Black Holes in the Dilatonic Einstein-Gauss-Bonnet Theory in Various Dimensions I–Asymptotically Flat Black Holes -Postprint

Authors: Zong-Kuan GUO, Nobuyoshi Ohta, Takashi Torii

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Full Text

Preamble

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Zong-Kuan Guo, Nobuyoshi Ohta, and Takashi Torii
Department of Physics, Kinki University, Higashi-Osaka, Osaka 577-8502, Japan
Department of General Education, Osaka Institute of Technology, Asahi-ku, Osaka 535-8585, Japan

Abstract

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field equations for a suitable ansatz in general D dimensions and construct black hole solutions of various masses numerically in D = 4, 5, 6, and 10 dimensional spacetime with a (D−2)-dimensional hypersurface of positive constant curvature. A detailed comparison with non-dilatonic solutions is made, and we examine the thermodynamic properties of the solutions. It is found that the dilaton has significant effects on the black hole solutions, and we discuss the physical consequences.

Introduction

One of the most important problems in theoretical physics is the formulation of a quantum theory of gravity and its application to physical systems to understand physics in strong gravity regimes. The leading candidates that include all fundamental forces of elementary particles are ten-dimensional superstring theories. Areas where such quantum gravity plays a significant role include cosmology and black hole physics.

There has been interest in applying string theory to these subjects. Since it is still difficult to study geometrical settings in superstring theories, most analyses have been performed using low-energy effective theories inspired by string theory. These effective theories are supergravities which typically involve not only the metric but also the dilaton field (as well as several gauge fields).

The first attempt at understanding black holes in the Einstein-Maxwell-dilaton system was made in Refs. [1, 2], in which a static spherically symmetric black hole solution with dilaton hair was found. After this, many solutions were discussed in various models. On the other hand, it is known that there are correction terms of higher orders in curvature to the lowest effective supergravity action coming from superstrings. The simplest correction is the Gauss-Bonnet (GB) term coupled to the dilaton field in the low-energy effective heterotic string [3]. (We ignore other gauge fields and forms for simplicity.) It is then natural to ask how black hole solutions are affected by the higher order terms in these effective theories.

When the dilaton is dropped or set to a constant, this correction is known as the first term (except for the cosmological constant and Einstein-Hilbert action) in Lovelock gravity [4, 5], which is the most general metric theory of gravity yielding conserved second-order equations of motion in an arbitrary number of dimensions D. It is a natural generalization of Einstein’ s general relativity (GR) to higher dimensions without ghosts, and for this reason it was conjectured [6] and indeed found to be the low-energy effective theory of strings. Motivated by these observations, there have been many works on black hole solutions in Lovelock theories [7-10]. In four-dimensional spacetime, the GB term does not give any contribution because it becomes a surface term and yields a topological invariant. Boulware and Deser [11] discovered static, spherically symmetric black hole solutions of such models in more than four dimensions. In systems with a negative cosmological constant, black holes can have horizons with non-spherical topology such as torus, hyperboloid, and other compactified submanifolds. These solutions were originally found in general relativity and are called topological black holes [12]. Topological black hole solutions were studied in GB gravity [13]. It is of interest to see how these are modified by the presence of a dilaton.

Another motivation for this work is the following. There has recently been renewed interest in these solutions for applications to the calculation of shear viscosity in strongly coupled gauge theories using black hole solutions in five-dimensional Einstein-GB theory via AdS/CFT correspondence [14]. Almost all these studies consider a pure GB term without a dilaton, or assume a constant dilaton, which is not a solution of the heterotic string. However, it is expected that AdS/CFT correspondence is valid within the effective theories of superstring. It is thus again important to investigate how the properties of black holes are modified when the dilaton is present. The inclusion of the dilaton was also considered by Boulware and Deser [15], but exact black hole solutions and their thermodynamic properties were not discussed. Callan et al. [16] considered black hole solutions in the theory with a higher-curvature term $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ and the dilaton field, and Refs. [17, 18] took both the GB term and the dilaton field into account in four-dimensional spacetime.

This is the first of a series of papers in which we study black holes in the dilatonic Einstein-GB theory in D dimensions with a (D−2)-dimensional Euclidean manifold of constant curvature and signature $k = 1, 0$. In this paper, we consider asymptotically flat black hole solutions in static spherically symmetric ($k = 1$) spacetime and discuss their thermodynamic properties. In subsequent publications, we intend to study $k = 0$ topological black hole solutions with possible cosmological constant, including asymptotically anti-de Sitter solutions, which should be useful for studying the dynamics of strongly coupled gauge theories.

This paper is organized as follows. In Section 2, we introduce the action of the dilatonic Einstein-GB theory and our metric for D-dimensional spacetime, then summarize the field equations. In Section 3, we focus on an asymptotically flat Schwarzschild-type metric and impose boundary conditions for the metric functions and dilaton field at the event horizon and spatial infinity. In Sections 4 through 8, we present black hole solutions for D = 4, 5, 6, and 10 cases. In Section 5, we discuss D = 4 black hole solutions for various dimensions with dilaton coupling $\gamma = 1$ and recover the results of Ref. [18] in subsection 5.1. In subsection 5.2, we discuss solutions for dilaton coupling $\gamma = 1/2$, which is the value adopted mainly in this paper, and find one notable difference from other dilaton couplings. That is, there are two solutions for a certain range of mass parameters at smaller values for dilaton coupling $\gamma = 1$, but no such region exists for coupling $\gamma = 1/2$. We find no other major differences between these cases in the behaviors of the metrics and dilaton fields. In particular, in both cases there is a lower bound on the mass and horizon for solutions to exist. On the other hand, we find that D = 5 solutions exhibit quite different properties, e.g., they exist even in the vanishing limit of the horizon radius, in contrast to four dimensions. This is presented in Section 6. We find that properties of solutions in D = 6 through D = 10 are very similar, so we present results only for D = 6 and 10 cases in Sections 7 and 8, respectively. In these sections, we also make detailed comparisons with the non-dilatonic case. In Section 9, we investigate the thermodynamic properties of dilatonic black holes with the GB term. Here again, we find that five-dimensional black holes have quite distinctive properties from other dimensional solutions. The heat capacity of the non-dilatonic black hole is negative for large mass but changes sign to be positive for smaller mass in five-dimensional solutions. However, it is always negative as the mass is varied if the dilaton field is added. In dimensions other than five, the heat capacity is always negative for both dilatonic and non-dilatonic black holes. Section 10 is devoted to conclusions.

2. Dilatonic Einstein-GB Theory

We consider the following low-energy effective action for the heterotic string:

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\partial_\mu \phi)^2 + \alpha' e^{-\gamma\phi} R^2_{\rm GB} \right] \tag{2.1}$$

where R is the scalar curvature, ϕ is the dilaton field, $R^2_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2$ is the Gauss-Bonnet combination, κ^2 is a numerical coefficient given in terms of the Regge slope parameter α', $G_D = \kappa^2/8\pi$ is the D-dimensional gravitational constant, and $\alpha_2 = \alpha'/8 (> 0)$. Here γ is the coupling constant of the dilaton field. There is ambiguity in the choice of this constant. If we first make dimensional reduction of the system in the string frame to D dimensions and then change to the Einstein frame, we would get $\gamma = 2/(D-2)$. This is the choice, for example, $\gamma = 1$ in Ref. [18] for $D = 4$. However, if we first go to the Einstein frame in ten dimensions with $\gamma = 1/2$ and then make the dimensional reduction, we get the value $\gamma = 1/2$ in any dimensions. Both choices have their own merit, but it is convenient to take the same value for our analysis of black holes in any dimensions. So in this paper we take the second viewpoint and choose $\gamma = 1/2$ mainly in our following study of black hole solutions. In order to see how the results depend on this choice, however, we also examine black hole solutions for both values of $\gamma = 1$ and $1/2$ in four dimensions. This also serves to check the consistency of our results with Ref. [18]. In fact we will find that the solutions exhibit quite similar behaviors although there are some differences in details. In order to make our formulae valid for any case, we will keep γ wherever possible.

Varying the action (2.1) with respect to $g_{\mu\nu}$, we obtain the gravitational equation:

$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2 + \partial_\mu \phi \partial_\nu \phi + \alpha_2 e^{-\gamma\phi} H_{\mu\nu} + 4(\gamma^2 \partial^\rho \phi \partial^\sigma \phi - \gamma \nabla^\rho \nabla^\sigma \phi) P_{\mu\rho\nu\sigma} = 0, \tag{2.2}$$

where

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{2.3}$$

$$H_{\mu\nu} \equiv 2\left[RR_{\mu\nu} - 2R_{\mu\rho}R^{\rho}{}_{\nu} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} + R_{\mu\rho\sigma\lambda}R_{\nu}{}^{\rho\sigma\lambda}\right] - \frac{1}{2} g_{\mu\nu} R^2_{\rm GB}, \tag{2.4}$$

$$P_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} + 2g_{\mu[\sigma}R_{\rho]\nu} + 2g_{\nu[\rho}R_{\sigma]\mu} + Rg_{\mu[\rho}g_{\sigma]\nu}. \tag{2.5}$$

$P_{\mu\nu\rho\sigma}$ is the divergence-free part of the Riemann tensor, i.e.,

$$\nabla^{\mu} P_{\mu\nu\rho\sigma} = 0. \tag{2.6}$$

The equation for the dilaton field is

$$\Box\phi + \alpha_2 \gamma e^{-\gamma\phi} R^2_{\rm GB} = 0, \tag{2.7}$$

where $\Box$ is the D-dimensional d' Alembertian.

To derive black hole solutions in this system, let us consider the line element in D-dimensional static spacetime

$$ds^2 = -e^{2u(r)}dt^2 + e^{2v(r)}dr^2 + r^2 h_{ij} dx^i dx^j, \tag{2.8}$$

where $h_{ij}dx^i dx^j$ represents the line element of a $(D-2)$-dimensional hypersurface with constant curvature of signature k and volume Σ_k for $k = 1, 0$. We consider $k = 1$ for the black hole solutions in this paper, but keep k wherever possible. The $(D-2)$-dimensional hypersurface with $k = 1$ is spherically symmetric in four-dimensional spacetime, but the $(D-2)$-dimensional hypersurface can have rich structure and is not necessarily homogeneous [19-21].

Our basic equations then give

$$\dot{u}^2 + \frac{2(D-3)}{r}\dot{u} - \frac{k}{r^2}(1 - e^{2v}) + \alpha_2 e^{-2v-\gamma\phi}(D-3)A(r) = 0, \tag{2.9}$$

$$\ddot{u} + \dot{u}^2 - \dot{u}\dot{v} + \frac{D-3}{r}(\dot{u} - \dot{v}) + \frac{k}{r^2}(1 - e^{2v}) + \frac{1}{2}\dot{\phi}^2 + \alpha_2 e^{-2v-\gamma\phi}(D-3)A(r) = 0, \tag{2.10}$$

$$\ddot{\phi} + \dot{\phi}(\dot{u} - \dot{v}) + \frac{D-2}{r}\dot{\phi} + \alpha_2 \gamma e^{2v-\gamma\phi} R^2_{\rm GB} = 0, \tag{2.11}$$

where the dot denotes derivative with respect to r, and we have defined

$$A(r) \equiv \frac{2(D-4)}{r} e^{-2v}(\dot{u}-\dot{v}) + \frac{(D-4)(D-5)}{r^2}(1-e^{-2v}), \tag{2.12}$$

and the GB term is expressed as

$$R^2_{\text{GB}} = (D-3)(D-4)e^{-4v}\left[\frac{2(D-5)}{r^3} e^{-2v}(\dot{u}-\dot{v}) + \frac{(D-5)(D-6)}{r^4}(1-e^{-2v})\right]. \tag{2.13}$$

The system (2.1) was considered in Ref. [22] for application to cosmological models with accelerating expansion. Time-dependent solutions for p- and q-dimensional external and internal spaces were studied. The field equations can be derived from the results given there by the replacement $v(r), u(r), p \to -v(r), u(r), p = 1, \sigma_p = 0, q = D-2, \sigma_q = k, \ln r$.

Eqs. (2.9)-(2.12) are not all independent but satisfy

$$\dot{u}\frac{d}{dr}\left[\text{Eq.}(2.9)\right] + \dot{v}\frac{d}{dr}\left[\text{Eq.}(2.10)\right] + \dot{\phi}\frac{d}{dr}\left[\text{Eq.}(2.11)\right] = 0. \tag{2.14}$$

This serves to check the consistency of the results.

3. Metrics and Boundary Conditions

3.1 Basic Equations

In order to discuss asymptotically flat solutions, we parametrize the metric as

$$ds^2 = -e^{-2\delta(r)}B(r)dt^2 + \frac{dr^2}{B(r)} + r^2 h_{ij}dx^i dx^j, \tag{3.1}$$

where we have defined

$$B(r) \equiv 1 - \frac{2m(r)}{r^{D-3}}. \tag{3.2}$$

The mass function $m = m(r)$ and the lapse function $\delta = \delta(r)$ depend only on the radial coordinate r. The field equations (2.9), (2.10), and (2.12) then give

$$m' = \frac{r^{D-2}}{2(D-2)}\left[\frac{1}{2}B\phi'^2 + \alpha_2 e^{-\gamma\phi}(D-3)(D-4)\frac{k-e^{-2\delta}B}{r^2}\right], \tag{3.3}$$

$$\delta' = \frac{r}{2(D-2)}\left[\phi'^2 + 2\alpha_2\gamma e^{-\gamma\phi}(D-3)(D-4)\frac{e^{-2\delta}B'}{r}\right], \tag{3.4}$$

$$\left(e^{-\delta}r^{D-2}B\phi'\right)' = \gamma\alpha_2 e^{-\gamma\phi}(D-3)(D-4)\left[(D-2)\frac{k-e^{-2\delta}B}{r^2}+e^{-2\delta}B'\right], \quad (3.5)$$

where the prime in the field equations denotes derivative with respect to r. Here we have also defined dimensionless variables

$$\tilde{r} \equiv \frac{r}{\sqrt{\alpha_2}}, \quad \tilde{m} \equiv \frac{m}{\alpha_2^{(D-3)/2}}, \quad \tilde{M} \equiv \tilde{m}(\infty), \quad (3.6)$$

and the prime in these equations denotes derivative with respect to $\tilde{r}$. We have also defined

$$C \equiv e^{-\gamma\phi_H}, \quad \tilde{h} \equiv e^{-2\delta_H}B'_H. \quad (3.7)$$

Note that these equations have a symmetry under

$$\phi \to \phi + \phi_\infty, \quad \tilde{r} \to e^{-\gamma\phi_\infty/2}\tilde{r}, \quad \delta \to \delta + \frac{\gamma\phi_\infty}{2}, \quad \tilde{m} \to e^{-(D-3)\gamma\phi_\infty/2}\tilde{m}. \quad (3.8)$$

This can be used to shift the asymptotic value of the dilaton field to zero even if we compute metric functions and the dilaton field for given boundary conditions at the horizon. Since the spacetime is time-independent, there is another shift symmetry under $t \to e^{-\delta_\infty}t$, which may be used to shift the asymptotic value of δ to zero.

3.2 Boundary Conditions

From now on, we consider $k = 1$. The horizon is defined by the condition $B(\tilde{r}_H) = 0$, i.e.,

$$2\tilde{m}(\tilde{r}_H) = \tilde{r}_H^{D-3}. \quad (3.9)$$

In what follows, quantities evaluated at the horizon are denoted with subscript H. We impose the following boundary conditions for the metric functions and the dilaton field at the event horizon and spatial infinity:

1. **Asymptotic flatness at spatial infinity** ($\tilde{r} \to \infty$):

$$\tilde{m}(\tilde{r}) \to \tilde{M}, \quad \delta(\tilde{r}) \to 0, \quad \phi(\tilde{r}) \to 0. \quad (3.10)$$

2. **Existence of a regular horizon at** $\tilde{r}_H$:

$$2\tilde{m}_H = \tilde{r}_H^{D-3}, \quad \delta_H < \infty, \quad \phi_H < \infty. \quad (3.11)$$

3. **The event horizon is the outermost one and the regularity of spacetime for $\tilde{r} > \tilde{r}_H$:**

$$2\tilde{m}(\tilde{r}) < \tilde{r}^{D-3}, \quad \delta(\tilde{r}) < \infty, \quad \phi(\tilde{r}) < \infty. \tag{3.12}$$

Looking at Eq. (3.4) for the dilaton, we see that it appears singular at the horizon $B = 0$ if we solve for ϕ''. In order to deal with this, we expand the equations and field variables in power series of $\tilde{r} - \tilde{r}_H$ to guarantee regularity at the horizon. From the zeroth-order term in the expansion of Eq. (3.2), we find

$$\tilde{h}_H = \frac{(D-3) + e^{-\gamma\phi_H}(D-5)\tilde{h}_H}{\tilde{r}_H}, \tag{3.13}$$

where we have used Eq. (3.7) in deriving the second equality. Using this in Eq. (3.4) at the horizon, we obtain the quadratic equation determining ϕ'_H:

$$\tilde{r}_H\phi'_H + \frac{(D-4)(3D-11)\gamma^2C^2 + 2(D-2)^5C - 3\gamma^2C^2 + 2(D-2)^5C^3\gamma^2}{(D-4)^5C^2 + 2(D-2)^5C^3\gamma^2} = 0, \tag{3.14}$$

where we have defined

$$C \equiv e^{-\gamma\phi_H}. \tag{3.15}$$

Eq. (3.14) shows that the values of the dilaton field and its derivatives are related, and ϕ'_H is obtained in terms of ϕ_H. From the first-order terms of $\tilde{r} - \tilde{r}_H$, we can express the second derivative ϕ''_H in terms of ϕ_H and ϕ'_H, and use their analytic solution for the first step of integration.

In the asymptotic region far from the horizon, the curvature of spacetime is small and the GB term is negligible. The dilaton field then behaves as

$$\phi(\tilde{r}) \sim \frac{\Sigma}{\tilde{r}^{D-3}}, \tag{3.16}$$

where Σ is the dilaton charge. This global charge is not a free parameter of the solution but is fixed by the mass of the black hole. In this sense, the dilaton charge is classified as secondary hair.

4. Non-dilatonic Black Hole Solutions

It will be instructive to compare our results with the non-dilatonic case. Let us derive physical quantities for this case first. When the dilaton field is absent (i.e., Einstein-GB system), we substitute $\phi = 0$ and $\gamma = 0$ into Eqs. (3.2) and (3.3). In the $D = 4$ case, the GB term is a total divergence and does not give any contribution to the field equations. As a result, the solution reduces to the Schwarzschild solution.

For $D \geq 5$, the field equations can be integrated to yield [9, 11]

$$\bar{B} = 1 + \frac{\tilde{r}^2}{2(D-3)(D-4)}\left(1 \mp \sqrt{1 + \frac{4(D-3)(D-4)\bar{M}}{\tilde{r}^{D-1}}}\right), \quad \delta = 0, \quad (4.1)$$

where $\bar{M}$ is an integration constant corresponding to the asymptotic value $\bar{m}(\infty)$ for the plus sign in Eq. (4.1). Throughout this paper, all quantities with a bar denote those of the non-dilatonic solution normalized by α_2 as in the dilatonic case. In the $\alpha_2 \to 0$ limit, the solutions with the plus sign approach the Schwarzschild solutions. This means they can be considered as solutions with GB correction to GR. On the other hand, the solutions with the minus sign do not have such a limit. For these reasons, we call the solutions with plus (minus) sign the (non-)GR branch.

For $\bar{M} = 0$, the metric function becomes

$$\bar{B} = \begin{cases} 1 & \text{(GR branch)} \\ 1 + \frac{\tilde{r}^2}{(D-3)(D-4)} & \text{(non-GR branch)} \end{cases} \quad (4.2)$$

Hence the spacetime is Minkowski in the GR branch while it is anti-de Sitter in the non-GR branch, although the pure cosmological constant Λ is absent.

For $\bar{M} \neq 0$, besides the central singularity at $\tilde{r} = 0$, there can be another known as the branch singularity at finite radius $\tilde{r}_b > 0$, which is obtained by the condition that the inside of the square root in Eq. (4.1) vanishes. We find the $\bar{M}$-$\tilde{r}_b$ relation

$$\tilde{r}_b^{D-1} = -\frac{4(D-3)(D-4)}{\bar{M}}. \quad (4.3)$$

This implies that the branch singularity appears for negative mass parameter.

It can be shown that there is no black hole solution in the non-GR branch. On the other hand, in the GR branch, Eq. (4.1) evaluated at the horizon $\bar{B} = 0$ gives

$$\bar{M} = \tilde{r}_H^{D-3} + \frac{(D-3)^2}{(D-4)}\tilde{r}_H^{D-5}. \quad (4.4)$$

This is the $\bar{M}$-$\tilde{r}_H$ relation for black holes without the dilaton field. This relation indicates that the $D = 5$ case is qualitatively different from other dimensional solutions. In four dimensions, the constant term in Eq. (4.4) vanishes and the relation between $\bar{M}$ and $\tilde{r}_H$ is linear, and it vanishes in the $\tilde{r}_H \to 0$ limit. In dimensions higher than five, the constant remains but the overall factor of $\tilde{r}_H$ has positive power, and it also vanishes as the size of the black hole becomes zero. In the five-dimensional case, however, $\bar{M}$ takes a nonzero finite value ($\bar{M} = 1$) in the zero horizon-radius limit. Since the derivative of the metric function $\bar{B}$ of such a tiny black hole goes to zero in this limit, it has an almost degenerate horizon at zero radius, i.e., a quasi-extreme black hole. In higher dimensions, the derivative of the metric function $\bar{B}$ does not vanish at the horizon and the solutions are non-degenerate.

In the context of Lovelock gravity [4, 5], the first, second, and third Lovelock actions are the cosmological constant, Einstein-Hilbert action, and the GB term, respectively. In four dimensions, the GB term is known as Euler density and does not contribute to the field equations at the classical level. In five and six dimensions, the GB term is the highest-order action in Lovelock gravity and would give significant deviation from the four-dimensional case. For this reason and the qualitative difference from the non-dilatonic case, we investigate the cases of $D = 4, 5, 6$, and 10 in the following sections.

5. D = 4 Black Hole Solutions

We first present black hole solutions for $D = 4$. Because there is ambiguity in the choice of γ as discussed in Section 2, we examine black hole solutions for two different choices: $\gamma = 1$ and $\gamma = 1/2$. The first case is examined to check the consistency of our solutions with those in Ref. [18]. The second case is examined to see how the results depend on the choice of the dilaton coupling.

5.1 $\gamma = 1$ Solutions

The condition of the regular horizon (3.14) reduces to

$$\tilde{r}_H \phi'_H + 6 = 0, \tag{5.1}$$

which can be shown to be equivalent to the condition given in Ref. [18]. Eq. (5.1) has two solutions

$$\phi'_{H,\pm} = \frac{24C^2\gamma^2 - 2C\gamma \pm \tilde{r}_H}{\tilde{r}_H}, \tag{5.2}$$

among which only the smaller solution gives regular black holes. The solutions are real only for

$$\tilde{r}_H \geq 4\sqrt{6}\gamma e^{-\frac{\gamma}{2}\phi_H}, \tag{5.3}$$

which gives the lower bound on $\tilde{r}_H$ for the regular solution to exist for any γ. We will see that this is in sharp contrast to higher-dimensional solutions.

For several boundary conditions on ϕ_H and δ_H at the horizon with the smaller solution for ϕ'_H in (5.2), we obtain the behaviors of the dilaton, mass, and lapse functions by integrating the basic equations (3.2)-(3.4) from the horizon. Using the symmetry (3.11), we set the asymptotic value of the dilaton to zero. The resulting configurations of these functions are depicted in Fig. 1 [Figure 1: see original paper] for $\tilde{r}_H = 4.44251, 4.6226, 5.32723$, and 6.15605. The masses $\tilde{M}$ for these cases are found to be $2.40541, 2.43848, 2.72056$, and 3.10867, respectively. We find that regular black hole solutions exist only for $\tilde{r}_H \geq 4.44142$, which is a consequence of condition (5.3).

In the present four-dimensional case, we find that the dilaton field increases monotonously from its value at the horizon to zero (Fig. 1(a)). The mass function decreases near the horizon and increases towards a finite value (Fig. 1(b)) as $\tilde{r}$ increases. In fact, it can be shown that

$$\tilde{m}'_H = -\frac{(D-2)(D-3)}{2}\tilde{r}_H^{D-4}e^{-\gamma\phi_H} < 0, \tag{5.4}$$

so the decrease near the horizon occurs for any solution. If we regard the dilaton terms and those proportional to α_2 in Eq. (2.2) as "matter terms", the effective energy density is given by $\rho_{\text{eff}} = \tilde{m}'/8\pi\tilde{r}^{D-2}$. In the region where the mass function decreases, we see that the effective energy density becomes negative. This does not mean that these solutions are unstable, but they are indeed stable [18, 27].

A notable feature in these solutions is that there are two solutions with different horizon radii for the range $2.40528 \lesssim \tilde{M} \lesssim 2.40546$ (Fig. 1(e)). In the non-dilatonic case, we have

$$\tilde{M} = \frac{\tilde{r}_H}{2}, \tag{5.5}$$

which is also displayed in Fig. 1(d), and there can be a black hole however small the mass is. But in the dilatonic case, we do not find analogous behavior. For the solution with smallest horizon radius, condition (5.3) is saturated, and the second derivative of the dilaton field diverges at the horizon. Moreover, it was shown that the smaller black hole solutions for $2.40528 \lesssim \tilde{M} \lesssim 2.40546$ are unstable, although the larger black hole solutions are stable [18, 27]. All these results are consistent with those in Ref. [18].

5.2 $\gamma = 1/2$ Solutions

We next examine the $\gamma = 1/2$ case. For several boundary conditions on ϕ_H and δ_H at the horizon with the smaller solution for ϕ'_H in (5.2), we obtain the behaviors of the dilaton, mass, and lapse functions by integrating the basic equations (3.2)-(3.4) from the horizon. Using the symmetry (3.11), we set the asymptotic value of the dilaton to zero. The resulting configurations of these functions are depicted in Fig. 2 [Figure 2: see original paper] for $\tilde{r}_H = 2.68697, 2.90965, 3.19148$, and 3.52851. The masses $\tilde{M}$ for these cases are found to be $1.47251, 1.53808, 1.65113$, and 1.80161, respectively. We find that regular black hole solutions exist only for $\tilde{r}_H \geq 1.47126$, which is a consequence of condition (5.3).

We note that for this choice of $\gamma = 1/2$, there are not two solutions close to the minimum value of the mass parameter, in contrast to $\gamma = 1$. Except for this, we find that the behaviors of the dilaton, mass, and lapse functions are quite similar. We expect that similar behaviors are obtained regardless of the value of γ also in other dimensions. So in the rest of this paper, we examine only the $\gamma = 1/2$ case.

6. D = 5 Solutions

Five dimensions is the lowest dimension in which the GB term makes a nontrivial contribution to the vacuum solution. Eq. (3.14) reduces to

$$C\gamma(1+C+3C^2\gamma^2)\tilde{r}_H^2+(1+C)(1+C-6C^2\gamma^2)\tilde{r}_H\phi'_H+3C(3-3C^2)\gamma = 0. \quad (6.1)$$

Here again, there are two solutions but only the smaller solution of Eq. (6.1) gives regular black holes. The discriminant of this quadratic equation is (for $\gamma = 1/2$)

$$\Delta = 18C^6 + 30C^5 + 5C^4 - 16C^3 - 12C^2 + 8C + 2, \quad (6.2)$$

which is always positive for $C > 0$, and hence there is no bound on the value of $\tilde{r}_H$ for the reality of the solution, unlike in four dimensions.

For various boundary conditions for ϕ_H and δ_H and the smaller solution ϕ'_H of Eq. (6.1) at the horizon, we obtain the configurations of the dilaton field, mass, and lapse functions, which are depicted in Fig. 3 [Figure 3: see original paper] for $\tilde{r}_H = 0.754129, 1.13599, 1.46193$, and 2.68391. The masses $\tilde{M}$ for these cases are found to be $0.573328, 1.05972, 1.66924$, and 5.0097, respectively. In contrast to the four-dimensional case, we find that regular black hole solutions exist for all $\tilde{r}_H > 0$, in accordance with the above observation.

For large black holes, the dilaton field ϕ increases monotonously just as in the four-dimensional case. (We note that the smallest black hole in four dimensions is the solution at the turning point C in Fig. 1, which has $\tilde{M} = 2.40528$.) From Eq. (6.1), we find that the smaller solution for ϕ'_H is positive when $C(> 0)$ satisfies both conditions

$$C < \frac{1+\sqrt{1+24\gamma^2}}{12\gamma^2} \tag{6.3}$$

and $C < 1+24\gamma^2$, and is non-positive otherwise. For $\gamma = 1/2$, the first condition is stronger than the second one, and the condition becomes $C \lesssim 0.72$. Since $\phi_H \to 0$ for large black holes, the dilaton field increases at the horizon. For small black holes, however, the GB term affects the configurations of the field functions significantly near the horizon, and the dilaton field takes a positive value at the horizon. Then it decreases asymptotically. This means that the dilaton charge is negative.

For black holes with large mass, the mass function increases monotonously, as in the four-dimensional case. It shows, however, peculiar behaviors for small black holes. It increases near the horizon but decreases in the intermediate region, which means that $\tilde{m}'/\tilde{r}^{D-2}$ becomes negative there. These can be intuitively explained as follows. There are two typical length scales for our solution. One is the string scale given by $\ell_s = \sqrt{\alpha_2}$ and the other is the horizon radius r_H. When the length scale we are interested in is longer than ℓ_s, the GB term is negligible in the field equations (2.2) and (2.7). On the other hand, when the length scale is shorter than ℓ_s, the effect of the GB term becomes dominant. Hence for small black holes, the field functions show qualitatively exotic behaviors in the region $r_H < r < \ell_s$. In fact, the log-linear plots of the dilaton field (Fig. 3(e)) show that the dilaton field behaves as $\phi \sim \log \tilde{r}$ in this region. Beyond this region, the behavior suddenly changes to power decay $\phi \sim \Sigma/\tilde{r}^2$. This is a characteristic feature of the solution in which spacetime is divided into the GB region and the GR region distinctly.

The fact that the effective energy density is negative in some intermediate region does not mean that the solutions are unstable. In fact, it has been checked that they are stable in four dimensions, and we expect the same stability here.

The perturbative approach gives us clearer and further understanding. Let us assume $\gamma \ll 1$ and expand the basic equations (2.2) and (2.7) with respect to γ. Then the zeroth-order equations reduce to the Einstein-GB-massless scalar system and give the non-dilatonic solution with $\phi = 0$. Since the terms with the dilaton field in the gravitational equation (2.2) are second order, the non-dilatonic solution is also a solution of Eq. (2.2) up to first order.

On the other hand, the first-order equation for the dilaton field gives

$$\bar{\phi}_1 = \alpha_2 \bar{R}^2_{\mathrm{GB}}, \tag{6.4}$$

where ϕ_1 is the first-order expansion of ϕ as $\phi = \phi_0 + \phi_1\gamma + \cdots$, and $\bar{R}^2_{\mathrm{GB}}$ is evaluated by the zeroth-order variables. Under our ansatz, Eq. (6.4) is written as

$$\left(\tilde{r}^{D-2}\bar{B}\phi_1'\right)' = \tilde{r}^{D-2}\bar{R}^2_{\rm GB}. \tag{6.5}$$

(It should be noted that the variables are normalized by $\sqrt{\alpha_2}$.) In the asymptotic region, the first term on the left-hand side is the leading term and we find $\phi_1 \sim -\Sigma/\tilde{r}^{D-3}$. We call this the GR region. In the intermediate region, this equation can be integrated as

$$\phi_1(\tilde{r}) = \tilde{r}^{-(D-2)}\bar{B}^{-1}\int \tilde{r}^{D-2}\bar{R}^2_{\rm GB}\, d\tilde{r} + \phi_H. \tag{6.6}$$

Near the event horizon, the behavior of the dilaton ϕ is nontrivial, and this is due to the effect of the GB term. We call this the GB region.

To see the boundary between the GR region and the GB region, the discussion of the effective energy density is helpful. In the perturbative approach, we find

$$\rho_{\rm eff} = \frac{1}{32\pi(D-3)^4}\left[1 + \frac{(\mu/\tilde{r})^{D-1}}{1+2(\mu/\tilde{r})^{D-1}}\right]^{-2}\frac{(D+3)^4}{\tilde{r}^{D-1}}, \tag{6.7}$$

where $\mu^{D-3} \equiv \tilde{M} + (D-3)^4$. When the condition $\tilde{r} \gg \mu$ is satisfied,

$$\rho_{\rm eff} \approx \frac{1}{64\pi(D-3)^4}\frac{(D+3)^4}{\tilde{r}^{D-1}}. \tag{6.8}$$

Since the effective energy density vanishes, this condition corresponds to the GR region. When the condition $\tilde{r} \ll \mu$ is satisfied,

$$\rho_{\rm eff} \approx \frac{1}{32\pi(D-3)^4}\frac{1}{\tilde{r}^{D-1}}. \tag{6.9}$$

Hence this region is significantly affected by the GB term, and it is the GB region. The boundary $\tilde{r}_*$ of the transition between the GR region and the GB region is simply obtained by $\tilde{r}_* = \mu$. In the five-dimensional case, this condition becomes

$$\tilde{r}_* = \sqrt{2 + \frac{2}{\tilde{r}_H^2}}. \tag{6.10}$$

For large black holes with $r_H \gg \ell_s$, the boundary is at $r_* = \sqrt{2}r_H\ell_s$. This is less than r_H and the transition occurs inside the event horizon. Hence the outer spacetime of the event horizon is well approximated by the Tangherlini solution (D-dimensional Schwarzschild solution) [28]. For "string size" black holes with $r_H \simeq \ell_s$, the transition occurs around $\sqrt{2}\ell_s$. The effect of the GB term extends to a region a few times the radius of the event horizon. For small black holes with $r_H \ll \ell_s$, the boundary is $r_* \simeq \ell_s$. Hence the string effect extends to the region of the string scale. These results are consistent with the intuitive consideration given above.

We have found that regular black hole solutions exist for all $\tilde{r}_H > 0$. In the four-dimensional case, $\phi'_{H,\pm}$ from Eq. (5.1) degenerates for finite horizon radius, and the solution disappears below the radius (5.3). On the other hand, $\phi'_{H,\pm}$ has two non-degenerate roots even in the $\tilde{r}_H \to 0$ limit in five dimensions, and there are regular black hole solutions for all $\tilde{r}_H$. The mass of the black hole approaches a nonzero constant $\tilde{M} = 0.288185$ as $\tilde{r}_H \to 0$.

It is instructive to compare this result with the non-dilatonic case. For five dimensions, Eq. (4.4) gives

$$\bar{M} = \tilde{r}_H^2 + 2. \tag{6.11}$$

As $\tilde{r}_H \to 0$, the mass of the black hole approaches a nonzero constant $\bar{M} = 2$. This property is very similar to our dilatonic case, although the constant value is different due to the effect of the dilaton.

The difference in masses between the dilatonic and non-dilatonic cases (Fig. 3(d)) can also be estimated in terms of length scales. For large black holes ($r_H > \ell_s$), the GB term can be neglected and the dilaton field contributes to the solutions mainly as an ordinary matter field in GR, and the mass becomes large compared to the non-dilatonic case. However, for small black holes ($r_H < \ell_s$), the effect of the GB term becomes large. If we regard the GB term as a matter term again, the dilaton field plays the role of the coupling between the GB term and gravity. Because of this nontrivial coupling with the GB term, the effective energy density becomes negative (see Fig. 3(b)), and the total mass decreases compared to the non-dilatonic case. In the zero horizon-radius limit, C and $\tilde{r}_H \phi'_H$ take constant values, which means that the derivative of the dilaton field diverges at the horizon. The dilaton field itself also diverges as $\phi_H \sim -\frac{2}{\gamma} \log \tilde{r}_H$. As a result, the energy density of the dilaton field diverges. Since the derivative of the mass function also diverges at the horizon, the zero-size black hole is singular. The difference between $\tilde{M}$ in the dilatonic case and $\bar{M}$ in the non-dilatonic case in the $\tilde{r}_H \to 0$ limit is due to the diverging energy density of the dilaton field at $\tilde{r}_H = 0$.

7. D = 6 Solutions

We now examine the six-dimensional solutions. We find that many properties in the $D = 6$ case are similar to higher-dimensional cases up to ten dimensions, where we come back to the original string theory. So we will be brief, leaving more detailed discussions to $D = 10$.

For $D = 6$, Eq. (3.14) reduces to

$$6+14C+4C^2(14\gamma^2+1)+48C^3\gamma^2-40C^2\gamma^2(1+C-3C^2)+(3+C)(1+2C)^2\tilde{r}_H\phi'_H+40C(2-7C^2)\gamma=0. \quad (7.1)$$

Here again, there are two solutions but only the smaller solution of Eq. (7.1) gives regular black holes. The discriminant of this quadratic equation is (for $\gamma = 1/2$)

$$\Delta = 100C^8+720C^7+1656C^6+1288C^5+280C^4-40C^3+85C^2+78C+9, \quad (7.2)$$

which is again always positive for $C > 0$, and hence there is no bound on the value of $\tilde{r}_H$ for the reality of the solution.

For various boundary conditions for ϕ_H and δ_H and the smaller solution ϕ'_H of Eq. (7.1) at the horizon, we find the behaviors of the dilaton, mass, and lapse functions, which are depicted in Fig. 4 [Figure 4: see original paper] for $\tilde{r}_H = 0.125367, 1.13596, 1.46199$, and 4.12369. The masses $\tilde{M}$ for these cases are found to be $0.311672, 2.6993, 4.25081$, and 49.2744, respectively.

For large black holes, the dilaton field ϕ increases monotonously as in the four-dimensional case, and for smaller black holes ($r_H \sim \ell_s$), ϕ_H becomes large and the dilaton field decreases asymptotically with negative dilaton charge as in the five-dimensional case. However, as the black hole becomes further smaller ($r_H \ll \ell_s$), the magnitude of the dilaton field becomes smaller and vanishes in the $r_H \to 0$ limit.

As for the mass function $\tilde{m}$, it has a peak and there is a region where the effective energy density is negative when the horizon radius is $r_H \sim \ell_s$. This means that the string effect is significant. However, as the black hole becomes smaller, the mass function has no peak and takes small values, which vanish in the zero horizon-radius limit.

We find that regular black hole solutions exist for all $\tilde{r}_H > 0$. The mass of the black hole approaches zero as $\tilde{r}_H \to 0$. This is different from the four- and five-dimensional cases, but it is in agreement with the non-dilatonic case. In fact, for six dimensions, Eq. (4.4) gives

$$\bar{M} = \tilde{r}_H(\tilde{r}_H^2 + 6). \quad (7.3)$$

As $\tilde{r}_H \to 0$, the mass of the black hole approaches zero.

8. D = 10 Solutions

The $D = 10$ case is the most interesting from a theoretical point of view since it is the critical dimension of string theory. In the Einstein-Maxwell-dilaton system, the spacetime structure of D-dimensional black hole solutions changes at ten dimensions [1, 2]. For $D = 10$, Eq. (3.14) reduces to

$$7+57C+6C^2(76\gamma^2+15)+1680C^3\gamma^2-432C^2\gamma^2(1+C-15C^2)+(1+6C)^2(7+15C)\tilde{r}_H\phi'_H+4(6C+99C^2)\gamma = 0. \quad (8.1)$$

Once again, the discriminant of this quadratic equation is (for $\gamma = 1/2$) always positive for $C > 0$, and there is no bound on the value of $\tilde{r}_H$ for the reality of the solution.

For various boundary conditions for ϕ_H and δ_H and the smaller solution ϕ'_H of Eq. (8.1) at the horizon, we find the behaviors of the dilaton, mass, and lapse functions, which are depicted in Fig. 5 [Figure 5: see original paper] for $\tilde{r}_H = 0.968549, 1.13596, 1.46194$, and 9.68119. The masses $\tilde{M}$ for these cases are found to be $16.7172, 36.8633, 129.489$, and 5.88035×10^6, respectively.

The configurations of the field functions are almost the same as those in the $D = 6$ case qualitatively. For large black holes, the dilaton field ϕ increases monotonically. From Eq. (3.14), we find that the smaller solution for ϕ'_H is positive when both conditions

$$(D-4)^2(D-1)C^2 - (D-4)^5C^2 - 4(D-4)(D+1) < 0, \quad (8.2)$$

and

$$(D-3) + (D-4)C < 0 \quad (8.3)$$

are satisfied, and is non-positive otherwise. Since $C \to 0$ in the large $\tilde{r}_H$ limit, both conditions are satisfied for large black holes, which means that the dilaton field increases around the horizon. This behavior is independent of the value of γ and dimensions. The difference from the $D = 6$ case is that we find no peak in the mass function $\tilde{m}$, and there is no region with negative effective energy density even for small black holes. This is in sharp contrast to other lower dimensions and may be related to the fact that ten dimensions is the dimension in which string theory lives.

As $\tilde{r}_H \to 0$, the mass of the black hole approaches zero. This is again in agreement with the non-dilatonic case, for which Eq. (4.4) gives

$$\bar{M} = \tilde{r}_H(\tilde{r}_H^2 + 42). \quad (8.4)$$

The behaviors of the functions can also be explained in terms of the effective energy density. As in the $D = 6$ case, μ is given by Eq. (6.8), and the boundary between the GR region and the GB region is

$$\tilde{r}_* = \left[2(D-3)^4 r_H^{D-3}\right]^{1/(D-1)} . \tag{8.5}$$

For large black holes with $r_H \gg \ell_s$, the boundary is $r_* = [2(D-3)^4]^{1/(D-1)} r_H$. This is less than r_H and the transition occurs inside the event horizon. Hence the outer spacetime of the event horizon is the GR region. For "string size" black holes with $r_H \simeq \ell_s$, the transition occurs around $r_* \simeq \ell_s$. (The factor is 2.5 for $D = 10$.) The effect of the GB term extends to a region a few times the radius of the event horizon. These are similar to the five-dimensional case.

For small black holes with $r_H \ll \ell_s$, the transition occurs around $r_* \simeq \ell_s$. Hence the string effect is confined to the region just around the event horizon, which is much smaller than the string scale. One would naturally expect that the string effect extends to the region of the string scale ℓ_s. However, this is not the case. Actually, if we assume that the dilaton field does not diverge in the $r_H \to 0$ limit for $D \geq 6$, from Eq. (3.14) we find

$$e^{\gamma\phi_H} \tilde{r}_H \to \text{constant}. \tag{8.6}$$

For $D = 10$, this gives

$$e^{\gamma\phi_H} \tilde{r}_H \to 0.15714, \tag{8.7}$$

which is consistent with what is obtained by numerical calculation where $\phi_H = 0$. As a result, the dilaton field vanishes throughout the whole spacetime and the solution approaches the non-dilatonic one in the $\tilde{r}_H \to 0$ limit. This behavior can be seen more clearly in six dimensions in Fig. 4(d). The $\tilde{M}$-$\tilde{r}_H$ curve rapidly approaches the non-dilatonic one in the $\tilde{r}_H \to 0$ limit.

9. Thermodynamics

Thermodynamic properties of black holes are one of the important issues, particularly when we discuss the evolution of black holes through Hawking radiation. It is expected that string effects become dominant in the final stage of black hole evaporation and give nontrivial processes. Although black hole thermodynamics in non-GR theories is not as well understood as in GR, we can define the temperature and entropy of black holes which obey the first law of thermodynamics in asymptotically flat spacetime in diffeomorphism-invariant theories [29]. The Hawking temperature is given by the periodicity of Euclidean time at the horizon

$$T = \frac{1}{4\pi r_H} \left(2\tilde{m}'_H \tilde{r}_H^{D-4}\right) . \tag{9.1}$$

We show the $\tilde{M}$-β relations in Fig. 6 [Figure 6: see original paper], where $\beta = 1/T$ is the inverse temperature.

In $D = 4$, we can see that the GB term has the tendency to raise the temperature compared to the non-dilatonic solution (Schwarzschild black hole). This behavior comes from the contribution of $\tilde{m}'_H$ from Eqs. (5.1) and (5.4) at the singular point. Since the temperature takes a nonzero finite value for all mass ranges, we expect that the black hole will not stop emitting radiation and continue evaporating until the solution reaches the minimum mass solution denoted by point C in Fig. 1(e), where $\tilde{m}'_H$ has the minimum value in Eq. (9.1).

In the five-dimensional case, the non-dilatonic black holes have interesting thermodynamic properties. The temperature increases as the mass of the black hole becomes small for large black holes, which means that the heat capacity is negative. Below the mass $\tilde{M} = 2.976072$, however, the temperature decreases as the mass becomes small. The sign of the heat capacity changes at this mass. This behavior is qualitatively the same as the Reissner-Nordström black hole solution and can be classified as a second-order phase transition. As the black hole becomes small through Hawking radiation, the temperature becomes extremely low, and the solution cannot reach the singularity with zero horizon radius. This is a favorable feature from the point of view of the cosmic censorship hypothesis, although more detailed analysis should be necessary for a definite answer.

By adding the dilaton field to the system, we find that the thermodynamic properties change drastically. The heat capacity is negative for all mass ranges, and the temperature blows up at the singular solution. This is due to the nontrivial coupling between the dilaton field and the GB term and the resultant divergence of the dilaton field at the horizon.

For $D \geq 6$, the behavior of the temperature is qualitatively the same as in the non-dilatonic case. The dilaton field has a tendency to lower the temperature for large black holes, while it raises the temperature for small black holes. In six dimensions and higher, our analysis shows that the temperature diverges for the zero-mass "solution" and the black hole continues evaporating.

In GR, the horizon radius of a black hole is related to entropy by $S = \pi r_H^2$. Hence the dependence of entropy on mass M can be easily estimated from the M-r_H diagram. In our case with GB gravity, however, entropy is not obtained by a quarter of the area of the event horizon. Along the definition of entropy in Ref. [29], which originates from the Noether charge associated with the diffeomorphism invariance of the system, we obtain

$$S = -2\pi \int_\Sigma \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma}, \tag{9.2}$$

where Σ is the event horizon $(D-2)$-surface, $\mathcal{L}$ is the Lagrangian density, and $\epsilon_{\mu\nu}$ denotes the volume element binormal to Σ. This entropy has desirable properties such that it obeys the first law of black hole thermodynamics and is expected to obey even the second law [30]. For our present model, this gives

$$S = \frac{A_H}{4G_D}\left[1 + 2(D-3)(D-4)\alpha_2 e^{-\gamma\phi_H}\frac{k}{r_H^2}\right], \tag{9.3}$$

where $A_H = 2\pi^{(D-1)/2}/\Gamma((D-1)/2)r_H^{D-2}$ is the area of the event horizon.

Fig. 7 [Figure 7: see original paper] shows the M-S plots of our solutions. Although there is no qualitative difference between the dilatonic and non-dilatonic cases except that the solution disappears at a nonzero finite mass for $D = 4$ (the dilatonic solution) and $D = 5$, it should be noted that the entropy of the dilatonic black hole is always larger than that of the non-dilatonic black hole with the same mass. We currently have no physical interpretation for this behavior at either the classical or quantum levels since they are different systems.

10. Conclusions and Discussions

We have studied black hole solutions in the dilatonic Einstein-Gauss-Bonnet theory in D-dimensional spacetime. We assumed that spacetime is static and spherically symmetric, and asymptotically flat. Our analysis is a direct extension of the four-dimensional dilatonic system to higher-dimensional spacetime and of the non-dilatonic black holes to the dilatonic ones in higher-dimensional solutions. Our black holes show remarkable properties that depend strongly on dimension. We focused on the $D = 4, 5, 6$, and 10 cases in numerical analysis and compared the solutions with known non-dilatonic solutions. We also studied their thermodynamic properties.

First, in four dimensions, we made a consistency check on our solutions with those previously investigated. Spacetime around the event horizon has regions where the effective energy density becomes negative. There is a minimum mass of black hole below which no regular solution exists. There is also a solution that has a minimum horizon radius.

In five dimensions, we find that the effects of the GB term are negligible for large black holes ($r_H \gg \ell_s$), and the solution can be well approximated by the Tangherlini solution with the dilaton field that decays with a power law. In contrast, for small black holes ($r_H \lesssim \ell_s$), spacetime is divided into the GR region and the GB region with a sharp transition. In the GB region, the dilaton field behaves logarithmically and the effective energy density becomes negative. Regular black hole solutions exist for all horizon radii. In the zero horizon-radius limit, the solution becomes singular. These properties are the same as those of the non-dilatonic solutions, while the mass in this limit is different due to diverging energy density at the center.

In higher dimensions ($D \geq 6$), for the spacetime of small black holes ($r_H \ll \ell_s$), the string effect extends to just around their event horizons, which are much smaller than the string scale. This is a remarkable property since one naturally expects that the string effect extends to ℓ_s in any situation. However, this is not the case. Regular solutions exist for any horizon radius. In the zero horizon-radius limit, the mass of the solution approaches zero, which is different from the lower-dimensional cases.

In the non-dilatonic case, the thermodynamic properties are similar for all dimensions except for five dimensions. In five dimensions, the specific heat is negative for large black holes but becomes positive for small mass solutions through a second-order phase transition. For dilatonic black holes, all of them have negative specific heat without phase transition and have nonzero temperature. This suggests that black holes continue evaporating to singular (in $D = 4$ and 5) and zero-mass solutions (in $D \geq 6$). Entropies of our solutions are larger than those of the non-dilatonic solutions.

There are some related issues concerning our solutions. All the solutions we have obtained in this paper correspond to the GR branch in the non-dilatonic solutions. We have found that there is no black hole solution in the non-GR branch for $\alpha_2 > 0$. Since our numerical analysis was limited to the outer spacetime of the event horizon, the global structures of the solutions such as the existence of inner horizons and (central or branch) singularities have not been clarified. This may be done by integrating the field equations inward numerically.

The ambiguity of frames is also important. In this paper, we have studied the system in the Einstein frame. There is, however, a possibility that the properties of solutions change drastically when transforming to the string frame. In particular, the conformal transformation may become singular.

For the choice $\gamma = 2/(D-2)$, the system in the string frame

$$\hat{S} = \int d^D x \sqrt{-\hat{g}} e^{-2\phi} \left[\hat{R} + 4(\partial_\mu \phi)^2 + \alpha' \hat{R}^2_{\mathrm{GB}} \right] \tag{10.1}$$

is obtained by the conformal transformation [22]

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega = e^{-\gamma\phi/2}. \tag{10.2}$$

The metric in the string frame becomes

$$d\hat{s}^2 = -Be^{-2\delta-\gamma\phi} dt^2 + B^{-1} e^{\gamma\phi} dr^2 + \hat{r}^2 h_{ij} dx^i dx^j, \tag{10.3}$$

where the prime denotes the r-derivative in the Einstein frame and

$$\hat{r} = re^{-\gamma\phi/2}. \tag{10.4}$$

The difference between the frames appears for small black holes since the dilaton field behaves nontrivially. In four dimensions, both ϕ_H and ϕ'_H are finite for all solutions (although ϕ''_H diverges for the solution at the singular point). This means there is no qualitative difference in both frames.

In five dimensions, $\phi_H \sim -\frac{1}{\gamma}\log\tilde{r}_H$ in the zero horizon-radius limit. Then, the prefactor of $\hat{g}_{rr}$, i.e., $(1-\gamma r\phi'/2)^{-2}$, takes a nonzero finite value, and the metric does not give a throat-like structure but ordinary black hole spacetime, unlike the extreme solution in the Einstein-Maxwell-dilaton system [2].

For $D \geq 6$, both ϕ_H and ϕ'_H vanish in the zero horizon-radius limit. Hence the spacetime structures in both frames are exactly the same.

The black hole solutions can be applied to brane world cosmology. By adopting our solutions as the bulk solution, we may obtain the dilatonic braneworld with the GB term. This is the dilatonic extension of the vacuum analysis [31]. Since the dilaton field shows nontrivial behavior near the event horizon, the early evolution of cosmology would be strongly affected.

The stability of our solutions is another important subject to study [18, 27, 32]. It would also be interesting to extend our analysis to dilatonic black holes (large and small) with charges [33]. We hope to report results on these issues as well as topological black holes elsewhere.

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