Cosmological Evolution of Dirac-Born-Infeld Field: Postprint

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Full Text

Preamble

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Abstract

We investigate the cosmological evolution of a system composed of a Dirac-Born-Infeld field plus a perfect fluid. We analyze the existence and stability of scaling solutions for the AdS throat and quadratic potential. We find that scaling solutions exist when the equation of state of the perfect fluid is negative and in the ultra-relativistic limit.

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1 Introduction

Inflation in the early universe provides a natural explanation for the homogeneity and isotropy of the universe and for the observed spectra of density perturbations. Recently, inflationary models from string theory have attracted much attention. One approach to string inflation is based on D-branes [?]. Of particular interest are scenarios where a type IIB orientifold is compactified on a Calabi-Yau three-fold, where the moduli fields are stabilized due to the presence of non-trivial flux. These fluxes generate local regions within the Calabi-Yau space with a warped geometry or "throat." In many settings, an anti-D3-brane is fixed at one location in the infrared tip of the throat and a mobile D3-brane experiences a small attractive force towards the anti-D3-brane. The distance between the branes plays the role of the inflaton field and, since this is an open string mode, its dynamics is determined by a Dirac-Born-Infeld (DBI) action. Such a DBI action with higher derivative terms gives rise to a variety of novel cosmological consequences [?, ?, ?, ?].

It is well known that in a universe containing a perfect fluid and a normal scalar field with an exponential potential, for a wide range of parameters the scalar field mimics the perfect fluid with the same equation of state [?]. Scaling solutions in which the ratio of the energy densities of the two components remains constant are realized in such systems and serve as attractors at late times. In tachyon cosmology, the inverse square potential for a tachyon field allows similar scaling solutions, just as the exponential potential does for a normal scalar field [?]. This type of scaling solution is useful for explaining the current acceleration of the universe. It is thus interesting to investigate whether scaling solutions are also present and stable in the DBI scenario.

In this paper, we undertake the first attempt to study a system of dimensionless dynamical variables for the DBI field plus a perfect fluid using the phase-plane analysis method that has been widely applied [?, ?, ?]. In the case of the AdS throat and quadratic potential, the system can be cast into an autonomous system. We find that in addition to the DBI inflationary solutions, scaling solutions exist in the ultra-relativistic case. We analyze their existence and stability.

2 Autonomous System

Consider the following effective action [?]:

$$f(\phi)(cid:20)1 + f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi f(\phi) - 1 + V(\phi) + S_m, (cid:21)$$

where g_{YM}^2 is the Yang-Mills coupling and $V(\phi)$ is a potential of the DBI field ϕ. In the case of the AdS throat, we have $f(\phi) = \lambda/\phi^4$, where λ is the ' t Hooft coupling which is related to $g_{YM}^2 N$ in the large-N limit of the field theory. In the action (1), we have also taken into account the contribution of a perfect fluid.

In a spatially-flat Friedmann-Robertson-Walker (FRW) metric, the energy density and pressure of the DBI field are given by:

$$+V(\phi), \quad f\gamma - V(\phi), \quad f(\phi)\dot{\phi}^2$$

where

$$\gamma = \frac{1}{\sqrt{1 - f(\phi)\dot{\phi}^2}}.$$

The field equations read:

$$+V(\phi)+\rho_m, \quad \gamma\dot{\phi}^2+(1+w_m)\rho_m, \quad H^2 = \frac{3}{f}, \quad \phi f^2+\dot{\rho}_m+3H(1+w_m)\rho_m = 0, \quad V_{,\phi}+\gamma^3 = 0,$$

where a dot denotes a derivative with respect to t and $\kappa^2 = 1/(g_p^2)$ with M_p being the reduced Planck mass. Note that ρ_m and P_m are the energy density and pressure of the fluid with an equation of state $w_m = P_m/\rho_m$.

We define the following variables:

$$\sqrt{3}Hs, \quad \kappa\dot{\phi}\sqrt{\gamma}, \quad \kappa f^{5/2}V^{3/2}, \quad \mu_1(\phi)\kappa f^{1/2}V^{3/2}, \quad \mu_2(\phi)$$

From the Friedmann equation (5), we have the constraint equation:

$$3H^2 = \frac{1}{\tilde{\gamma}}x^2, \quad \frac{1}{\gamma} = \frac{y^2}{3x^2}.$$

The energy fraction and equation of state of the DBI field ϕ are given by:

$$\Omega_\phi = (1-\tilde{\gamma})x^2 + z^2, \quad \tilde{\gamma}x^2, \quad \tilde{\gamma}x^2 + z^2.$$

From Eq. (6) we obtain:

$$(1+w_m)\tilde{\gamma}x^2, \quad P_m + P_\phi, \quad \rho_m + \rho_\phi, \quad w_{\text{eff}} = y^2 + (1+w_m)\tilde{\gamma}x^2,$$

where a prime represents a derivative with respect to the number of e-foldings $N = \ln a$.

The effective equation of state, $w_{\text{eff}} = 2H'/3H$, is:

$$(1+w_m)\tilde{\gamma}x^2, \quad P_m + P_\phi, \quad \rho_m + \rho_\phi, \quad w_{\text{eff}} = y^2 + (1+w_m)\tilde{\gamma}x^2.$$

Taking the derivative of x, y, z, $\mu_1(\phi)$ and $\mu_2(\phi)$ with respect to N, we obtain the following equations:

$$(1+w_m)1+\tilde{\gamma}^2\tilde{\gamma}x^2(\mu_1+\mu_2)\mu_1+(1-\tilde{\gamma})x^2, \quad (1+w_m)\tilde{\gamma}x^2 1+\tilde{\gamma}^2, \quad h\left[1 = \mu_2^2 = \mu_2 V V_{,\phi\phi} f f_{,\phi\phi} f_{,\phi} - \right],$$

$$(1+w_m)\tilde{\gamma}x^2 e^{\alpha\phi},$$

where α is a constant. The set of Eqs. (16)-(18) becomes an autonomous system if both μ_1 and μ_2 are constants. Actually, when μ_1 is a constant, the potential is obtained by integrating Eq. (9):

$$\left(\frac{f^{1/2}d\phi}{V^{3/2}}\right).$$

For the AdS throat ($f = \lambda/\phi^4$), Eq. (21) gives:

$$V(\phi) = 1 + c\phi!,$$

where c is an integration constant. In the region where this potential reduces to the quadratic one: $V(\phi) \approx 1$, this potential reduces to the quadratic one: $V(\phi) = m^2\phi^2/2$.

In what follows, we specialize to the case of the AdS throat, $f(\phi) = \lambda/\phi^4$, and the quadratic potential, $V(\phi) = m^2\phi^2/2$. In this case, μ_1 is a constant and $\mu_2 = 2\mu_1\tilde{\gamma}x^2z^{-2}$. The evolution Eqs. (16)-(18) can be written as the following autonomous system:

$$1+\tilde{\gamma}^2, \quad 2\tilde{\gamma}(1-\tilde{\gamma})x^2, \quad (1+w_m)1+\tilde{\gamma}^2, \quad [\,], \quad (1+w_m)\tilde{\gamma}x^2, \quad (1+w_m)\tilde{\gamma}x^2, \quad \tilde{\gamma}x^2,$$

where $\mu_1 = 2\sqrt{2}/(\kappa\sqrt{\lambda}m)$.

3 Scaling Solutions

One can derive the fixed points of the system (23)-(25) by setting $x' = 0$, $y' = 0$ and $z' = 0$. The fixed points correspond to an expanding universe with a scale factor $a(t) \propto t^p$, where $p = 2/[y^2 + 3(1 + w_m)\Omega_m]^{-1}$. From Eq. (25) we find that there are two cases: (i) $z = 0$ and (ii) $y^2 + 3(1 + w_m)[1 - \mu_1 yz/x]z^2 = \tilde{\gamma}x^2$. We will study the case $\dot{\phi} < 0$, i.e., $y < 0$.

In case (i) we have the following fixed points:

(A) Fluid-dominated solutions $(x, y, z) = (0, 0, 0)$, $\Omega_m = 1$, $w_{\text{eff}} = w_m$.

(B) Kinetic-dominated solutions $(x, y, z) = (1, \sqrt{3}, 0)$, $\Omega_m = 0$, $w_{\text{eff}} = 0$.

The fixed point (A) represents fluid-dominated solutions since $\Omega_m = 1$. The fixed point (B) corresponds to kinetic-dominated solutions that behave like dust (i.e., non-relativistic matter), which are power-law expanding solutions with $a \propto t^{2/3}$.

In case (ii) we have either $\mu_1(z^2 - \tilde{\gamma}x^2)z + \mu_1 x^2 z + xy = 0$ or $y = 0$ from Eqs. (23)-(25). In the former situation, we obtain either $y^2 = 3x^2$ (i.e., $\tilde{\gamma} = 0$) or $x^2(1 - 2\tilde{\gamma}) = 0$ by using Eq. (24). When $y^2 = 3x^2$, the fixed points are given by:

(C) Accelerated solutions $x = [\mu_1(1 + 12\mu_1)/6]^{1/2}$, $z = \sqrt{3}(1 + 12\mu_1)/6$, $\Omega_m = 0$, $w_{\text{eff}} = 1 + \mu_1(1 + 12\mu_1)/6$.

(D) Scaling solutions $x = [3(1+w_m)^3/(w_m\mu_1^2 - 1)]^{1/2}$, $z = \sqrt{3}(1+w_m)/\mu_1$, $\Omega_m = 1 + 3(1+w_m)^2/(w_m\mu_1^2 - 1)$, $w_{\text{eff}} = w_m$.

Both fixed points (C) and (D) exist in the ultra-relativistic region: $\gamma \gg 1$. These solutions are selected by the condition that $x > 0$, $z > 0$ and $\Omega_m \geq 0$ in the expanding universe. This requires $-1 < w_m < 0$ and $\mu_1 > \sqrt{-3/w_m(1+w_m)}$ for point (D). The fixed point (C) leads to accelerated expansion for $\mu_1 < 2$, which was proposed as an alternative to slow-roll inflation [**?**, **?**]. In such models, inflation may proceed even when the field is rolling relatively fast. The fixed point (D) corresponds to scaling solutions in which the ratio of their densities is a non-trivial constant. Note that even when μ_1 changes with time, the fixed points (C) and (D) can be regarded as "instantaneous" fixed points.

Under the condition $\tilde{\gamma} = 0$, the relation $x^2(1 - 2\tilde{\gamma}) = 0$ gives a fixed point that is not significantly different from point (A). Since accelerated expansion is not realized in this case, it is not of interest to us.

To analyze their stability, we substitute linear perturbations about the fixed points into the field equations (23)-(25). To first order in the perturbations, we obtain two independent equations of motion for $\tilde{\gamma} = 0$. If their eigenvalues are both negative, the fixed point is stable. For fixed point (A), we obtain two eigenvalues $\lambda_1 = 3(1+w_m)/2$ and $\lambda_2 = 3w_m$, which indicate that it is unstable for $-1 < w_m < 1$. For fixed point (B), we obtain two eigenvalues that indicate it is also unstable. For fixed point (C), the two eigenvalues are $\lambda_1 = 3/2$ and $\lambda_2 = 3w_m/2$, which indicate that it is stable for $\mu_1 < \sqrt{-3/w_m(1+w_m)}$. For point (D), the two eigenvalues are:

$$\frac{4}{1+(3w_m+1)^2}\left[24(1+w_m)^3/\mu_1^2 - 1 + (3w_m+1)^2\right],$$

$$\frac{4}{1+(3w_m+1)^2}\left[24(1+w_m)^3/\mu_1^2 - 3/w_m(1+w_m)\right].$$

Thus the scaling solutions are always stable when they exist for $\mu_1 > \sqrt{-3/w_m(1+w_m)}$.

Different regions in the (w_m, μ_1) parameter space lead to different qualitative evolution, as shown in Fig. 1 [Figure 1: see original paper]. In region I, all four fixed points exist and fixed point (D) is the attractor solution. In region II, fixed point (D) does not exist and fixed point (C) is the attractor solution.

4 Conclusions and Discussions

We have investigated the cosmological evolution for a spatially-flat FRW universe containing a Dirac-Born-Infeld field and a perfect fluid. We find that the field equations can be cast into an autonomous system (23)-(25) in the case of the AdS throat and quadratic potential. In addition to the DBI inflationary solutions (C), there exist scaling solutions (D) in which the ratio of the energy densities of the two components is a constant. We have analyzed the existence and stability of the fixed points and shown that the scaling solutions (D) exist and are stable when the equation of state of the perfect fluid satisfies $-1 < w_m < 0$, for $\mu_1 > \sqrt{-3/w_m(1+w_m)}$, and in the ultra-relativistic regime (i.e., $\tilde{\gamma} = 0$).

There is another string-motivated choice of the warp factor, namely a constant f. This corresponds to the case where inflation proceeds in the angular directions instead of the radial [**?**]. In this case μ_2 vanishes and μ_1 becomes a constant for an inverse square potential. In the ultra-relativistic region, all results are the same as those derived above.

Given a warp factor $f(\phi)$ and a potential term $V(\phi)$, the set of equations (16)–(18) can in principle be written as an autonomous system since both $\mu_1(\phi)$ and $\mu_2(\phi)$ in the equation set can be expressed in terms of the variables x and z. It is worth studying further the cosmological dynamics of general functions $f(\phi)$ and $V(\phi)$ to explain the present acceleration of the universe. We note that the dynamics of tachyon actions with a runaway potential contain caustics with multi-valued regions because high-order spatial derivatives of the tachyon field become divergent [**?**]. Here we do not consider a runaway potential but a quadratic one, which may stabilize the system (1). Checking whether this expectation is truly valid is an interesting problem that we leave for future study.

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