Supersymmetry and Vector-like Extra Generation (Postprint)

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Full Text

Preamble

Supersymmetry and Vector-like Extra Generation

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Abstract

Within the framework of supersymmetry, the particle content is extended in a way that each Higgs doublet is in a full generation. Namely, in addition to ordinary three generations, there is an extra vector-like generation, and it is the extra slepton SU(2)L doublets that are taken to be the two Higgs doublets. R-parity violating interactions contain ordinary Yukawa interactions. Breaking of supersymmetry and gauge symmetry are analyzed. Fermion and boson spectra are calculated. Phenomenological constraints and relevant new physics at Large Hadron Collider are discussed.

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Introduction

The mainstream spirit of physics beyond the Standard Model (SM) is as follows. SM gauge interactions unify at a high scale (10^{16} GeV) [?]. Supersymmetry (SUSY) [?, ?, ?, ?, ?, ?, ?, ?] is then a must to stabilize the SM Higgs mass. The minimal SUSY extension of SM (MSSM), which necessarily involves two Higgs doublets, reinforces the idea of grand unification theory (GUT) because of LEP data [?, ?, ?, ?]. Further assuming R-parity conservation, dark matter (DM) is provided [?, ?, ?].

However, things can be different. Before verification of SUSY GUT, other ideas should be pursued whenever they have reasonable points. In this Large Hadron Collider (LHC) era, discoverable ideas besides MSSM are of particular interest.

In this paper, we still use weak-scale SUSY to extend the SM. Our consideration is as follows. The SUSY SM needs two Higgs doublets in order to provide masses to both up- and down-type quarks. Fermions in each doublet cause anomaly. In MSSM, anomalies of the two doublets would cancel each other. The two Higgs doublets would be vector-like matter in MSSM. They are very different from the three generation chiral matter in which the anomaly of each generation automatically vanishes. To treat the Higgs and the matter on equal footing, we introduce additional fields associated with each Higgs doublet, making them like one generation of leptons and quarks, so the anomaly due to each SU(2) Higgsino cancels those of the newly introduced fields. In such an extension, for example, the down-type Higgs is in the same position as a lepton doublet in a full matter generation. Immediately we find that, except for the lepton and baryon numbers, the down-type Higgs and its associated fields can be identified as another full matter generation, that is the fourth generation.

The fourth chiral generation may have been generally disfavored because of Z^0 decays which show only three generations of light neutrinos. There were many discussions about the fourth generation [?, ?, ?, ?, ?, ?]. Ref. [?] introduced the fourth generation and identified its slepton as the Higgs. The partial role of sleptons in electroweak symmetry breaking (EWSB) was discussed previously [?]. Different from Ref. [?], we take the weak scale as a low energy one, therefore we still have two Higgs doublets. As we will see, when introducing SUSY, the fourth generation neutrino is automatically heavy because of the existence of the generation associated with the up-type Higgs.

The extra generations in this model are vector-like. There are quite a few studies of vector-like fermionic generations [?, ?, ?]. Vector-like fermions may have interesting physical implications [?, ?, ?, ?, ?]. Within SUSY, Ref. [?] studied one mirror generation case. We introduce a vector-like generation pair which contains the two Higgs doublets required for EWSB.

This paper is organized as follows. In Sect. II, the model is constructed. SUSY breaking, EWSB and particle spectra are presented. In Sect. III, phenomenological constraints are discussed. LHC phenomenology is given in Sect. IV. We summarize the model and further discuss its other aspects in the final section.

II. Model

In this paper, we consider a SUSY SM with a vector-like generation. The particle contents are given below. In addition to the fourth generation superfields, L_4, E_4^c, Q_4, U_4^c, D_4^c, the up-type Higgs H_u and its associated matter (E_H^c, Q_H, U_H^c, D_H^c), which compose an anomaly-free chiral generation, are introduced. Their quantum numbers under $SU(2)_L \times SU(3)_c \times U(1)_Y$ and the global baryon number are:

$$L_m(2,1,1,0), \quad E_m^c(1,2,1,0), \quad Q_m(2,3,\frac{1}{3},\frac{1}{3}), \quad U_m^c(1,\bar{3},-\frac{4}{3},-\frac{1}{3}), \quad D_m^c(1,\bar{3},\frac{2}{3},-\frac{1}{3}),$$

$$H_u(2,1,1,0), \quad E_H^c(1,2,1,0), \quad Q_H(2,3,\frac{1}{3},\frac{1}{3}), \quad U_H^c(1,\bar{3},-\frac{4}{3},-\frac{1}{3}), \quad D_H^c(1,\bar{3},\frac{2}{3},-\frac{1}{3}),$$

where $m = 1,2,3,4$. In fact, the up-type Higgs family is in the anti-particle representation compared to particles in the other four ordinary generations. It will be massive after combining with one of the ordinary families.

The superpotential is written as follows. Instead of the R-parity, baryon number conservation is assumed,

$$\mathcal{W} = \mu_m L_m H_u + \mu_e E_m^c E_H^c + \mu_Q Q_m Q_H + \mu_U U_m^c U_H^c + \mu_D D_m^c D_H^c + \lambda_{lmn} L_l L_m E_n^c + \lambda'_{lmn} Q_l L_m D_n^c + y_{mn} Q_m H_u U_n^c + y$$

where μ_m' s are mass parameters, $\lambda^{(\prime)}$, $y^{(\prime)}$ and $\tilde{y}^{(\prime)}$' s are coefficients. Note, $l,m,n = 1,2,3,4$.

By redefining the down-type Higgs and the other fourth generation fields,

$$L_4 \equiv \mu_e E_m^c, \quad E_4^c \equiv \mu_Q Q_m, \quad Q_4 \equiv \mu_U U_m^c, \quad U_4^c \equiv \mu_D D_m^c, \quad D_4^c \equiv \mu_D D_m^c,$$

where $H_d \equiv \mu_m L_m$, the superpotential is

$$\mathcal{W} = \mu H_d H_u + \mu_e E_4^c E_H^c + \mu_Q Q_4 Q_H + \mu_U U_4^c U_H^c + \mu_D D_4^c D_H^c + \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} Q_i L_j D_k^c + y_{ij} Q_i H_u U_j^c + \tilde{\lambda}_{ij} E_i^c L_j H_u +$$

where field decomposition have been generally written as follows with i being $1,2,3$,

$$L_m = c_{mi} L_i + c_{m4} H_d, \quad E_m^c = c_{mi}^E E_i^c + c_{m4}^E E_4^c, \quad Q_m = c_{mi}^Q Q_i + c_{m4}^Q Q_4,$$

and the coefficients are

$$\lambda_{ijk} = \lambda_{lmn} c_{li}^L c_{mj}^L c_{nk}^E, \quad \lambda'_{ijk} = \lambda'_{lmn} c_{li}^Q c_{mj}^L c_{nk}^D, \quad y_{ij} = y_{mn} c_{mi}^Q c_{nj}^U,$$

$$\tilde{\lambda}_{ij} = \tilde{y}_{mn} c^E_{mi} c^L_{nj}, \quad \lambda'_{i4} = \lambda'_{lmn} c^Q_{li} c^L_{m4} c^D_{n4}, \quad y_{i4} = y_{mn} c^Q_{mi} c^U_{n4},$$
$$\tilde{\lambda}_{i4} = \tilde{y}_{mn} c^E_{mi} c^L_{n4}, \quad \lambda'_{4j} = \lambda'_{lmn} c^Q_{l4} c^L_{mj} c^D_{n4}, \quad y_{4j} = y_{mn} c^Q_{m4} c^U_{nj},$$
$$\tilde{\lambda}_{4j} = \tilde{y}_{mn} c^E_{m4} c^L_{nj}, \quad \lambda'_{44} = \lambda'_{lmn} c^Q_{l4} c^L_{m4} c^D_{n4}, \quad y_{44} = y_{mn} c^Q_{m4} c^U_{n4}, \quad \tilde{\lambda}_{44} = \tilde{y}_{mn} c^E_{m4} c^L_{n4}.$$

From the superpotential (4), we see that because of Dirac mass terms of up-type Higgs and the four generations, one of the four generations, namely the fourth generation (H_d, E^c_4, Q_4, U^c_4, D^c_4), is always heavy which is identified as the one containing the down-type Higgs. The fourth generation neutrino together with the "neutrino" in H_u consists of Higgsinos.

After the mass terms, the next five terms in Eq. (4) are ordinary Yukawa interactions and trilinear lepton number (R-parity) violating terms, where i, j, k stand for three light generations. The other terms in (4) are new which involve extra generations. Many of these new terms violate lepton numbers. Note that by taking $\mu_{e,Q,U,D} \sim \mathcal{O}(\text{TeV})$, we obtain MSSM as a low energy effective theory.

A. SUSY breaking

Soft SUSY breaking mass terms should be included into the Lagrangian. In addition to gaugino masses, they include mass-squared terms of scalars and $B\mu$-type terms corresponding to those μ-terms in superpotential (1),

$$\mathcal{L}_{\text{soft}} = M^2_{\tilde{L}} \tilde{L}^\dagger_m \tilde{L}_m + M^2_{\tilde{E}} \tilde{E}^{c*}_m \tilde{E}^c_m + M^2_{\tilde{Q}} \tilde{Q}^\dagger_m \tilde{Q}_m + M^2_{\tilde{U}} \tilde{U}^{c*}_m \tilde{U}^c_m + M^2_{\tilde{D}} \tilde{D}^{c*}_m \tilde{D}^c_m$$

$$+(B_{\mu m} \tilde{L}_m h_u + B_e \mu_e \tilde{E}^c_m \tilde{E}^c_H + B_Q \mu_Q \tilde{Q}_m \tilde{Q}_H + B_U \mu_U \tilde{U}^c_m \tilde{U}^c_H + B_D \mu_D \tilde{D}^c_m \tilde{D}^c_H + \text{h.c.}),$$

where tildes stand for scalars. We have assumed universality of the mass-squared terms and the alignment of the B terms, namely the mass parameters $B_{e,Q,U,D}$ do not depend on the subscript m. In terms of three light generations of Eq. (4), universality of these soft mass terms is easily seen,

$$\mathcal{L}_{\text{soft}} = M^2_{\tilde{L}} \tilde{L}^\dagger_i \tilde{L}_i + M^2_h h^\dagger_d h_d + M^2_{\tilde{E}} \tilde{E}^{c*}_i \tilde{E}^c_i + M^2_{\tilde{Q}} \tilde{Q}^\dagger_i \tilde{Q}_i + M^2_{\tilde{U}} \tilde{U}^{c*}_i \tilde{U}^c_i + M^2_{\tilde{D}} \tilde{D}^{c*}_i \tilde{D}^c_i$$

$$+(B_\mu h_d h_u + B_e \mu_e \tilde{E}^c_4 \tilde{E}^c_H + B_Q \mu_Q \tilde{Q}_4 \tilde{Q}_H + B_U \mu_U \tilde{U}^c_4 \tilde{U}^c_H + B_D \mu_D \tilde{D}^c_4 \tilde{D}^c_H + \text{h.c.}).$$

Numerically soft masses M' s, B' s and gaugino masses are assumed to be $\mathcal{O}(100)$ GeV.

Soft trilinear terms corresponding to Eq. (1) are

$$\mathcal{L}_{\text{tri}} = \bar{\lambda}_{lmn} \tilde{L}_l \tilde{L}_m \tilde{E}^c_n + \bar{\lambda}'_{lmn} \tilde{Q}_l \tilde{L}_m \tilde{D}^c_n + \bar{y}_{mn} \tilde{Q}_m h_u \tilde{U}^c_n + \bar{y}'_{mn} \tilde{Q}_H \tilde{L}_m \tilde{U}^c_n + \bar{\tilde{y}}_m \tilde{E}^c_m \tilde{E}^c_H + \bar{\tilde{y}}_{mn} \tilde{E}^c_m \tilde{L}_n h_u + \text{h.c.},$$

where the following coupling alignment will be assumed,

$$\bar{\lambda}^{(\prime)}_{lmn} = \lambda^{(\prime)}_{lmn} m_0, \quad \bar{y}_{mn} = y_{mn} m_0, \quad \bar{y}'_m = y'_m m_0,$$

with m_0 being of the order of soft masses $\mathcal{O}(100)$ GeV.

B. EWSB

Let us look at gauge symmetry breaking. From the Lagrangian, the scalar potential can be written down straightforwardly. To get EWSB, one needs a negative determinant of the Higgs mass-squared matrix, namely

$$\det \begin{pmatrix} M_h^2 + \mu^2 & B_\mu \\ B_\mu & M_h^2 + \mu^2 \end{pmatrix} < 0,$$

with the ordinary condition $M_h^2 + M_h^2 + 2\mu^2 + 2B_\mu > 0$. This requirement can be realized when the renormalization group is considered. M_h^2 will become negative at the weak scale, due to the large top quark Yukawa coupling. Therefore, everything of EWSB here will be the same as that in MSSM. The MSSM analysis of EWSB applies here. EWSB in this model occurs at the weak scale.

In addition to B_μ, other $B\mu$-terms in Eqs. (7) and (8) might complicate the gauge symmetry breaking analysis. The experience from MSSM shows that if a $B\mu$-term is large enough, gauge symmetry breaking always occurs. The new $B_{Q,U,D}\mu_{Q,U,D}$-terms could result in color symmetry breaking and the $B_e\mu_e$-term could cause purely $U(1)_Y$ symmetry breaking. Such unwanted gauge symmetry breaking should be avoided. To be concrete, besides Eq. (11), correct EWSB also requires

$$M_X^2 + \mu_X^2 > 0, \quad |B_X\mu_X|^2 > (M_X^2 + \mu_X^2)^2,$$

for $X = e, Q, U, D$.

Then the remaining analysis of EWSB is identical to that of MSSM with same Higgs and Higgsino spectra. Eq. (12) can be satisfied easily. Careful thinking of EWSB conditions Eqs. (11) and (12), we see that if $\mu < \mu_X$, EWSB occurs naturally. This point will be discussed later. The fact that pure $U(1)_Y$ breaking does not occur can be simply due to a large enough μ_e compared to μ.

In the scalar potential, quartic terms are determined by SUSY gauge interactions. From the point of view of quartic terms, the larger the coefficients of certain quartic terms, the more difficult the gauge symmetry breaking is. Therefore EWSB is easier to be obtained compared to that of the color symmetry. Traditional $SU(2)_L \times U(1)_Y$ breaking via Higgs doublet fields contains the quartic term

$$\frac{g^2 + g'^2}{8}(h_u^\dagger h_u - h_d^\dagger h_d)^2 + \cdots.$$

Whereas for purely $U(1)_Y$ symmetry breaking, the relevant quartic term is

$$\frac{g'^2}{2}(h_u^\dagger h_u - h_d^\dagger h_d)^2 + \cdots.$$

Because $g'^2/2$ is comparable to $(g^2 + g'^2)/8$ numerically at the weak scale, pure $U(1)_Y$ symmetry breaking is not really favored.

C. Fermion spectra

We write down relevant matter superfields in $SU(2)_L$ components explicitly,

$$L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}, \quad H_d = \begin{pmatrix} \nu_4 \\ e_4 \end{pmatrix}, \quad Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}, \quad Q_4 = \begin{pmatrix} u_4 \\ d_4 \end{pmatrix}.$$

Because fourth generation doublet leptons have been taken as Higgsinos, and EWSB is the same as that in MSSM, gaugino and Higgsino spectra are identical to those in MSSM. In the following we first look at the lepton spectrum.

As we have seen, the first three generation sneutrinos do not obtain vacuum expectation values (VEVs). $SU(2)_L$ doublet leptons, therefore, do not mix with the down-type Higgsino and chargino. Lepton masses are due to ordinary Yukawa couplings and Yukawa couplings between $SU(2)_L$ doublet leptons with the fourth generation singlet lepton, as well as the μ_e term,

$$\mathcal{L} \supset -e_i^c(m_{ij}^l e_j + m_{i4}^l e_4) - e_4^c(m_{4j}^l e_j + m_{44}^l e_4) - \mu_e e_4^c e_4 + \text{h.c.},$$

where small letters denote fermionic components, the 4×4 charged lepton mass matrix is given as

$$M_l = \begin{pmatrix} m_{ij}^l & m_{i4}^l \\ m_{4j}^l & m_{44}^l \end{pmatrix},$$

where $m_{ij}^l \equiv y_{ij}^l v\cos\beta$, $m_{i4}^l \equiv y_{i4}^E v\cos\beta$, $m_{4j}^l \equiv y_{4j}^E v\cos\beta$, and $m_{44}^l \equiv \mu_e$. Consider typically the 3rd and 4th generation case, namely that of i, j being 3, taking $m_{34}^l \sim m_{43}^l \sim m_{33}^l \sim m_\tau \cos\beta$, lepton masses are obtained,

$$M_l \simeq \begin{pmatrix} m_\tau\cos\beta & m_\tau\cos\beta \\ m_\tau\cos\beta & \mu_e \end{pmatrix}.$$

The unitary matrix diagonalizing $M_l M_l^\dagger$ is then

$$U_l = \begin{pmatrix} \frac{\mu_e}{\sqrt{\mu_e^2+m_\tau^2}} & \frac{m_\tau}{\sqrt{\mu_e^2+m_\tau^2}} \\ -\frac{m_\tau}{\sqrt{\mu_e^2+m_\tau^2}} & \frac{\mu_e}{\sqrt{\mu_e^2+m_\tau^2}} \end{pmatrix}.$$

This implies that there is an $\mathcal{O}(m_\tau/\mu_e)$ unitarity deviation among the three generation leptons.

For the down-type quark spectrum, introduction of additional two generations makes the full down quark mass matrix a 5×5 one,

$$\mathcal{L} \supset -d_i^c(m_{ij}^d d_j + m_{i4}^d d_4 + m_{iH}^d d_H) - d_4^c(m_{4j}^d d_j + m_{44}^d d_4 + m_{4H}^d d_H) - d_H^c(m_{Hj}^d d_j + m_{H4}^d d_4 + m_{HH}^d d_H) + \text{h.c.},$$

where $m_{ij}^d \equiv y_{ij}^d v\cos\beta$, $m_{i4}^d \equiv y_{i4}^{QD} v\cos\beta$, $m_{iH}^d \equiv \mu_D\delta_{iH}$, $m_{4j}^d \equiv y_{4j}^{QD} v\cos\beta$, $m_{44}^d \equiv \mu_Q$, $m_{4H}^d \equiv y_{44}^{QD} v\cos\beta$, $m_{Hj}^d \equiv \mu_D\delta_{Hj}$, $m_{H4}^d \equiv y_{44}^{QD} v\cos\beta$, and $m_{HH}^d \equiv \mu_Q$. The 3×3 sub-matrix m_{ij}^d is the ordinary down quark mass matrix which is now not necessarily unitary. Focusing on its unitarity deviation due to extra generations, we consider the sub-mass-matrix of the 3rd generation and extra generations, that is

$$M_d = \begin{pmatrix} m^d_{33} & m^d_{34} & \mu_D \\ m^d_{43} & \mu_Q & m^d_{44} \\ \mu_D & m^d_{44} & \mu_Q \end{pmatrix},$$

where $m^d_{33} \equiv y^d_{33} v\cos\beta$, $m^d_{34} \equiv y^{QD}_{34} v\cos\beta$, $m^d_{43} \equiv y^{QD}_{43} v\cos\beta$, $m^d_{44} \equiv y^{QD}_{44} v\cos\beta$. It is natural to take $|m^d_{34}| \sim |m^d_{43}| \sim |m^d_{33}| \sim m_b\cos\beta$ with m and n being 3 and 4, the absolute mass eigenvalues are then $|m_{d_{1,2}}| \sim |\mu_{Q,D}|$ and $|m_{d_3}| \sim m_b \cos\beta$ to the first order of $m^d_{mn}/\mu_{D,Q}$ respectively. The mass matrix is diagonalized by unitary matrices U_d and V_d, where

$$U_d^\dagger M_d V_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mu_Q & 0 \\ 0 & 0 & \mu_D \end{pmatrix},$$

$$U_d = \begin{pmatrix} 1 & \frac{m^d_{34}}{\mu_D} & \frac{m^{d*}_{43}}{\mu_Q} \\ -\frac{m^{d*}_{34}}{\mu_D} & 1 & 0 \\ -\frac{m^{d*}_{43}}{\mu_Q} & 0 & 1 \end{pmatrix}, \quad V_d = \begin{pmatrix} 1 & \frac{m^{d*}_{34}}{\mu_Q} & \frac{m^d_{43}}{\mu_D} \\ -\frac{m^d_{34}}{\mu_Q} & 1 & 0 \\ -\frac{m^d_{43}}{\mu_D} & 0 & 1 \end{pmatrix}.$$

Therefore there is generally an $\mathcal{O}((m^d_{34}/\mu_D)^2)$ contribution to unitarity deviation of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Similarly, the up-type quark mass matrix is given in the following

$$\mathcal{L} \supset -u^c_i(m^u_{ij}u_j + m^u_{i4}u_4 + m^u_{iH}u_H) - u^c_4(m^u_{4j}u_j + m^u_{44}u_4 + m^u_{4H}u_H) - u^c_H(m^u_{Hj}u_j + m^u_{H4}u_4 + m^u_{HH}u_H) + \text{h.c.},$$

where $m^u_{ij} \equiv y^u_{ij} v\sin\beta$, $m^u_{i4} \equiv y^{QU}_{i4} v\sin\beta$, $m^u_{iH} \equiv \mu_U \delta_{iH}$, $m^u_{4j} \equiv y^{QU}_{4j} v\sin\beta$, $m^u_{44} \equiv \mu_Q$, $m^u_{4H} \equiv y^{QU}_{44} v\sin\beta$, $m^u_{Hj} \equiv \mu_U\delta_{Hj}$, $m^u_{H4} \equiv y^{QU}_{44} v\sin\beta$, and $m^u_{HH} \equiv \mu_Q$. Taking $|m^u_{34}| \sim |m^u_{43}| \sim |m^u_{33}| \sim m_t\sin\beta$ and $|\mu_{U,Q}| \gg m_t\sin\beta$, respectively. The absolute mass eigenvalues are $|m_{u_{1,2}}| \sim |\mu_{U,Q}|$, and $|m_{u_3}| \sim m_t\sin\beta$. The diagonalizing matrices to Eq. (25) are U_u and V_u,

$$U_u^\dagger M_u V_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mu_Q & 0 \\ 0 & 0 & \mu_U \end{pmatrix},$$

where

$$U_u = \begin{pmatrix} 1 & \frac{m^u_{34}}{\mu_U} & \frac{m^{u*}_{43}}{\mu_Q} \\ -\frac{m^{u*}_{34}}{\mu_U} & 1 & 0 \\ -\frac{m^{u*}_{43}}{\mu_Q} & 0 & 1 \end{pmatrix}, \quad V_u = \begin{pmatrix} 1 & \frac{m^{u*}_{34}}{\mu_Q} & \frac{m^u_{43}}{\mu_U} \\ -\frac{m^u_{34}}{\mu_Q} & 1 & 0 \\ -\frac{m^u_{43}}{\mu_U} & 0 & 1 \end{pmatrix}.$$

The contribution to unitarity deviation of the CKM matrix can be as large as $(m_t/\mu_U)^2$.

D. Boson spectra

Boson masses are more complicated than the fermion case because of soft terms as well as SUSY kinetic terms. Higgs bosons, and therefore gauge bosons, in this model have exactly the same spectra as those in MSSM. So we will only consider slepton and squark masses.

SUSY kinetic terms contribute sfermion masses in the same manner as that in MSSM,

$$\mathcal{L}_{\rm D} \supset \frac{g^2+g'^2}{8}\cos 2\beta \sum_{\tilde{f}} \tilde{f}^\dagger \tilde{f}(T_3 - Q\sin^2\theta_W),$$

where $\tilde{f}$ denotes $\tilde{L}_i$, $\tilde{Q}_m$, $\tilde{Q}_H$, $\tilde{E}_{Ri}$, $\tilde{E}_4$, $\tilde{E}_H$, $\tilde{U}^c_m$, $\tilde{D}^c_m$, $\tilde{D}^c_H$, $\tilde{U}^c_4$, and θ_W is the Weinberg angle.

Trilinear terms in Eqs. (9) and (10) contribute

$$\mathcal{L}_{\rm tri} \supset -m_0 \Big[\lambda_{ijk}\tilde{L}_i\tilde{L}_j\tilde{E}^c_k + \lambda'_{ijk}\tilde{Q}_i\tilde{L}_j\tilde{D}^c_k + y_{ij}\tilde{Q}_i h_u \tilde{U}^c_j + \tilde{\lambda}_{ij}\tilde{E}^c_i\tilde{L}_j h_u + \lambda'_{i4}\tilde{Q}_i\tilde{L}_4\tilde{D}^c_H + y_{i4}\tilde{Q}_i h_u\tilde{U}^c_4 + \tilde{\lambda}_{i4}\tilde{E}^c_i\tilde{L}_4 h_u +$$

The superpotential which involves μ terms and Yukawa interactions contributes

$$\mathcal{L}_\mu \supset -\Big[\mu\tilde{L}_i h_u + \mu_e\tilde{E}^c_4\tilde{E}^c_H + \mu_Q\tilde{Q}_4\tilde{Q}_H + \mu_U\tilde{U}^c_4\tilde{U}^c_H + \mu_D\tilde{D}^c_4\tilde{D}^c_H + {\rm h.c.}\Big]$$

$$-\Big[\tilde{L}_i^\dagger m_l^\dagger m_l\tilde{L}_i + \tilde{Q}_i^\dagger m_d^\dagger m_d\tilde{Q}_i + \tilde{Q}_H^\dagger m_u^\dagger m_u\tilde{Q}_H + \tilde{E}^{c*}_i m_e^\dagger m_e\tilde{E}^c_i + \tilde{E}^{c*}_4 m_e^\dagger m_e\tilde{E}^c_4 + \tilde{E}^{c*}_H m_e^\dagger m_e\tilde{E}^c_H + \tilde{D}^{c*}_i m_d^\dagger m_d\tilde{D}^c_i + \tilde{D}$$

$$-\Big[(\mu\tan\beta)\tilde{L}_i m_l\tilde{E}^c_i + (\mu\tan\beta)\tilde{Q}_i m_d\tilde{D}^c_i + (\mu\cot\beta)\tilde{Q}_H m_u\tilde{U}^c_H + (\mu\tan\beta)m_u\tilde{U}^{c*}_i\tilde{U}^c_4 + \mu_D^* m_d\tilde{D}^{c*}_i\tilde{D}^c_4 + \mu_Q^* m_u\tilde{U}^{c*}_i\tilde{U}$$

where $m_{l,d,u}$ are matrices with elements defined before.

Together with $B\mu$ terms given in Eq. (8), full mass-squared matrices of sfermions are obtained. In the Appendix, the charged slepton, up squark and down squark mass-squared matrices will be given explicitly.

III. Phenomenological Constraints

The direct experimental search of extra generation particles at LEP requires that they should be heavier than 100 GeV, and direct search of extra generation quarks at Tevatron requires they are heavier than 270 GeV [?]. This result can be simply satisfied if μ_X' s are larger than 100 GeV or 270 GeV. Note that we do not have extra neutrinos which, in this model, consist of Higgsinos.

The electroweak precision measurement generally has constraints on extra matters [?, ?]. Current constraints are [?]

$$S = 0.10, \quad T = 0.11, \quad U = 0.15.$$

For one extra chiral generation, oblique parameters S, T, U still allow the existence of the 4th generation [?] provided that there is certain mass splitting in extra $SU(2)_L$ doublets. In our case, the vector-like generation contributes to the parameters in the way of $1/\mu_X^2$ as expected from the decoupling theorem [?]. Typically,

$$\Delta S \sim \Delta T \sim \Delta U \sim \frac{m_t^2}{\mu_X^2}.$$

The effect of the extra generation can be small enough if we take $\mu_X \gtrsim 1$ TeV.

Important constraints come from the unitarity of the 3×3 CKM quark mixing matrix of three chiral generations [?]. This unitarity is consistent with current data within experimental errors. In this model, extra generations mix with ordinary three chiral generations which necessarily break the unitarity of the CKM mixing matrix. As we have observed following Eqs. (20) and (25), unitarity violation is about $(m_{i4}^{d(u)}/\mu_{D(U)})^2$. This μ_X dependence is generally expected in the case of extra vector-like generations. Hierarchical or small mixing masses $m_{i4}^{d(u)}$ can easily make the CKM matrix approximately unitary within errors. For an example, $(m_{14}^{d(u)}/\mu_X)^2 \lesssim 10^{-3}$. Assuming only the third generation mixes with extra generations, the constraint is still loose, $(m_{34}^{d(u)}/\mu_{D(U)})^2 \lesssim 0.39$. The quantity $m_{34}^{d(u)}$ is at most about m_t. This gives that the parameter $\mu_{D,U} \gtrsim 280$ GeV.

From Eqs. (22), (23), (27) and (28), it can be seen that there are new phases in fermion mixing matrices. However, these new matrix elements are of order of m_t/μ_X at most. So new CP violation effects are generally suppressed.

One of the characteristic properties of the superpotential in this model is that there are many μ-parameters. In the following analysis, we prefer to take all the μ-parameters approximately equal. They are expected to have a common origin. On the other hand, by taking $|\mu| \ll |\mu_X|$, such as $\mu = 100$ GeV and $\mu_X = 500$ GeV, this model will have MSSM as its low energy effective theory. In order to make this model distinct, considering the above discussed constraints, we will take μ close to μ_X, and $\mu_X \sim 500$ GeV.

We bear in mind that for correct EWSB, it may be necessary to require that μ is somewhat smaller than μ_X, and μ_X cannot be too large. Of course, values of soft masses and $B\mu$' s are also important to EWSB. They can affect the choice of values of μ-parameters.

In the following, we give a numerical illustration. μ parameters can have the following values,

$$\mu = 300 \text{ GeV}, \quad \mu_e = 400 \text{ GeV}, \quad \mu_{Q,U,D} = 500 \text{ GeV}.$$

Correct EWSB happens if we take

$$B_X\mu_X = (260 \text{ GeV})^2, \quad M_{\tilde{L}}^2 = (219 \text{ GeV})^2, \quad M_h^2 = -(200 \text{ GeV})^2,$$

$$M_{\tilde{E}}^2 = (243 \text{ GeV})^2, \quad M_{\tilde{Q}}^2 = (200 \text{ GeV})^2, \quad M_{\tilde{U}}^2 = M_{\tilde{D}}^2 = (200 \text{ GeV})^2.$$

Note that we have taken M_h^2 negative. This is expected due to the large top quark Yukawa coupling [?]. It is straightforward to see that this set of parameters results in consistent EWSB, and it fixes that $\tan\beta = 2$ and results in the Higgs spectrum,

$$m_{h^0}^2 = (124\text{ GeV})^2, \quad m_{H^0}^2 = (420\text{ GeV})^2, \quad m_{A^0}^2 = (413\text{ GeV})^2, \quad m_{H^\pm}^2 = (424\text{ GeV})^2,$$

where the quantum correction to $m_{h^0}^2$ has been included.

IV. LHC Phenomenology

This model can be tested at LHC. In addition to the (super)particle content of MSSM, it predicts the following vector-like particles: one lepton singlet (E_4^c, E_H^c), one quark doublet (Q_4, Q_H), one up-type quark singlet (U_4^c, U_H^c) and one down-type quark singlet (D_4^c, U_H^c), but there is no extra doublet leptons which are already identified as the Higgs doublets.

Let us look at fermions. Because the effect of EWSB is much smaller than μ-parameters, these new Weyl fermions form several Dirac fermions with masses $\sim \mu_X \sim 500$ GeV,

$$\Psi_e \equiv \begin{pmatrix} e_4 \\ e_4^c \end{pmatrix}, \quad \Psi_Q \equiv \begin{pmatrix} q_4 \\ q_H^c \end{pmatrix}, \quad \Psi_u \equiv \begin{pmatrix} u_4^c \\ u_H \end{pmatrix}, \quad \Psi_d \equiv \begin{pmatrix} d_4^c \\ d_H \end{pmatrix}.$$

Note that in this case, mass splitting in the $SU(2)_L$ doublet q_4 or q_H is also negligible.

Considering gauge kinetic terms, the Lagrangian describing these pseudo-Dirac fermions can be written as

$$\mathcal{L} \supset \sum_X \bar{\Psi}_X \gamma^\mu D_\mu \Psi_X + \mu_X \bar{\Psi}_X \Psi_X,$$

where the covariant derivative is self-evident.

Decay signals of these new particles can be easily identified. From trilinear Yukawa interactions given in Eq. (4), it is seen that they decay into SM first three generation matters. Ψ_e can decay into e_i and a neutral Higgs, the decay rate is

$$\Gamma(\Psi_e \to e_i h^0) \simeq \frac{|y_{i4}^E|^2}{16\pi}\mu_e.$$

Alternative to the Higgs, scalar neutrinos can be also the decay product which is expected at least heavier than the lighter neutral Higgs. Similarly, new quarks Ψ_u and Ψ_d decay into ordinary quarks and the Higgs,

$$\Psi_u \to u_i(c,t)h^0, \quad \Psi_d \to d_i(s,b)h^0,$$

with decay rates

$$\Gamma(\Psi_u \to u_i h^0) \simeq \frac{|y_{i4}^{QU}|^2}{16\pi}\mu_U, \quad \Gamma(\Psi_d \to d_i h^0) \simeq \frac{|y_{i4}^{QD}|^2}{16\pi}\mu_D.$$

And decays of new quarks q_4 and q_H have following results,

$$\Gamma(\Psi_d \to u_i W^-) \simeq \frac{|y_{i4}^{Q'}|^2}{16\pi}\mu_D, \quad \Gamma(\Psi_u \to d_i W^+) \simeq \frac{|y_{i4}^{Q}|^2}{16\pi}\mu_U.$$

Taking relevant Yukawa coefficients y_i' s $\sim 10^{-2}$, decay rates in Eqs. (40)-(43) are $\sim$ 500 MeV.

Taking EWSB into consideration, Ψ_X mixes with SM fermions. The 5×5 generalized CKM matrix is derived from Eqs. (22) and (27),

$$V_{\rm CKM} = U_u^\dagger U_d \simeq \begin{pmatrix} V_{\rm CKM}^{3\times3} & \frac{m_{i4}^u}{\mu_U} \\ \frac{m_{4j}^{d*}}{\mu_D} & 1 \end{pmatrix}.$$

Note that $(m/\mu_X)^2$ terms have been omitted. We see that decays $\Psi_d \to \bar{t}W^+$ and $\Psi_u \to \bar{b}W^-$ occur via the $SU(2)_L$ gauge interaction at the level of (m/μ_X),

$$\Gamma(\Psi_d \to \bar{t}W^+) \simeq \frac{G_F m_t^2}{8\sqrt{2}\pi}\mu_D \left(1 + 2\frac{m_t^2}{\mu_D^2}\right)\sqrt{1 - \frac{m_t^2}{\mu_D^2}},$$

$$\Gamma(\Psi_u \to \bar{b}W^-) \simeq \frac{G_F m_t^2}{8\sqrt{2}\pi}\mu_U \left(1 + 2\frac{m_t^2}{\mu_U^2}\right)\sqrt{1 - \frac{m_t^2}{\mu_U^2}},$$

where the phase space factors were given in Refs. [**?**, **?**]. Taking $m/\mu_X \sim 1/3$, these Γ' s are about 1 GeV.

All above decays are fast enough that they occur inside detectors. With the invariant mass method, decayed new fermions will be reconstructed.

These new quarks can be produced at LHC via gluon fusion processes,

$$gg \to Q_4 \bar{Q}_H, \quad gg \to U_4^c \bar{U}_H^c, \quad gg \to D_4^c \bar{D}_H^c.$$

The production mechanism is essentially the same as that of the top quark [**?**] with an estimated cross section $\sim$ hundreds fb by taking $\mu_X \sim 500$ GeV and $\sqrt{s} = 14$ TeV.

For the new lepton, the Drell-Yan process is the main production mechanism,

$$pp \to E_4^c \bar{E}_H^c.$$

The cross section is estimated to be few fb which means a few tens events in one year [**?**, **?**]. Considering the detector efficiency, new lepton observation may be challenging at LHC. However, once they are produced, their decay signals are easy to be identified.

V. Summary and Discussion

Within the framework of SUSY, we have extended the matter content in a way that each Higgs doublet is in a full generation. Namely, in addition to ordinary three generations, there is an extra vector-like generation, and it is the extra slepton $SU(2)_L$ doublets that are taken as two Higgs doublets. R-parity violating interactions contain ordinary Yukawa interactions. SUSY and gauge symmetry breaking have been analyzed. Fermion and boson spectra have been calculated. Phenomenological constraints and relevant LHC physics have been discussed.

Finally, we discuss some aspects of this model. We are motivated by trying to naturally understand the Higgs. Within SUSY, Higgs can be considered as certain slepton doublets. Thus they are nothing special compared to three ordinary generations, the anomaly cancels within each generation.

It might be amusing to note that the first generation composes the ordinary matter, the second generation provides fermion mixing, the third gives CP violation, and the fourth and fifth (the vector one) give out EWSB because they contain Higgs doublets.

Like in traditional R-parity violating models, baryon number conservation is required. While the requirement of any first principle global symmetry is a drawback compared to SM, it is possible to consider baryon number conservation as a result of the so-called discrete gauge symmetry [**?**, **?**].

R-parity violation implies that neutralinos cannot be DM. Of course, the TeV particle theory does not necessarily provide DM; there are many alternative scenarios, like the axion, the sterile neutrino, or even modified gravity. Nevertheless, the thermally produced weakly interacting massive particle (WIMP) is one of the most attractive. To this model, it is still possible to introduce additional matter playing the role of WIMP. We note that a recent DM proposal by Arkani-Hamed et al. [**?**] can be directly combined with our model. Their proposal is an effort to explain all recent DM experiments [**?**, **?**], where WIMP DM lies in a new sector.

Neutrino masses can be generated. This model has lepton number violation which contributes to neutrino masses [**?**, **?**]. Although the original superpotential (1) is simple, its expression in terms of ordinary three light generations given in Eq. (4) is complicated. There are many new lepton number violating sources. All of them involve the vector-like extra generation which is heavier than soft SUSY breaking masses. Therefore, except for the ordinary R-parity violating terms with couplings λ_{ijk} and λ'_{ijk}, the new lepton number violating terms are less stringently constrained at low energies. The full phenomenological analysis of lepton number violation is rather involved, and will be considered in a separate work. Furthermore, the see-saw mechanism can be introduced to get a fully realistic neutrino mass pattern.

It seems that we have lost GUT. In MSSM, running gauge coupling constants meet together at the energy $\sim 10^{16}$ GeV. This is regarded as a result of GUT. By adding new matter which is charged under the SM, gauge coupling unification would be lost generally. However, if the new matter composes complete representations of GUT, gauge coupling unification is still kept at least to the one-loop level [?, ?, ?]. Compared to MSSM, the new matter contents we have added in this model are $(E_4^c, Q_4, U_4^c, D_4^c)$ and $(E_H^c, Q_H, U_H^c, D_H^c)$. They do not compose complete representations of GUT. Giving up GUT while keeping SUSY sounds bizarre. However, GUT relation of gauge coupling constants may be finally restored after additional matter, that is DM, is included. This SUSY model has already introduced the new matter $5 \oplus \bar{5}$ and $10 \oplus \bar{10}$ in $SU(5)$ representation. Gauge couplings still do not reach their Landau poles in the GUT energy scale [?, ?].

We have ignored the mixing of first two chiral generations with the vector generation. Detailed consideration of such possibly small mixing may give interesting observable phenomena [?].

SUSY breaking and its mediation to our sector should be considered systematically. This is closely related to EWSB and LHC phenomenology.

All above discussed aspects deserve further and separate studies.

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Appendix: Scalar Mass-Squared Matrices

In writing sfermion mass matrices, for simplicity and without losing generality, we will omit the first two generations. Neglecting the mixing between $(1,2)$ generations and $(3,4)$ generations, the first two generation sfermions themselves are the same as those in MSSM.

The charged slepton mass-squared matrix is

$$\mathcal{M}_{\tilde{l}}^2 = \begin{pmatrix} \tilde{L}_3^\dagger & \tilde{L}_4^\dagger & \tilde{E}_3^{c*} & \tilde{E}_4^{c*} \end{pmatrix} \begin{pmatrix} (\mathcal{M}_{\tilde{l}}^2)_{11} & (\mathcal{M}_{\tilde{l}}^2)_{12} & (\mathcal{M}_{\tilde{l}}^2)_{13} & (\mathcal{M}_{\tilde{l}}^2)_{14} \\ (\mathcal{M}_{\tilde{l}}^2)_{21} & (\mathcal{M}_{\tilde{l}}^2)_{22} & (\mathcal{M}_{\tilde{l}}^2)_{23} & (\mathcal{M}_{\tilde{l}}^2)_{24} \\ (\mathcal{M}_{\tilde{l}}^2)_{31} & (\mathcal{M}_{\tilde{l}}^2)_{32} & (\mathcal{M}_{\tilde{l}}^2)_{33} & (\mathcal{M}_{\tilde{l}}^2)_{34} \\ (\mathcal{M}_{\tilde{l}}^2)_{41} & (\mathcal{M}_{\tilde{l}}^2)_{42} & (\mathcal{M}_{\tilde{l}}^2)_{43} & (\mathcal{M}_{\tilde{l}}^2)_{44} \end{pmatrix} \begin{pmatrix} \tilde{L}_3 \\ \tilde{L}_4 \\ \tilde{E}_3^c \\ \tilde{E}_4^c \end{pmatrix},$$

where $\mathcal{M}_{\tilde{l}}^2$ is given by the following matrix elements:

$$(\mathcal{M}_{\tilde{l}}^2)_{11} = M_{\tilde{L}}^2 + \frac{g^2 + g'^2}{8} \cos 2\beta + m_\tau^2,$$

$$(\mathcal{M}_{\tilde{l}}^2)_{12} = (m_0 - \mu \tan\beta) m_\tau^*, \quad (\mathcal{M}_{\tilde{l}}^2)_{13} = (m_0 - \mu \tan\beta) m_\tau^*, \quad (\mathcal{M}_{\tilde{l}}^2)_{14} = \mu_e m_\tau^*,$$

$$(\mathcal{M}^2_{\tilde{l}})_{21} = (m_0 - \mu\tan\beta)m_\tau, \quad (\mathcal{M}^2_{\tilde{l}})_{22} = M_h^2 + \frac{g^2+g'^2}{8}\cos 2\beta + m_\tau^2,$$
$$(\mathcal{M}^2_{\tilde{l}})_{23} = m_\tau m_\tau^*, \quad (\mathcal{M}^2_{\tilde{l}})_{24} = 0,$$
$$(\mathcal{M}^2_{\tilde{l}})_{31} = (m_0-\mu\tan\beta)m_\tau, \quad (\mathcal{M}^2_{\tilde{l}})_{32} = m_\tau^* m_\tau, \quad (\mathcal{M}^2_{\tilde{l}})_{33} = M^2_{\tilde{E}} + \frac{g'^2}{4}\cos 2\beta + m_\tau^2,$$
$$(\mathcal{M}^2_{\tilde{l}})_{34} = B_e\mu_e, \quad (\mathcal{M}^2_{\tilde{l}})_{41} = \mu_e^* m_\tau, \quad (\mathcal{M}^2_{\tilde{l}})_{42} = 0, \quad (\mathcal{M}^2_{\tilde{l}})_{43} = B_e^*\mu_e^*,$$
$$(\mathcal{M}^2_{\tilde{l}})_{44} = M^2_{\tilde{E}} + \frac{g'^2}{2}\cos 2\beta + m_\tau^2.$$

The down quark mass-squared matrix is

$$\mathcal{M}^2_{\tilde{d}} = \begin{pmatrix} \tilde{Q}_3^\dagger & \tilde{Q}_4^\dagger & \tilde{D}_3^{c*} & \tilde{D}_4^{c*} & \tilde{Q}_H^\dagger & \tilde{D}_H^{c*} \end{pmatrix} \begin{pmatrix} (\mathcal{M}^2_{\tilde{d}})_{11} & (\mathcal{M}^2_{\tilde{d}})_{12} & (\mathcal{M}^2_{\tilde{d}})_{13} & (\mathcal{M}^2_{\tilde{d}})_{14} & (\mathcal{M}^2_{\tilde{d}})_{15} & (\mathcal{M}^2_{\tilde{d}})_{16} \\ (\mathcal{M}^2_{\tilde{d}})_{21} & (\mathcal{M}^2_{\tilde{d}})_{22} & (\mathcal{M}^2_{\tilde{d}})_{23} & (\mathcal{M}^2_{\tilde{d}})_{24} & (\mathcal{M}^2_{\tilde{d}})_{25} & (\mathcal{M}^2_{\tilde{d}})_{26} \\ (\mathcal{M}^2_{\tilde{d}})_{31} & (\mathcal{M}^2_{\tilde{d}})_{32} & (\mathcal{M}^2_{\tilde{d}})_{33} & (\mathcal{M}^2_{\tilde{d}})_{34} & (\mathcal{M}^2_{\tilde{d}})_{35} & (\mathcal{M}^2_{\tilde{d}})_{36} \\ (\mathcal{M}^2_{\tilde{d}})_{41} & (\mathcal{M}^2_{\tilde{d}})_{42} & (\mathcal{M}^2_{\tilde{d}})_{43} & (\mathcal{M}^2_{\tilde{d}})_{44} & (\mathcal{M}^2_{\tilde{d}})_{45} & (\mathcal{M}^2_{\tilde{d}})_{46} \\ (\mathcal{M}^2_{\tilde{d}})_{51} & (\mathcal{M}^2_{\tilde{d}})_{52} & (\mathcal{M}^2_{\tilde{d}})_{53} & (\mathcal{M}^2_{\tilde{d}})_{54} & (\mathcal{M}^2_{\tilde{d}})_{55} & (\mathcal{M}^2_{\tilde{d}})_{56} \\ (\mathcal{M}^2_{\tilde{d}})_{61} & (\mathcal{M}^2_{\tilde{d}})_{62} & (\mathcal{M}^2_{\tilde{d}})_{63} & (\mathcal{M}^2_{\tilde{d}})_{64} & (\mathcal{M}^2_{\tilde{d}})_{65} & (\mathcal{M}^2_{\tilde{d}})_{66} \end{pmatrix} \begin{pmatrix} \tilde{Q}_3 \\ \tilde{Q}_4 \\ \tilde{D}_3^c \\ \tilde{D}_4^c \\ \tilde{Q}_H \\ \tilde{D}_H^c \end{pmatrix},$$

where

$$(\mathcal{M}^2_{\tilde{d}})_{11} = M^2_{\tilde{Q}} + \frac{g^2+g'^2}{24}\cos 2\beta + \frac{g^2-g'^2}{12}\cos 2\beta + m_b^2,$$
$$(\mathcal{M}^2_{\tilde{d}})_{12} = (m_0-\mu\tan\beta)m_b^*, \quad (\mathcal{M}^2_{\tilde{d}})_{13} = (m_0-\mu\tan\beta)m_b^*, \quad (\mathcal{M}^2_{\tilde{d}})_{14} = \mu_D m_{43}^{d*} + m_{34}^{d*} m_{44}^d,$$
$$(\mathcal{M}^2_{\tilde{d}})_{15} = m_{43}^{d*} + m_{34}^{d*}, \quad (\mathcal{M}^2_{\tilde{d}})_{16} = 0,$$
$$(\mathcal{M}^2_{\tilde{d}})_{21} = (m_0-\mu\tan\beta)m_b, \quad (\mathcal{M}^2_{\tilde{d}})_{22} = M^2_{\tilde{Q}} + \frac{g^2+g'^2}{24}\cos 2\beta + \frac{g^2-g'^2}{12}\cos 2\beta + m_b^2,$$
$$(\mathcal{M}^2_{\tilde{d}})_{23} = m_b m_{43}^{d*}, \quad (\mathcal{M}^2_{\tilde{d}})_{24} = 0, \quad (\mathcal{M}^2_{\tilde{d}})_{25} = (m_0-\mu\tan\beta)m_{44}^{d*}, \quad (\mathcal{M}^2_{\tilde{d}})_{26} = \mu_Q^* m_{44}^d,$$
$$(\mathcal{M}^2_{\tilde{d}})_{31} = (m_0-\mu\tan\beta)m_b, \quad (\mathcal{M}^2_{\tilde{d}})_{32} = m_{34}^{d*} m_b, \quad (\mathcal{M}^2_{\tilde{d}})_{33} = M^2_{\tilde{D}} + \frac{g'^2}{6}\cos 2\beta + m_b^2,$$
$$(\mathcal{M}^2_{\tilde{d}})_{34} = B_D\mu_D, \quad (\mathcal{M}^2_{\tilde{d}})_{35} = (m_0-\mu\tan\beta)m_{44}^{d*}, \quad (\mathcal{M}^2_{\tilde{d}})_{36} = \mu_Q^* m_{44}^d,$$
$$(\mathcal{M}^2_{\tilde{d}})_{41} = \mu_D^* m_{34}^d + m_{44}^{d*} m_{34}^d, \quad (\mathcal{M}^2_{\tilde{d}})_{42} = 0, \quad (\mathcal{M}^2_{\tilde{d}})_{43} = B_D^*\mu_D^*,$$
$$(\mathcal{M}^2_{\tilde{d}})_{44} = M^2_{\tilde{D}} + \frac{g'^2}{3}\cos 2\beta + m_{DH}^2, \quad (\mathcal{M}^2_{\tilde{d}})_{45} = \mu_D^* m_{44}^d, \quad (\mathcal{M}^2_{\tilde{d}})_{46} = 0,$$
$$(\mathcal{M}^2_{\tilde{d}})_{51} = m_b m_{43}^{d*}, \quad (\mathcal{M}^2_{\tilde{d}})_{52} = (m_0-\mu\tan\beta)m_{44}^{d*}, \quad (\mathcal{M}^2_{\tilde{d}})_{53} = (m_0-\mu\tan\beta)m_{44}^{d*},$$
$$(\mathcal{M}^2_{\tilde{d}})_{54} = \mu_D m_{44}^{d*}, \quad (\mathcal{M}^2_{\tilde{d}})_{55} = M^2_{\tilde{Q}} + \frac{g^2+g'^2}{24}\cos 2\beta - \frac{g^2-g'^2}{6}\cos 2\beta + m_{QH}^2,$$
$$(\mathcal{M}^2_{\tilde{d}})_{56} = B_Q^*\mu_Q^*,$$
$$(\mathcal{M}^2_{\tilde{d}})_{61} = 0, \quad (\mathcal{M}^2_{\tilde{d}})_{62} = \mu_Q m_{44}^{d*}, \quad (\mathcal{M}^2_{\tilde{d}})_{63} = \mu_Q m_{44}^{d*}, \quad (\mathcal{M}^2_{\tilde{d}})_{64} = 0,$$

$$(\mathcal{M}^2_{\tilde{d}})_{65} = B_Q\mu_Q, \quad (\mathcal{M}^2_{\tilde{d}})_{66} = M^2_{\tilde{D}} + \frac{g'^2}{3}\cos 2\beta + m^2_{DH}.$$

The up quark mass-squared matrix is

$$\mathcal{M}^2_{\tilde{u}} = \begin{pmatrix} \tilde{Q}_3^\dagger & \tilde{Q}_4^\dagger & \tilde{U}_3^{c*} & \tilde{U}_4^{c*} & \tilde{Q}_H^\dagger & \tilde{U}_H^{c*} \end{pmatrix} \begin{pmatrix} (\mathcal{M}^2_{\tilde{u}})_{11} & (\mathcal{M}^2_{\tilde{u}})_{12} & (\mathcal{M}^2_{\tilde{u}})_{13} & (\mathcal{M}^2_{\tilde{u}})_{14} & (\mathcal{M}^2_{\tilde{u}})_{15} & (\mathcal{M}^2_{\tilde{u}})_{16} \\ (\mathcal{M}^2_{\tilde{u}})_{21} & (\mathcal{M}^2_{\tilde{u}})_{22} & (\mathcal{M}^2_{\tilde{u}})_{23} & (\mathcal{M}^2_{\tilde{u}})_{24} & (\mathcal{M}^2_{\tilde{u}})_{25} & (\mathcal{M}^2_{\tilde{u}})_{26} \\ (\mathcal{M}^2_{\tilde{u}})_{31} & (\mathcal{M}^2_{\tilde{u}})_{32} & (\mathcal{M}^2_{\tilde{u}})_{33} & (\mathcal{M}^2_{\tilde{u}})_{34} & (\mathcal{M}^2_{\tilde{u}})_{35} & (\mathcal{M}^2_{\tilde{u}})_{36} \\ (\mathcal{M}^2_{\tilde{u}})_{41} & (\mathcal{M}^2_{\tilde{u}})_{42} & (\mathcal{M}^2_{\tilde{u}})_{43} & (\mathcal{M}^2_{\tilde{u}})_{44} & (\mathcal{M}^2_{\tilde{u}})_{45} & (\mathcal{M}^2_{\tilde{u}})_{46} \\ (\mathcal{M}^2_{\tilde{u}})_{51} & (\mathcal{M}^2_{\tilde{u}})_{52} & (\mathcal{M}^2_{\tilde{u}})_{53} & (\mathcal{M}^2_{\tilde{u}})_{54} & (\mathcal{M}^2_{\tilde{u}})_{55} & (\mathcal{M}^2_{\tilde{u}})_{56} \\ (\mathcal{M}^2_{\tilde{u}})_{61} & (\mathcal{M}^2_{\tilde{u}})_{62} & (\mathcal{M}^2_{\tilde{u}})_{63} & (\mathcal{M}^2_{\tilde{u}})_{64} & (\mathcal{M}^2_{\tilde{u}})_{65} & (\mathcal{M}^2_{\tilde{u}})_{66} \end{pmatrix} \begin{pmatrix} \tilde{Q}_3 \\ \tilde{Q}_4 \\ \tilde{U}_3^c \\ \tilde{U}_4^c \\ \tilde{Q}_H \\ \tilde{U}_H^c \end{pmatrix}$$

where

$$(\mathcal{M}^2_{\tilde{u}})_{11} = M^2_{\tilde{Q}} + \frac{g^2+g'^2}{24}\cos 2\beta + \frac{g^2-g'^2}{12}\cos 2\beta + m_t^2,$$

$$(\mathcal{M}^2_{\tilde{u}})_{12} = (m_0 - \mu\cot\beta)m_t^*, \quad (\mathcal{M}^2_{\tilde{u}})_{13} = (m_0 - \mu\cot\beta)m_t^*, \quad (\mathcal{M}^2_{\tilde{u}})_{14} = \mu_U m_{43}^{u*} + m_{34}^{u*}m_{44}^u,$$

$$(\mathcal{M}^2_{\tilde{u}})_{15} = m_{43}^{u*} + m_{34}^{u*}, \quad (\mathcal{M}^2_{\tilde{u}})_{16} = 0,$$

$$(\mathcal{M}^2_{\tilde{u}})_{21} = (m_0 - \mu\cot\beta)m_t, \quad (\mathcal{M}^2_{\tilde{u}})_{22} = M^2_{\tilde{Q}} + \frac{g^2+g'^2}{24}\cos 2\beta + \frac{g^2-g'^2}{12}\cos 2\beta + m_t^2,$$

$$(\mathcal{M}^2_{\tilde{u}})_{23} = m_t m_{43}^{u*}, \quad (\mathcal{M}^2_{\tilde{u}})_{24} = 0, \quad (\mathcal{M}^2_{\tilde{u}})_{25} = (m_0 - \mu\cot\beta)m_{44}^{u*}, \quad (\mathcal{M}^2_{\tilde{u}})_{26} = \mu_Q^* m_{44}^u,$$

$$(\mathcal{M}^2_{\tilde{u}})_{31} = (m_0 - \mu\cot\beta)m_t, \quad (\mathcal{M}^2_{\tilde{u}})_{32} = m_{34}^{u*}m_t, \quad (\mathcal{M}^2_{\tilde{u}})_{33} = M^2_{\tilde{U}} + \frac{g'^2}{3}\cos 2\beta + m_t^2,$$

$$(\mathcal{M}^2_{\tilde{u}})_{34} = B_U\mu_U, \quad (\mathcal{M}^2_{\tilde{u}})_{35} = (m_0 - \mu\cot\beta)m_{44}^{u*}, \quad (\mathcal{M}^2_{\tilde{u}})_{36} = \mu_Q^* m_{44}^u,$$

$$(\mathcal{M}^2_{\tilde{u}})_{41} = \mu_U^* m_{34}^u + m_{44}^{u*}m_{34}^u, \quad (\mathcal{M}^2_{\tilde{u}})_{42} = 0, \quad (\mathcal{M}^2_{\tilde{u}})_{43} = B_U^*\mu_U^*,$$

$$(\mathcal{M}^2_{\tilde{u}})_{44} = M^2_{\tilde{U}} + \frac{2g'^2}{3}\cos 2\beta + m^2_{UH}, \quad (\mathcal{M}^2_{\tilde{u}})_{45} = \mu_U^* m_{44}^u, \quad (\mathcal{M}^2_{\tilde{u}})_{46} = (m_0 - \mu\cot\beta)m_{44}^{u*},$$

$$(\mathcal{M}^2_{\tilde{u}})_{51} = m_t m_{43}^{u*}, \quad (\mathcal{M}^2_{\tilde{u}})_{52} = (m_0 - \mu\cot\beta)m_{44}^{u*}, \quad (\mathcal{M}^2_{\tilde{u}})_{53} = (m_0 - \mu\cot\beta)m_{44}^{u*},$$

$$(\mathcal{M}^2_{\tilde{u}})_{54} = \mu_U m_{44}^{u*}, \quad (\mathcal{M}^2_{\tilde{u}})_{55} = M^2_{\tilde{Q}} + \frac{g^2+g'^2}{24}\cos 2\beta - \frac{g^2-g'^2}{6}\cos 2\beta + m^2_{QH},$$

$$(\mathcal{M}^2_{\tilde{u}})_{56} = B_Q^*\mu_Q^*,$$

$$(\mathcal{M}^2_{\tilde{u}})_{61} = 0, \quad (\mathcal{M}^2_{\tilde{u}})_{62} = \mu_Q m_{44}^{u*}, \quad (\mathcal{M}^2_{\tilde{u}})_{63} = \mu_Q m_{44}^{u*}, \quad (\mathcal{M}^2_{\tilde{u}})_{64} = (m_0 - \mu\cot\beta)m_{44}^{u*},$$

$$(\mathcal{M}^2_{\tilde{u}})_{65} = B_Q\mu_Q, \quad (\mathcal{M}^2_{\tilde{u}})_{66} = M^2_{\tilde{U}} + \frac{2g'^2}{3}\cos 2\beta + m^2_{UH}.$$

As we have expected, taking the $3-4$ mixing mass to be small $m_{34}^{l(u,d)} \to 0$, the third generation also decouples from the two extra generations.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.