}$. Due to the heaviness of glueballs, such t -channel glueball exchange might be suppressed except at short distances, which is also required by confinement.

The second case involves meson exchanges. When a quark and an antiquark share a common flavor in the baryonium, the interaction shown in Fig. 2 [Figure 2: see original paper] [3] becomes relevant. This interaction is interpreted as meson exchange. If any quark can annihilate with any antiquark—that is, when all quarks have identical flavors in the baryonium—Fig. 2 contributes at the same order in N_c as Fig. 1. Realistic situations involve an interplay of both interactions depicted in Figs. 1 and 2. The baryon-antibaryon scattering amplitude described by Fig. 2 for $N_f = 1$ [3] scales as N_c , matching the N_c -dependence of baryon kinetic energies and demonstrating the strong interaction between baryon and antibaryon.

Assuming baryonia exist, interesting observations about hybrids and glueballs emerge in the large- N_c limit. For a baryonium where all quarks share the same flavor ($N_f = 1$), a hybrid state with valence content of (N_c-1) quarks, (N_c-1) antiquarks, and one gluon is large- N_c enhanced for the same reason that baryon interactions are strong, as seen by cutting Fig. 2 in half. This state and the baryonium constantly transform into each other through strong dynamics, making them physically indistinguishable in the large- N_c limit. For the same reason, this state can be equally identified as composed of (N_c-2) quarks, (N_c-2) antiquarks, and two gluons. Remarkably, a glueball composed of N_c valence gluons is also indistinguishable from the baryonium. From Fig. 2, a valence gluon has a mass equivalent to two valence quarks, approximately $2\Lambda_{\text{QCD}}$. Therefore, a glueball with two valence gluons has a mass of about $4\Lambda_{\text{QCD}}$, and a glueball with N_c valence gluons has the same mass as a baryonium. We conclude that in $N_c \rightarrow \infty$, $N_f = 1$ QCD, the baryonium, the hybrid, and the glueball represent the same state. Although this is not true at the quark-gluon level, confinement prevents distinguishing them at the hadron level.

Conversely, this reasoning implies that glueball existence supports baryonium existence. In the $N_f = 0$ case, confinement requires glueballs as hadrons. Light glueballs are massive, with masses several times Λ_{QCD} . Adding a single flavor introduces new hadrons including the meson from chiral symmetry breaking and anomaly, and the Δ^{++} baryon. For $N_c \rightarrow \infty$, glueballs and baryonia (Δ^{++} , $^-\Delta^{--}$) with identical quantum numbers become indistinguishable, leading us to generally expect baryonia. The constituent quark mass is determined to be half

the constituent gluon mass. With additional flavors, many new baryons with various flavor quantum numbers appear, though baryonium existence beyond $N_f = 1$ is less certain than in the $N_f = 1$ case.

Note that a glueball of N_c valence gluons can transition to a glueball of $(N_c - 1)$ valence gluons, but this transition rate is $O(1)$. As seen in Fig. 3 [Figure 3: see original paper], the three-gluon vertex carries a factor of $1/\sqrt{N_c}$, and there are N_c possible transition paths. However, considering the color quantum numbers of the gluons in Fig. 3, these N_c paths do not add coherently, giving the rate of Fig. 3 as $1/N_c$. The total transition rate from an N_c valence gluon state to an $(N_c - 1)$ valence gluon state is therefore $O(1)$, which is $1/N_c$ suppressed compared to the transition rate from a baryonium to a one-gluon hybrid state. Consequently, the N_c valence gluon state distinguishes itself from the $(N_c - 1)$ valence gluon state, with only $O(1)$ mixing amplitudes. In other words, the number of valence gluons within a glueball is well-defined in the large- N_c limit.

III. Diquark-Antidiquarks

Unlike Ref. [3], we distinguish baryonium states from diquark-antidiquark states. Removing a color-singlet quark pair from a baryonium always yields a color-singlet $(N_c - 1)$ -quark- $(N_c - 1)$ -antiquark state, whose existence was argued in [3]. The $(N_c - 1)$ quarks form a \bar{N}_c representation, while the $(N_c - 1)$ antiquarks form an N_c representation. Such a state has a mass of approximately $2(N_c - 1)\Lambda_{\text{QCD}}$, representing the large- N_c extension of the tetra-quark state. Taking $N_c = 3$ gives the diquark-antidiquark configuration. In a $(N_c - 1)$ -quark- $(N_c - 1)$ -antiquark system where all quark flavors are identical, the hybrid state of $(N_c - 2)$ quarks, $(N_c - 2)$ antiquarks, and one gluon is physically indistinguishable from the $(N_c - 1)$ -quark- $(N_c - 1)$ -antiquark state.

Let us examine the $(N_c - 1)$ -quark- $(N_c - 1)$ -antiquark system in greater detail. For $N_f = 1$, processes similar to those in Fig. 2 occur. In most cases where the gluon is formed from quarks of different colors, the interaction maintains the $(N_c - 1)$ quarks in the \bar{N}_c representation and $(N_c - 1)$ antiquarks in the N_c representation. This process amplitude scales as N_c . When the gluon forms from quarks of the same color, the final $(N_c - 1)$ quarks generally do not remain in the \bar{N}_c representation, but this transition amplitude is $O(1)$. Similar results obtain for t-channel gluon exchanges. Therefore, the quark configuration of a state with $(N_c - 1)$ quarks in the \bar{N}_c representation and $(N_c - 1)$ antiquarks in the N_c representation remains meaningful in the large- N_c limit. In the realistic case ($N_c = 3$), this means the $3\bar{3}$ tetraquark configuration does not mix with the $6\bar{6}$ tetraquark configuration when $N_c = 3$ is considered large.

Further removing a color-singlet quark pair yields a $(N_c - 2)$ -quark- $(N_c - 2)$ -antiquark state with mass approximately $2(N_c - 2)\Lambda_{\text{QCD}}$. The $(N_c - 2)$ quarks form a $\bar{N}_c \bar{N}_c$ representation, while the $(N_c - 2)$ antiquarks form an $N_c N_c$ representation, making the situation more complicated.

This procedure can continue iteratively, eventually forming a valence quark-

antiquark state with mass approximately $2\Lambda_{\text{QCD}}$ —precisely a meson. (Note that chiral symmetry breaking cannot be accounted for in this framework.) As discussed for baryonium and glueballs, the single-flavor (N_c-1) -quark and (N_c-1) -antiquark system is indistinguishable from a glueball composed of N_c-1 valence gluons, and the two-quark-two-antiquark system is indistinguishable from a glueball composed of two valence gluons.

With additional flavors, the large- N_c considerations become more complex. For example, the (N_c-1) quarks occupy the lowest energy state when they carry (N_c-1) different flavors. In this case, the (N_c-1) -quark- (N_c-1) -antiquark state transition to the hybrid state of (N_c-2) quarks, (N_c-2) antiquarks, and one gluon is $1/N_c$ suppressed compared to the single-flavor case. These states cannot be identical in the large- N_c limit, though they have $O(1)$ mixing amplitudes.

IV. Decays and Binding Energies

Decays of baryoniums and (N_c-1) quark- (N_c-1) antiquark states were discussed in Ref. [3]. A baryonium decays into one meson and a (N_c-1) quark- (N_c-1) antiquark state, which in turn decays into one meson and a (N_c-2) quark- (N_c-2) antiquark state. Such cascade decays continue until the final 4-quark state decays into two mesons. These decays are slow.

The decay rates are $O(1)$. This follows from the fact that a color-singlet quark pair drops out of a baryonium or a (N_c-1) quark- (N_c-1) antiquark state with amplitude $O(1)$. Theoretically, baryon-antibaryon systems might be difficult to treat because their typical interaction energy scales as N_c [3], the same N_c -dependence as baryon masses $N_c\Lambda_{\text{QCD}}$. However, as argued in Section II, the baryon-antibaryon interaction energy is expected to be smaller than $N_c\Lambda_{\text{QCD}}$ due to confinement.

Furthermore, large- N_c QCD suggests that baryon-baryon interaction energies also scale as N_c , yet phenomenologically nuclear physics shows that the typical binding energy of a baryon inside a nucleus is only a few MeV. Therefore, we expect baryonium binding energies to be substantially smaller than baryon masses, making the molecular description (in the QCD sense) of baryonium states meaningful. Such systems can be well described by non-relativistic quantum mechanics, and consequently baryon spins decouple from baryonium dynamics.

To state this more clearly: in the large- N_c limit, baryon-baryon binding energy is N_c , and baryon-antibaryon binding energy is N_c . The confinement argument gives $< \Lambda_{\text{QCD}}$ and $< \Lambda_{\text{QCD}}$.

The slow transition from a baryonium to a (N_c-1) quark- (N_c-1) antiquark state plus one meson implies that the interaction strengths inside baryoniums and (N_c-1) quark- (N_c-1) antiquark states are identical. In other words, in the large- N_c limit the constituent quark mass (Λ_{QCD}) can be taken as the same in both hadron types.

V. Analysis

We provide a numerical illustration by analyzing realistic situations. In this semi-quantitative analysis, we consider only ground-state hadrons. To be precise, we must specify the meaning of Λ QCD used here. This quantity describes the energy of an individual quark inside baryons. We define $\bar{\Lambda}$ QCD to replace Λ QCD via Mground state baryon $Nc\bar{\Lambda}$ QCD, identifying $\bar{\Lambda}$ QCD as the constituent quark mass in baryons. Taking $Nc = 3$, we obtain $\bar{\Lambda}$ QCD = (362 \pm 50) MeV by averaging the nucleon and Δ^{++} masses.

The analysis crucially depends on the baryonium binding energy magnitude. Different binding energies correspond to different physical pictures of hadrons, determining which particles in the Particle Data Book are identified as 4-quark states. We primarily adopt a 10 MeV binding energy, though estimating its error is difficult. Some phenomenological works [12] use about 300 MeV binding energy, which we briefly consider for comparison.

The numerical analysis is most appropriate for the $N_f = 1$ case, though we extend it to $N_f = 2$ and 3 without further noting that these cases are more assumption-dependent.

A. 10 MeV Binding Energy

Taking the binding energy as 10 MeV means $\bar{\Lambda}$ or slightly larger, since both $\bar{\Lambda}$ and Λ are essentially determined by Λ QCD and confinement. Nuclear physics tells us Nc is a few MeV, so we expect $Nc \bar{\Lambda} \approx 10$ MeV typically. A recent Skyrme model study indeed finds baryonium binding energy around 10 MeV [11].

For a single flavor, the lowest baryonium is an s-wave ($\Delta^{++}\bar{\Delta}^{--}$) state. Taking the up-quark as an example, its mass is approximately $2M_{\Delta^{++}} - 10$ MeV = 2450 MeV. As argued, this state can also be identified as the 0^{-+} ground state of a three-gluon glueball in the large- N_c limit. The inferred constituent gluon mass is consistent with other methods [13]. The actual glueball mass may be somewhat lower because the real flavor number exceeds one.

With an additional light flavor, the lowest proton-antiproton molecular state mass is estimated as $2MN - 10$ MeV = 1866 MeV. This molecular state mixes with the 0^{-+} glueball. In the large- N_c limit, this mixing depends on the large- N_c generalization of the nucleon, which we do not consider here.

Experimentally, the BES collaboration has found two baryonium candidates, X(1860) [14] and X(1835) [15], which were subsequently studied theoretically [11, 16]. Considering binding energy uncertainties, these states are consistent with our expectations. Furthermore, corresponding 0^{-+} states exist due to different baryon spin combinations, with approximate degeneracy resulting from baryon spin decoupling. As a check, $p\bar{n}$, $n\bar{p}$, and $n\bar{n}$ states should be degenerate with $p\bar{p}$ at about 1835 MeV or 1860 MeV. In the three-light-flavor case, the lowest baryonium ($p, \bar{\Lambda}$) or ($n, \bar{\Lambda}$) is expected to have mass $MN + M\Lambda - 10$ MeV = 2220 MeV, consistent with experiment.

An (N_c-1) quark- (N_c-1) antiquark state has mass approximately $2(N_c-1)^{-1}\Lambda_{\text{QCD}}$. Based on the previous section's argument, we reasonably take the constituent quark mass in tetra-quark states to equal that in baryoniums. For a single flavor, the 4-quark state is $(uu^{-}u^{-}u)$ with lowest mass estimated as $(2M_{\Delta}++ - 10 \text{ MeV}) - 2^{-1}\Lambda_{\text{QCD}} \approx 1710 \text{ MeV}$. This state can also be regarded as a $(u^{-}ug)$ hybrid or a 0^{++} ground-state two-gluon glueball, which we identify as $f_0(1710)$. Our estimate is consistent with lattice calculations [18].

With an additional flavor, the lowest mass can be written as $[MX(1835) \text{ or } MX(1860)] - 2^{-1}\Lambda_{\text{QCD}} \approx (1110-1140) \pm 100 \text{ MeV}$. This represents the 4-quark ground state $(ud^{-}u^{-}d)$ with both spin and isospin 0, identified as $f_0(1370)$ which has a mass ranging from 1200 to 1500 MeV [19]. As noted, it mixes with the hybrid state $(q^{-}qg)$ where q denotes u - or d -quarks.

Our 4-quark state mass estimation carries an uncertainty of $\pm 200 \text{ MeV}$, limiting its practical utility. Our main point is that within this uncertainty, 4-quark states should exist and indeed have experimental counterparts. After introducing the strange quark, three types of lowest diquarks can form: ud , us , and ds . The 4-quark states form a nonet, studied by many authors [20, 21, 22, 23, 24]. In our work, their mass differences are determined by the strange quark mass, not subject to large- N_c approximation uncertainties. The ground 4-quark states are naturally identified as $f_0(1370)$, $a_0(1450)$, $K^*_0(1430)$, and $f_0(1500)$. More explicitly: $f_0(1370)(ud^{-}u^{-}d)$, $K^*_0(1430)(ud^{-}u^{-}s, ud^{-}d^{-}s, us^{-}u^{-}d, ds^{-}u^{-}d)$, $a_0(1450)(us^{-}d^{-}s, ds^{-}u^{-}s)$, and $f_0(1500)(s(n^{-}n)^{+}s)/\sqrt{2}$ where $(n^{-}n)^{\pm} = (u^{-}u \pm d^{-}d)/\sqrt{2}$. Mixing among the f_0 states is an $O(1)$ effect. Note that our 4-quark state identification differs from most previous studies [20, 21, 22, 23] which take ρ , ω , $a_0(980)$, and $f_0(980)$ as the lowest 0^{++} 4-quark states.

B. Large Binding Energy

For comparison, consider large baryon-antibaryon interaction energies. While Λ_{QCD} is known, Λ_{baryon} remains an assumption. If annihilation effects are significant, Λ_{baryon} could be much larger; Refs. [12] take it as about 200 MeV, which still permits a molecular picture of baryoniums. We fix Λ_{baryon} by requiring 4-quark ground states to correspond to ρ , ω , $a_0(980)$, and $f_0(980)$. For large binding energy, the diquark-antidiquark mass is written as $2(N_c-1)^{-1}\Lambda_{\text{QCD}}$, while the baryonium mass is $2N_c^{-1}\Lambda_{\text{QCD}}$. It is reasonable that $\Lambda_{\text{baryon}} \approx \Lambda_{\text{QCD}} - \Lambda_{\text{QCD}} - \Lambda_{\text{QCD}}$ for ground states, as we can imagine an $O(1)$ process generating a ground-state diquark-antidiquark from a ground-state baryonium by emitting a meson of mass $2^{-1}\Lambda_{\text{QCD}}$. The interaction energy between a diquark-antidiquark and a meson is $1/N_c$ suppressed relative to N_c^{-1} and is neglected, as done previously. Taking the strange quark mass $m_s = 150 \text{ MeV}$, from $M_{f_0(980)} - \Lambda_{\text{QCD}} + 2m_s$ we obtain $\Lambda_{\text{QCD}} \approx 680 \text{ MeV}$. The lowest 0^{-+} baryoniums should then be around 1020 MeV, implying a 960 MeV binding energy. However, no such baryonium appears in the Particle Data Book.

Therefore, considering the practical situation, large- N_c QCD analyses favor a

small baryonium binding energy, and we will not consider the large binding energy case further.

C. Decay Widths

A baryonium decays into one meson and one tetra-quark state, while a tetra-quark state decays into two mesons [3]. The decay rates are $O(1)$. Thus, from the large- N_c perspective, diquark-antidiquark states decay slowly, distinguishing them from two-meson states. The 0^-+ baryonium $X(1835)$ or $X(1860)$ decays in p-wave into $f_0(1370)$ and ρ . Note that it cannot decay to s-wave $f_0(1370)$ and ρ due to phase space constraints. Therefore, the dominant decay products are ρ and $f_0(1370)$, which can be checked by future experiments.

VI. Heavy Hadrons

Now we consider including a single heavy quark. Heavy quark effective theory (HQET) [25] provides a systematic framework for investigating hadrons containing a single heavy quark. It is an effective field theory of QCD for such heavy hadrons, where heavy quark spin-flavor symmetry becomes explicit in the limit $m_Q/\Lambda_{\text{QCD}} \rightarrow \infty$. The hadron mass is $M_H = m_Q + \bar{\Lambda}_H + O(1/m_Q)$.

To obtain the universal heavy hadron mass $\bar{\Lambda}_H$ defined in HQET, non-perturbative QCD methods are required. We can apply the large- N_c method, which has been used to study heavy baryons [10, 26, 27, 28, 29]. First, consider the relation between $\bar{\Lambda}_H$ for a ground-state heavy baryon and the nucleon mass. Heavy baryons contain (N_c-1) light quarks and one “massless” heavy quark (modulo m_Q). The baryon mass or energy is determined by summing individual quark energies. The heavy quark kinetic energy is typically $\bar{\Lambda}_{\text{QCD}}$, like that of light quarks. The interaction energy between the heavy quark and each light quark is typically $\bar{\Lambda}_{\text{QCD}}/N_c$, so the total interaction energy between the heavy quark and the light quark system scales as $\bar{\Lambda}_{\text{QCD}}$. However, the total interaction energy among the light quark system itself scales as $N_c \bar{\Lambda}_{\text{QCD}}$. Therefore, in the large- N_c limit, $\bar{\Lambda}_H = MN + O(\bar{\Lambda}_{\text{QCD}})$ in $1/N_c$ expansion.

In practice, the uncertainty in the mass relation between heavy baryons and corresponding light baryons under the large- N_c limit is smaller than $\bar{\Lambda}_{\text{QCD}}$, because the heavy quark constituent mass (modulo m_Q) does not deviate much from $\bar{\Lambda}_{\text{QCD}}$. For example, the Λ_Q baryon mass $\bar{\Lambda}_{\Lambda_Q}$ is about 0.80 GeV [30]. It is more reasonable to take ± 0.15 GeV as the uncertainty in the following analysis.

Heavy baryoniums containing a heavy quark are analyzed in the same large- N_c spirit as the light quark case. The 0^- ground-state baryoniums (Λ_c, \bar{N}) and $(\Lambda_c, \bar{\Lambda})$ have masses:

$$\begin{aligned} M(\Lambda_c, \bar{N}) &= M_{\Lambda_c} + M_N - 10 \text{ MeV} \quad 3.21 \text{ GeV} \\ M(\Lambda_c, \bar{\Lambda}) &= M_{\Lambda_c} + M_{\Lambda} - 10 \text{ MeV} \quad 3.50 \text{ GeV} \end{aligned}$$

where m_c is taken as 1.43 GeV [30]. This is consistent with naive estimates. These states also have corresponding degenerate 1^- states due to baryon spin decoupling.

For heavy diquarks, heavy quark spin symmetry implies that a spin-zero diquark Q_q exists alongside a spin-one diquark [23]. The lowest 4-quark charm state spectrum is:

$$\begin{aligned} M(\bar{c}d, \bar{u}^-d) &= M(\bar{c}u, \bar{u}^-d) = m_c + M_{f0}(1370) = 2.54 \pm 0.15 \text{ GeV} \\ M(\bar{c}d, \bar{u}^-s) &= M(\bar{c}u, \bar{u}^-s) = M(\bar{c}d, \bar{d}^-s) = M(\bar{c}u, \bar{d}^-s) = M(\bar{c}s, \bar{u}^-d) = m_c + \\ &M_{a0}(1450) = 2.57 \pm 0.15 \text{ GeV} \\ M(\bar{c}s, \bar{u}^-s) &= M(\bar{c}s, \bar{d}^-s) = m_c + M_{f0}(1500) = 2.60 \pm 0.15 \text{ GeV} \end{aligned}$$

In the heavy quark limit, we expect degeneracy among 0^+ , 1^+ , and 2^+ 4-quark states. Therefore, we anticipate a rich charm hadron spectrum ranging from 2.54 GeV to 3.50 GeV. The $1/m_Q$ uncertainty is about $\Lambda_{\text{QCD}}^2/m_c \approx 60$ MeV. For smaller $1/m_Q$ corrections, the uncertainty is about $\Lambda_{\text{QCD}}^2/2m_c \approx 30$ MeV due to heavy quark flavor symmetry.

The bottom case is similar:

$$\begin{aligned} M(\bar{b}d, \bar{u}^-d) &= M(\bar{b}u, \bar{u}^-d) = m_b + M_{f0}(1370) = 6.80 \pm 0.15 \text{ GeV} \\ M(\bar{b}d, \bar{u}^-s) &= M(\bar{b}u, \bar{u}^-s) = M(\bar{b}d, \bar{d}^-s) = M(\bar{b}u, \bar{d}^-s) = M(\bar{b}s, \bar{u}^-d) = m_b + \\ &M_{a0}(1450) = 6.83 \pm 0.15 \text{ GeV} \\ M(\bar{b}s, \bar{u}^-s) &= M(\bar{b}s, \bar{d}^-s) = m_b + M_{f0}(1500) = 6.86 \pm 0.15 \text{ GeV} \end{aligned}$$

where $m_b = 4.83$ GeV [30]. $M(\bar{b}d, \bar{u}^-d)$ and $M(\bar{b}d, \bar{u}^-s)$ are consistent with $M_{\Lambda b} + M_n - 10$ MeV and $M_{\Lambda b} + M_{\Lambda} - 10$ MeV, respectively.

For hadrons containing a heavy quark pair, the two heavy quarks tend to form a tighter object described by non-relativistic QCD. However, if the heavy quarks are separated by $1/\Lambda_{\text{QCD}}$ or more in certain hadrons, the above HQET procedure applies. For example, the following lowest baryoniums and 4-quark states may exist:

$$\begin{aligned} M(\bar{c}c, \bar{u}^-c) &= 2M_{\Lambda c} - 10 \text{ MeV} = 4.83 \text{ GeV} \\ M(\bar{c}u, \bar{c}^-u) &= M(\bar{c}u, \bar{c}^-d) = M(\bar{c}d, \bar{c}^-u) = M(\bar{c}d, \bar{c}^-d) = 2m_c + M_{f0}(1370) \\ &= 3.97 \pm 0.15 \text{ GeV} \\ M(\bar{c}u, \bar{c}^-s) &= M(\bar{c}d, \bar{c}^-s) = M(\bar{c}s, \bar{c}^-u) = M(\bar{c}s, \bar{c}^-d) = 2m_c + M_{a0}(1450) = 4.00 \\ &\pm 0.15 \text{ GeV} \\ M(\bar{c}s, \bar{c}^-s) &= 2m_c + M_{f0}(1500) = 4.03 \pm 0.15 \text{ GeV} \end{aligned}$$

The state $(\bar{c}s, \bar{c}^-s)$ is consistent with $Y(4260)$ [22], which cannot be identified as $(\bar{\Lambda}c, \bar{\Lambda}c)$ [31] in our scheme. Considering $1/N_c$ uncertainties, $X(3940)$ could be $(\bar{c}q\bar{c}^-q)$ ($q = u, d$). In that case, a charged $(\bar{c}u\bar{c}^-d)$ state is also expected around 3940 MeV.

VII. Summary and Discussion

From the large- N_c QCD perspective, we have considered baryoniums, four-quark states, hybrids, and glueballs. We argued for baryonium existence based on nuclear existence and constructed these hadrons from baryons. In $N_f = 1$ large- N_c QCD, a baryonium is always identical to a glueball with N_c valence gluons. We identified $f_0(1370)$, $a_0(1450)$, $K^*_0(1430)$, and $f_0(1500)$ as the lowest four-quark nonet. The three-valence-gluon glueball has mass about 2450 MeV, while $f_0(1710)$ is identified as the two-valence-gluon glueball. Combining with HQET, we predicted spectra for heavy baryoniums and heavy four-quark states.

This work extends the Fermi-Yang model to large- N_c QCD, providing a hadron classification based on large- N_c QCD. We constructed hadron spectra from baryoniums because our starting point is baryons; the reverse procedure is not necessarily true. For example, the one-gluon hybrid state in this scheme can always be generated from or identified as a diquark-antidiquark state. The large- N_c QCD arguments and consequent estimates help us understand relevant experimental results, though they do not yield a precise mathematical description for the studied hadrons. Finding such a systematic description remains a task in solving non-perturbative QCD.

We should discuss the analysis uncertainties. Of course, baryonium existence and the 10 MeV binding energy remain assumptions, though supported by large- N_c analysis. Even for $N_c = 3$, baryoniums are still expected, as large- N_c QCD's qualitative conclusions often hold when $N_c = 3$. The strong baryon-baryon interaction in large- N_c QCD implies baryon bound states; indeed, nuclei exist in real QCD with $N_c = 3$. Meson-meson interactions vanish in the large- N_c limit, so no meson molecular states exist, and in the real world meson molecular states appear non-existent.

It is important to discuss $1/N_c$ corrections to our numerical analysis. The estimated masses of baryoniums and diquark-antidiquark states would have $O(\Lambda_{\text{QCD}})$ uncertainties, but the relative masses of these hadrons have smaller uncertainties. For example, once the $p\bar{p}$ binding energy is fixed at 10 MeV, the $\Lambda\bar{\Lambda}$ binding energy is 10 MeV with about 30% uncertainty from $SU(3)$ violation. Thus, the mass difference between $p\bar{p}$ and $\Lambda\bar{\Lambda}$ baryoniums does not suffer large $1/N_c$ corrections. More accurate baryonium treatment can follow Refs. [9, 10] by incorporating baryonium binding energies. Similarly, diquark-antidiquark state mass differences have no large $1/N_c$ uncertainties. For instance, mass differences among $(cd, \bar{u}\bar{d})$, $(cd, \bar{u}\bar{s})$, and $(cs, \bar{u}\bar{s})$ states are not subject to Λ_{QCD} uncertainty. Discovery of baryoniums and diquark-antidiquark states with a single heavy quark, along with our mass predictions in Eqs. (3-5), will test our understanding in the near future.

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