

## Four-quark Operators Relevant to B Meson Lifetimes from QCD Sum Rules Postprint

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### Abstract

At the order of  $1/m^3_b$ , the B meson lifetimes are controlled by the hadronic matrix elements of some four-quark operators. The nonfactorizable magnitudes of these four-quark operator matrix elements are analyzed by QCD sum rules in the framework of heavy quark effective theory. The vacuum saturation for color-singlet four-quark operators is justified at hadronic scale, and the nonfactorizable effect is at a few percent level. However for color-octet four-quark operators, the vacuum saturation is violated sizably that the nonfactorizable effect cannot be neglected for the B meson lifetimes. The implication to the extraction of some of the parameters from B decays is discussed. The B meson lifetime ratio is predicted as  $\tau(B^-)/\tau(B^0) = 1.09 \pm 0.02$ . However, the experimental result of the lifetime ratio  $\tau(B^-)/\tau(B^0)$  still cannot be explained.

### Full Text

### Preamble

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### Four-quark Operators Relevant to B Meson Lifetimes from QCD Sum Rules

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### Abstract

At order  $1/m^3_b$ , B meson lifetimes are governed by the hadronic matrix elements of certain four-quark operators. The nonfactorizable magnitudes of these

matrix elements are analyzed using QCD sum rules within the framework of Heavy Quark Effective Theory (HQET). Vacuum saturation for color-singlet four-quark operators is justified at the hadronic scale, with nonfactorizable effects at the few-percent level. However, for color-octet four-quark operators, vacuum saturation is significantly violated, making the nonfactorizable effect non-negligible for B meson lifetimes. The implications for extracting certain parameters from B decays are discussed. The predicted B meson lifetime ratio is  $\tau(B_c)/\tau(B) = 1.09 \pm 0.02$ , though the experimental result for  $\tau(\Lambda_b)/\tau(B)$  remains unexplained.

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Heavy hadron lifetimes provide a testing ground for the Standard Model, particularly for QCD, because they can be systematically calculated within the heavy quark expansion framework. Theoretically, assuming local duality holds, heavy hadron lifetime differences appear at most at order  $1/m^2_Q$ . Recent experimental results on the  $\Lambda_b$  baryon to B meson lifetime ratio have shown deviations from theoretical expectations, attracting considerable attention. The current experimental values for the relevant lifetime ratios are:

$$\begin{aligned}\tau(B_c)/\tau(B) &= 1.06 \pm 0.04, \\ \tau(\Lambda_b)/\tau(B) &= 0.79 \pm 0.06.\end{aligned}$$

This suggests that  $O(1/m^2_Q)$  contributions may be insufficient to explain the difference between heavy baryon and heavy meson lifetimes. The  $1/m^3_b$  corrections to hadron lifetimes have been studied since the mid-1980s, and their potential importance has been noted. The hadronic matrix elements of four-quark operators appearing at order  $1/m^3_b$  are generally parameterized as:

$$\begin{aligned}F_B^2 \langle B | \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) b | B \rangle, \\ F_B^2 \langle B | \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 + \gamma_5) b | B \rangle, \\ F_B^2 \langle B | \bar{b} \gamma_\mu (1 - \gamma_5) \hat{t}^a q \bar{q} \gamma^\mu (1 - \gamma_5) \hat{t}^a b | B \rangle, \\ F_B^2 \langle B | \bar{b} \gamma_\mu (1 - \gamma_5) \hat{t}^a q \bar{q} \gamma^\mu (1 + \gamma_5) \hat{t}^a b | B \rangle,\end{aligned}$$

r,  
-B,

where the parameters  $F_{B_i}$ ,  $r_i$  ( $i = 1, 2$ ),  $F_B$ , r, and B must be calculated using nonperturbative QCD methods. In these equations, the renormalization scale is arbitrary, and the parameters depend on it. It can naturally be taken at the low hadronic scale to apply heavy quark expansion. On the other hand, if the scale is taken at  $m_b$ , the parameter  $F_B(m_b)$  is simply the well-defined physical quantity—the B meson decay constant  $f_B$ .

QCD sum rules, regarded as a nonperturbative method rooted in QCD itself, have been successfully used to calculate properties of various hadrons. In Ref. [8], the baryonic parameters r and B were calculated in the HQET framework, yielding  $r = 0.1-0.3$  and  $B = 1$ . For a complete analysis, the mesonic parameters

$B_{\text{I}}$  and  $\alpha_{\text{I}}$  must also be calculated from QCD sum rules. The four-quark operators, and consequently  $B_{\text{I}}$ ,  $\alpha_{\text{I}}$ ,  $r$ , and  $B$ , are scale-dependent when QCD radiative corrections are included. Shifman and Voloshin proposed that at low hadronic scales, the vacuum saturation approximation ( $B_{\text{I}} = 1$  and  $\alpha_{\text{I}} = 0$ ) makes sense. However, this approach cannot explain the measured  $(\Lambda_{\text{b}})/B$  ratio. Some arguments suggest that vacuum saturation may be a poor approximation, particularly from naive large  $N_{\text{c}}$  analysis indicating  $\alpha_{\text{I}} \sim 1/N_{\text{c}} \sim 0.3$ . We explore the violation of vacuum saturation in detail using QCD sum rules within HQET.

We first consider the parameters  $B_{\text{I}}$ . The following three-point Green's function is constructed:

$$\Gamma_{\text{O}}(\omega, \mathbf{v}) = i^2 \int d^4x d^4y e^{i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k} \cdot \mathbf{y}} \langle 0 | T[ \bar{q}(\mathbf{x}) \gamma_{\text{I}} \hat{h}_{\mathbf{v}}(\mathbf{b})(\mathbf{x}) ] O(0) [ \bar{q}(\mathbf{y}) \gamma_{\text{I}} \hat{h}_{\mathbf{v}}(\mathbf{b})(\mathbf{y}) ]^\dagger | 0 \rangle,$$

where  $\omega = 2\mathbf{v} \cdot \mathbf{k}$ ,  $\mathbf{v} = 2\mathbf{v} \cdot \mathbf{k}$ ;  $\hat{h}_{\mathbf{v}}(\mathbf{b})$  is the b-quark field in HQET with velocity  $\mathbf{v}$ . The operator  $O$  denotes the color-singlet operators from Eq. (2):

$$O = \bar{b} \Gamma_{\text{I}} q \bar{q} \Gamma_{\text{I}} b,$$

with  $\Gamma_{\text{I}} = \Gamma_{\text{I}} = \gamma_{\text{I}} (1 - \gamma_5)$  for  $B_{\text{I}}$  and  $\Gamma_{\text{I}} = 1 - \gamma_5$ ,  $\Gamma_{\text{I}} = 1 + \gamma_5$  for  $B$ . In terms of hadronic contributions, the parameter  $B_{\text{I}}$  appears in the ground-state contribution:

$$\Gamma_{\text{O}}(\omega, \mathbf{v}) = B_{\text{I}} / [(2\Lambda_{\text{b}} - \omega)(2\Lambda_{\text{b}} + \omega)] + \text{resonances},$$

where  $\Lambda_{\text{b}} = m_{\text{B}} - m_{\text{b}}$ . The resonance contribution is simulated by perturbative QCD contributions above some threshold energy based on local duality.

On the other hand, this Green's function is calculated using quarks and gluons via the operator product expansion in QCD. The essential feature of QCD sum rules is that vacuum condensates of quarks and gluons must be included. Practically, the calculation is performed at around 1 GeV, where only a few lowest-dimension condensates are important. This calculation is reinforced by Borel transformation, and its consistency is verified by finding a sum rule window, as will be explained in detail for the  $\alpha_{\text{I}}$  parameters.

The calculation of  $\Gamma_{\text{O}}(\omega, \mathbf{v})$  in HQET is straightforward. The fixed-point gauge is adopted, and factorizable four-quark condensates are assumed (violations will be discussed later). The dominant non-vanishing Feynman diagrams are shown in Fig. 1 [Figure 1: see original paper], where double lines denote the heavy quark. However, all these diagrams are factorizable—they do not produce any deviation from vacuum saturation. This remains true even when  $O(\alpha_s)$  radiative corrections are included, simply because  $O$  is a color-singlet operator. For the same reason, there is no nonfactorizable gluon condensate contribution. Tadpole diagrams with contracted light quark lines from the four-quark vertex have been subtracted. Generally, nonfactorizable diagrams for a color-singlet four-quark operator are listed in Fig. 2 [Figure 2: see original paper]. These would be the leading diagrams that could give  $O(1)$  corrections to  $B_{\text{I}} = 1$ , but

they vanish due to the special structure of  $\Gamma$  and  $\Gamma$  in  $O$  (Eq. (5)). Thus, vacuum saturation is valid at hadronic scale for color-singlet operators:  $B_i = 1$  ( $i = 1, 2$ ) through leading-order consideration.

To what extent do nonfactorizable effects cause  $B_i$  to deviate from unity? The next possibility is two-gluon exchange, as shown in Fig. 3 [Figure 3: see original paper]. Interestingly, the four-gluon condensate  $\langle s^2 G^2 \rangle$  contribution to  $\Gamma_O(\mu)$ , such as Fig. 3(b), vanishes. The perturbative two-gluon exchange diagram, Fig. 3(a), may provide the leading non-vanishing nonfactorizable contribution (other diagrams with similar magnitude, like Fig. 3(c), also exist). This four-loop diagram contributes to  $B_i$  at order  $(\langle s(1 \text{ GeV}) \rangle)^2$ , numerically a few percent—too small to significantly affect hadron lifetimes. We obtain:

$$B_i = 1 + O(10^{-2}), \quad (i = 1, 2).$$

The parameters  $\langle \dots \rangle_i$  ( $i = 1, 2$ ), as our analysis shows, deviate from vacuum saturation expectations at hadronic scale by amounts that cannot be neglected for B meson lifetimes. The procedure parallels that for  $B_i$ . The three-point Green's function is constructed as:

$$\Gamma_T(\mu) = i^2 \int d^4x d^4y e^{i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k} \cdot \mathbf{y}} \langle 0 | T[ \bar{q}(x) \hat{h}_v(b)(x) ] T(0) [ \bar{q}(y) \hat{h}_v(b)(y) ]^\dagger | 0 \rangle,$$

where  $T$  represents the color-octet operators from Eq. (2):

$$T = \bar{b} \Gamma \hat{t}^a q \bar{q} \Gamma \hat{t}^a b.$$

In hadronic language, the parameter  $\langle \dots \rangle_i$  appears in the ground-state contribution:

$$\Gamma_T(\mu) = \langle \dots \rangle_i / [(2\Lambda - \mu)(2\Lambda + \mu)] + \text{resonances}.$$

In calculating  $\Gamma_T(\mu)$ , all condensates with dimension lower than 6 are retained. The dominant diagrams are those in Fig. 4 [Figure 4: see original paper], which are non-vanishing. The four-quark condensate  $\langle \bar{q}q \rangle^2$  diagram vanishes. Perturbative three-loop diagrams of order  $\langle s \rangle$  are neglected, meaning QCD radiative corrections to  $\langle \dots \rangle_i$  are not included. From general QCD sum rule experience, condensate diagrams dominate over corresponding perturbative contributions. This neglect is expected to be viable. Condensates parameterize nonperturbative effects that at the 1 GeV scale are small enough to be treated as power corrections in  $1/\mu$  and  $1/\mu^2$  in the operator product expansion. Since perturbative contributions are neglected, resonances in Eq. (10) are also neglected per duality. While the calculation is justified for large  $(-)$  and  $(-)$ , hadron ground-state properties require small values. These contradictory requirements are reconciled through double Borel transformation in  $\mu$  and  $\mu^2$ . Two Borel parameters,  $T$  and  $T^2$ , correspond to  $\mu$  and  $\mu^2$ , respectively. They appear symmetrically, so we take  $T = T^2$ .

The sum rules for parameters  $\langle \dots \rangle_i$  are:

$$= [m^2 \bar{q}q / (4F_B^2)] e^{\{4\Lambda/T\}},$$

$$= -[ \chi_s GG / (12F_B^2)] T + [m^2 \bar{q}q / (4F_B^2)] e^{\{4\Lambda/T\}}.$$

There is no gluon condensate contribution to  $\chi_s$ . Numerically, we use:

$$\bar{q}q = -(0.23 \text{ GeV})^3,$$

$$\chi_s GG = 0.04 \text{ GeV}^4,$$

$$g \bar{q} \chi_s = \chi_s G^2 = m^2 \bar{q}q, m^2 = 0.8 \text{ GeV}^2.$$

For consistency, the HQET sum rule for parameter  $F_B$  [15, 16] is used. The result from Ref. [16] is:

$$F_B^2 e^{\{-2\Lambda/T\}} = (3/8) \chi_c d^2 e^{\{-\Lambda/T\}} - [\bar{q}q / 4] (1 + m^2/2T^2),$$

where  $\chi_c$  is twice the continuum threshold, determined as  $\chi_c = 2.0 \pm 0.3 \text{ GeV}$ . The Borel parameter  $T$  range in Eq. (11) should be similar to that for  $F_B$ , as evident from the  $B_i$  sum rules where  $B_i = 1$  only if the Borel parameter matches  $F_B$ 's when all non-factorizable contributions are neglected. Practically, we take the window as  $0.7 < T < 1.0 \text{ GeV}$ . The  $\chi_i$  show no  $\Lambda$  dependence as it cancels out. Numerical results for  $\chi_i$  and  $\chi_s$  are given in Fig. 5 [Figure 5: see original paper]. The figures show poor stability in the  $T$  window because perturbative diagrams are omitted. The final results are:

$$-(4.1 \pm 2.2) \times 10^{-2},$$

$$(6.1 \pm 3.5) \times 10^{-2}.$$

Despite large uncertainties, these values significantly affect B meson lifetime differences. Vacuum saturation for color-octet matrix elements is indeed violated, with  $\chi_i$  magnitudes reaching order 0.1.

The factorization hypothesis for four-quark condensates requires discussion. We used this assumption in analyzing  $B_i$  and  $\chi_i$ . While standard in QCD sum rule calculations, its violation could imply new contributions to nonfactorizable effects in four-quark operator matrix elements. This contribution is estimated as:

$$B_i \chi_i = [\bar{q}q^2 / (4F_B^2)] e^{\{4\Lambda/T\}},$$

where  $\bar{q}q^2$  denotes deviation of four-quark condensates from factorization. Large  $N_c$  expansion arguments suggest this approximation is accurate to within  $1/N_c^2 \approx 10\%$  [17]. Numerically, given its success in baryon calculations [18], we take  $\bar{q}q^2$  as less than 30% of  $\bar{q}q^2$ . Then  $B_i$  and  $\chi_i$  are below  $10^{-2}$ , insignificantly affecting our  $B_i$  and  $\chi_i$  results.

Note that the vacuum saturation and its violation analyzed here are at hadronic scale, not  $m_b$  scale. Working in HQET, the natural scale is  $\Lambda_{had} \sim m_b$ . The renormalization group evolution of relevant operators was calculated in Refs. [19, 7]. Information on parameters  $B_i$  and  $\chi_i$  at scale  $m_b$  is needed for hadron lifetime analysis using Ref. [7]. There is a notational difference:  $F_B$  in Eq. (13) is scale-dependent when renormalization effects are considered. In our analysis,  $F_B$  takes values at hadronic scale, whereas in Ref. [7] it is at  $m_b$

scale. Accounting for this, with  $\alpha_s(\mu_{\text{had}}) = 0.5$  (corresponding to  $\mu_{\text{had}} = 0.67$  GeV), the evolution relations are:

$$\begin{aligned} B_i(m_b) &= B_i(\mu_{\text{had}}) - 0.24 \alpha_i(\mu_{\text{had}}), \\ \alpha_i(m_b) &= -0.05 B_i(\mu_{\text{had}}) + 0.72 \alpha_i(\mu_{\text{had}}). \end{aligned}$$

From Eq. (14), we obtain:

$$\begin{aligned} B(m_b) &= 1.01 \pm 0.01, \\ B(m_b) &= 0.99 \pm 0.01, \\ \alpha(m_b) &= -0.08 \pm 0.02, \\ \alpha(m_b) &= -0.01 \pm 0.03. \end{aligned}$$

For greater relevance, we discuss a more detailed parameterization for four-quark operators proposed in Ref. [6], motivated by model-independent determination of hadronic matrix elements from the lepton spectrum in the endpoint region of semileptonic B decays. The parameterization is:

$$\begin{aligned} B|\bar{b}(1-\gamma_5)q\bar{q}(1-\gamma_5)b|B &= (v_{\text{singl}}v_{\text{singl}} - g_{\text{singl}}g_{\text{singl}})F^2, \\ B|\bar{b}(1-\gamma_5)\hat{t}^a q\bar{q}(1-\gamma_5)\hat{t}^a b|B &= (v_{\text{oct}}v_{\text{oct}} - g_{\text{oct}}g_{\text{oct}})F^2, \end{aligned}$$

where  $v_{\text{singl}}$ ,  $g_{\text{singl}}$ ,  $v_{\text{oct}}$ ,  $g_{\text{oct}}$  are parameters (tadpole contributions subtracted). They relate to Eq. (2) parameters as:

$$\begin{aligned} v_{\text{singl}} - 4g_{\text{singl}} &= B, \\ v_{\text{oct}} - 4g_{\text{oct}} &= \alpha. \end{aligned}$$

In vacuum saturation,  $v_{\text{singl}} = 1$  and  $g_{\text{singl}} = v_{\text{oct}} = g_{\text{oct}} = 0$ . QCD sum rule calculations give:

$$\begin{aligned} v_{\text{singl}} &= 1 + O(10^{-2}), \\ g_{\text{singl}} &= O(10^{-2}). \end{aligned}$$

The color-octet operator sum rules yield:

$$\begin{aligned} v_{\text{oct}} &= -[\alpha_s GG / (48F_B^2)] T + [m^2 \bar{q}q / (4F_B^2)] e^{\{4\Lambda/T\}}, \\ g_{\text{oct}} &= -[\alpha_s GG / (192F_B^2)] T + [m^2 \bar{q}q / (16F_B^2)] e^{\{4\Lambda/T\}}. \end{aligned}$$

Numerical results for  $v_{\text{oct}}$  and  $g_{\text{oct}}$  are shown in Fig. 6 [Figure 6: see original paper], giving:

$$\begin{aligned} v_{\text{oct}} &= (1.35 \pm 0.76) \times 10^{-1}, \\ g_{\text{oct}} &= (0.74 \pm 0.40) \times 10^{-1}, \end{aligned}$$

typically of order 0.1. From the QCD sum rule perspective, vacuum saturation for color-singlet four-quark operator matrix elements in Eq. (5) or the first equation of (18) holds to a few percent. However, for color-octet operators, it is violated at the ten percent level. The more significant nonfactorizable effect for color-octet operators is qualitatively understandable from perturbative diagrams: the effect appears at three-loop level for color-octet versus four-loop for color-singlet operators.

The  $1/m_b$  corrections to these results can be analyzed in principle. While formally  $O(1/m_b)$  effects for b-hadron lifetimes, they have little influence on our arguments and calculations.

Ref. [6] discusses extracting parameters from B decays. Parameter  $g_{\text{singl}}$  might significantly affect comparisons of  $B \rightarrow X_u$  and  $B \rightarrow X_s$  decay rates. Our analysis suggests the rate difference should be small since  $g_{\text{singl}}$  is very small even at  $m_b$  scale. The ratio  $g_{\text{singl}}/g_{\text{oct}}$  can be obtained from the lepton spectrum in these decays. While the sign of  $g_{\text{singl}}$  is undetermined, our analysis favors a smaller ratio  $|g_{\text{singl}}/g_{\text{oct}}| \sim 1/10$  than Ref. [6]. These points will be tested experimentally soon.

The b-hadron lifetime ratios are expressed as [7]:

$$\begin{aligned} \tau(B_c)/\tau(B) &= 1 + 0.03B - 0.71 + 0.20 + O(1/m_b), \\ (\Lambda_b)/\tau(B) &= 0.98 - 0.17 + 0.20 - (0.013 + 0.022B)r + O(1/m_b), \end{aligned}$$

where parameters are at  $m_b$  scale and the B meson decay constant  $f_B$  is taken as 200 MeV. Using Eq. (17) results, we obtain:

$$\begin{aligned} \tau(B_c)/\tau(B) &= 1.09 \pm 0.02, \\ (\Lambda_b)/\tau(B) &= 0.98 \pm 0.01, \end{aligned}$$

where baryonic parameters  $r$  and  $B$  from QCD sum rules [8] are also evaluated at  $m_b$  scale. Eq. (23) shows that while  $\tau(B_c)/\tau(B)$  agrees with measurement, the QCD sum rule result for  $(\Lambda_b)/\tau(B)$  still contradicts experiment after including  $1/m_b^3$  corrections.

In summary, nonfactorizable contributions to hadronic matrix elements of four-quark operators relevant to B meson lifetimes have been studied via QCD sum rules in HQET. Vacuum saturation for color-singlet four-quark operators is justified at hadronic scale with nonfactorizable effects at a few percent level. However, vacuum saturation for color-octet four-quark operators is significantly violated, making nonfactorizable effects non-negligible for B meson lifetimes. Implications for extracting parameters from B decays have been discussed. The predicted B meson lifetime ratio  $\tau(B_c)/\tau(B) = 1.09 \pm 0.02$  is consistent with experiment, but the  $(\Lambda_b)/\tau(B)$  discrepancy remains unresolved. Higher  $1/m_b$  corrections are unlikely to provide a solution. If future experiments confirm the  $(\Lambda_b)/\tau(B)$  data, this may imply either local duality failure or new physics.

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## Figure Captions

- Fig. 1. Dominant non-vanishing Feynman diagrams for  $\Gamma_{\text{O}}(\ , \ )$ .  
Fig. 2. Nonfactorizable diagrams for a general color-singlet four-quark operator.  
Fig. 3. Nonfactorizable diagrams with two-gluon exchange for  $\Gamma_{\text{O}}(\ , \ )$ .  
Fig. 4. Condensate contributions to  $\Gamma_{\text{T}}(\ , \ )$ .  
Fig. 5. Sum rules for (a) and (b). The sum rule window is  $T = 0.7\text{--}1.0$  GeV.  
Fig. 6. Sum rules for  $v_{\text{oct}}$  (a) and  $g_{\text{oct}}$  (b).

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