1/Nc Expansion of the Heavy Baryon Isgur-Wise Functions Postprint

Authors: Chun Liu

Date: 2017-09-17T00:00:00+00:00

Abstract

The 1/Nc expansion of the heavy baryon Isgur-Wise functions is discussed. Because of the contracted SU(2Nf) light quark spin-flavor symmetry, the universality relations among the Isgur-Wise functions of b ! #3;c and #6;(∗) b ! #6;(∗)c are valid up to the order of 1/N2 c.

Full Text

Preamble

1/Nc Expansion of the Heavy Baryon Isgur-Wise Functions

Chun Liu
Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea
Email: liuc@ctp.snu.ac.kr

September 1997

Abstract

The $1/N_c$ expansion of the heavy baryon Isgur-Wise functions is discussed. Due to the contracted $SU(2N_f)$ light quark spin-flavor symmetry, the universality relations among the Isgur-Wise functions for Λ_c and $\Sigma^{(*)}$ decays are valid up to order $1/N_c^2$.

PACS: 11.15.Pg, 11.30.Hv, 12.39.Hg, 13.30.-a

Heavy baryons provide a testing ground for the Standard Model, particularly for Quantum Chromodynamics (QCD) in certain aspects. With the accumulation of experimental data on heavy baryons, important Standard Model parameters such as the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{cb} can be extracted by comparing experiments with theoretical calculations. The main difficulties in these calculations arise from our poor understanding of nonperturbative QCD. In this Brief Report, we discuss the $1/N_c$ expansion [?] for heavy baryon weak decay form factors and point out that it can be applied to relate different baryon Isgur-Wise functions with comparative accuracy.

Heavy baryon weak decays can be systematically studied using heavy quark effective theory (HQET) [?]. The classification of form factors parameterizing the hadronic matrix elements of weak currents is significantly simplified [?]. In the heavy quark limit, only one universal form factor remains to be determined for Λ_c transitions, and two for $\Sigma^{(*)}$ transitions. These universal form factors are called Isgur-Wise functions and must be calculated using some nonperturbative method.

The large N_c limit is one of the most important model-independent methods in nonperturbative QCD. Nonperturbative properties of mesons can be understood from the analysis of planar diagrams, and those of baryons from a Hartree-Fock picture. Recently, there has been renewed interest in large N_c applications to baryons [?]. It has been pointed out that there exists a contracted $SU(2N_f)$ light quark spin-flavor symmetry in the baryon sector in the large N_c limit. This observation of light quark spin-flavor symmetry has led to many quantitative applications [?]. In the large N_c limit, relations among the baryon Isgur-Wise functions have been studied [?]. With the definitions

$$\langle \Lambda_c(v',s')|\bar{c}\Gamma b|\Lambda_b(v,s)\rangle = \eta(y)\,\bar{u}_{\Lambda_c}(v',s')\Gamma u_{\Lambda_b}(v,s), \tag{1}$$

$$\langle \Sigma_c^{(*)}(v',s')|\bar{c}\Gamma b|\Sigma_b^{(*)}(v,s)\rangle = [\xi(y)g_{\mu\nu} + \zeta(y)v_\nu v'_\mu]\,\bar{u}^\nu(v',s')\Gamma u^\mu(v,s), \tag{2}$$

where $y = v\cdot v'$, and $u^\mu_{\Sigma_Q}(v,s)$ is the Rarita-Schwinger spinor defined by

$$u^\mu_{\Sigma_Q}(v,s) = (\gamma^\mu + v^\mu)\gamma_5\, u_{\Sigma_Q}(v,s), \tag{3}$$

the Isgur-Wise functions $\eta(y)$, $\xi(y)$ and $\zeta(y)$ satisfy the following large N_c relations [?, ?]:

$$\eta(y) = \xi(y) = (y+1)\zeta(y). \tag{4}$$

We study the $1/N_c$ expansion of the relations among the baryon Isgur-Wise functions. This is interesting because of the observed light quark spin-flavor symmetry in the baryon sector in the large N_c limit. It is instructive to illustrate the large N_c baryon Isgur-Wise functions in a naive way that can be directly applied to discussions of $1/N_c$ corrections. In the heavy quark limit, the heavy quark in a heavy hadron has a fixed velocity identical to the heavy hadron velocity, and the heavy quark spin decouples from its strong interaction with the light quark system in the hadron. The light quark system cannot see any properties of the heavy quark except its velocity. In this case, the spin J_l and isospin I_l

of the light quark system become good quantum numbers for describing heavy baryons in which quarks have no orbital angular momentum excitations in the constituent picture and there are only two flavors of light quarks. For N_c colors, the baryons of interest have $(I_l, J_l) = (0,0), (1,1), \ldots, (\frac{N_c-1}{2}, \frac{N_c-1}{2})$. When the heavy quark velocity changes from v to v' due to weak decay, the "brown muck" must undergo a transition through strong interaction from the heavy quark. The Isgur-Wise functions defined in Eqs. (1) and (2) measure the amplitudes of these brown muck transfers, though they cannot be determined further from HQET. It is at this stage that the large N_c method is applied.

In the large N_c limit, there is an $SU(4)$ light quark spin-flavor symmetry for baryons [?]. During the transition, the spin of any light quark in the brown muck is conserved; in other words, the light quark spin individually decouples from the strong interaction in the brown muck transition. The Isgur-Wise functions are independent of the light quark spin configuration in the brown muck in the large N_c limit and therefore admit an $SU(4)$ expansion. At leading order in the $SU(4)$ expansion, the relations given by Eq. (3) still hold. This is the essential point in deriving the large N_c universality of the baryon Isgur-Wise functions for Λ_c and $\Sigma^{(*)}$ decays in Ref. [?] based on $SU(4)$ symmetry.

Let us now consider corrections to this contracted $SU(4)$ light quark spin-flavor symmetry result. These corrections appear only for finite N_c. We note that the baryon spectrum satisfies the relation $I_l = J_l$. Therefore the Isgur-Wise functions have the following expansions:

$$\eta(y) = \tilde{\eta}_0(y), \tag{5}$$

$$\xi(y) = \tilde{\eta}_0(y) + \tilde{\xi}(y)\,\frac{J_l^2}{N_c^2}, \tag{6}$$

$$(y+1)\zeta(y) = \tilde{\eta}_0(y) + \tilde{\zeta}(y)\,\frac{J_l^2}{N_c^2}, \tag{7}$$

where $\tilde{\eta}_0(y)$ is the leading $SU(4)$ symmetry result that is independent of the brown muck spin or isospin, while $\tilde{\xi}(y)$ and $\tilde{\zeta}(y)$ parameterize the $SU(4)$ breaking effects and are normalized to be of order 1 in the large N_c limit. The factor N_c^2 must be included to maintain the correct N_c scaling for the Isgur-Wise functions. In the extreme case where all light quark spins in the baryon align in the same direction, J_l^2 scales as $N_c^2/4$. Only by dividing by a factor of N_c^2 do the terms proportional to J_l^2 in the above equations have the correct N_c scaling. Note that there are no terms with linear dependence on J_l in the corrections. This is simply because there is no way to combine J_l with $(v-v')^\mu$ into a Lorentz and CP invariant quantity. Generally, the Isgur-Wise function should depend on J_l/N_c or I_l/N_c of the brown muck undergoing the transition. However, it is interesting to note that the Isgur-Wise function $\eta(y)$ has no corrections in the $SU(4)$ expansion because the Λ_Q baryon is an $SU(4)$ singlet. From Eq. (4), we see that the $SU(4)$ symmetry relations (3) are valid up to order $1/N_c^2$.

It is helpful to compare this framework with the heavy quark Skyrme model [**?**, **?**], which is often believed to be the large N_c HQET. The model predicted an exponential form for the Isgur-Wise function $\eta(y)$ [**?**], but a method to calculate its $1/N_c$ corrections is not available. On the other hand, as is well-known, there are $O(1/N_c)$ contributions in $\tilde{\eta}_0(y)$. While we have no knowledge about $\tilde{\eta}_0(y)$ itself, the predicted universality relations of the Isgur-Wise functions, Eq. (4), are valid up to order $1/N_c^2$. (The analogous situation holds for the case of baryon masses [**?**, **?**].) If we focus on the relations among the Isgur-Wise functions, the $SU(4)$ expansion provides a better framework. In the near future, the Isgur-Wise function $\eta(y)$ can be extracted from experimental data on $\Lambda_b \to \Lambda_c$ semileptonic decay. With this information, the weak decays $\Omega_c^{(*)}$ can be predicted to a comparatively accurate level in the chiral $SU(3)$ symmetry limit.

Finally, we add a remark on the $1/m_Q$ expansion for the form factors. HQET provides a systematic method for this expansion [**?**]. What we have discussed in this note is only the leading order of the heavy quark expansion. At order $1/m_Q$, there is an additional universal form factor for the $\Lambda_b \to \Lambda_c$ transition. This form factor vanishes in the limit $N_c \to \infty$ and is subject to an uncertainty of order $\Lambda_{\mathrm{QCD}}/N_c m_Q$, which is numerically of the same order as $1/N_c^2$. Therefore, when we extract $\eta(y)$ from Λ_c semileptonic decay, this uncertainty does not spoil the accuracy we hope to achieve.

I am grateful to Mark Wise for helpful discussions. This work was supported by the Korea Science and Engineering Foundation through the SRC program.

References

[1] G. ’ t Hooft, Nucl. Phys. B72 (1974) 461; E. Witten, Nucl. Phys. B160 (1979) 57.
[2] N. Isgur and M.B. Wise, Phys. Lett. B 232 (1989) 113, B 237 (1990) 527; H. Georgi, Phys. Lett. B 240 (1990) 447.
[3] N. Isgur and M.B. Wise, Nucl. Phys. B 348 (1991) 276; H. Georgi, Nucl. Phys. B 348 (1991) 293; T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B 355 (1991) 38; Y.B. Dai, X.H. Guo and C.S. Huang, Nucl. Phys. B412 (1994) 277.
[4] R.F. Dashen and A.V. Manohar, Phys. Lett. B315 (1993) 425, 438; E. Jenkins, Phys. Lett. B315 (1993) 431, 441, 447; R.F. Dashen, E. Jenkins and A.V. Manohar, Phys. Rev. D49 (1994) 4713, D51 (1995) 3697.
[5] C. Carone, H. Georgi and S. Osofsky, Phys. Lett. B322 (1994) 227.
[6] M.A. Luty and J. March-Russell, Nucl. Phys. B426 (1994) 71.
[7] J.-L. Gervais and B. Sakita, Phys. Rev. D30 (1984) 1795.
[8] E. Jenkins and R.F. Lebed, Phys. Rev. D52 (1995) 282; J. Dai, R.F. Dashen, E. Jenkins and A.V. Manohar, Phys. Rev. D53 (1996) 273; E. Jenkins, Phys. Rev. D53 (1996) 2625.
[9] E. Jenkins, Phys. Rev. D54 (1996) 4515.
[10] C. Carone, H. Georgi, L. Kaplan and D. Morin, Phys. Rev. D50 (1994) 5793.

[11] C.K. Chow, Phys. Rev. D51 (1995) 1224.
[12] C.K. Chow, Phys. Rev. D54 (1996) 873.
[13] D.E. Brahm and J. Walden, Mod. Phys. Lett. A12 (1997) 357.
[14] C. Liu, Phys. Lett. B389 (1996) 347; Seoul Nat. Univ. preprint Report No. SNUTP.
[15] E. Jenkins, A.V. Manohar and M.B. Wise, Nucl. Phys. B396 (1993) 27.
[16] C.G. Callan and I. Klebanov, Nucl. Phys. B262 (1985) 222; M. Rho, D.O. Riska and N.N. Scoccola, Phys. Lett. B251 (1990) 597; Z. Phys. A341 (1992) 343; Y. Oh, D.-P. Min, M. Rho and N.N. Scoccola, Nucl. Phys. A534 (1991) 493.
[17] E. Jenkins, A.V. Manohar and M.B. Wise, Nucl. Phys. B396 (1993) 38.
[18] M.E. Luke, Phys. Lett. B 252 (1990) 447.

Note: Figure translations are in progress. See original paper for figures.