Uncorrelated estimates of the primordial power spectrum postprint

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Full Text

Uncorrelated Estimates of the Primordial Power Spectrum

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Abstract

We use localized principal component analysis to detect deviations from scale invariance of the primordial power spectrum of curvature perturbations. With this technique we make uncorrelated estimates of the primordial power spectrum with five wavenumber bins. Within the framework of a minimal ΛCDM model, using the latest cosmic microwave background data from the WMAP and ACT experiments we find that more than 95% of the preferred models are incompatible with the assumption of scale invariance, but remain compatible with a power-law primordial spectrum. We also forecast the sensitivity and constraints achievable by the Planck experiment by performing Monte Carlo studies on simulated data. Planck could significantly improve the constraints on the primordial power spectrum, especially at small scales by roughly a factor of 4.

Keywords: inflation, cosmological parameters from CMBR

Introduction

Measurements of anisotropies in the cosmic microwave background (CMB) have played an essential role in constraining basic cosmological parameters, particularly in probing the dynamics of the inflationary phase in the early Universe [**?**]. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite has measured the CMB over the full sky down to 0.2° resolution [**?**, **?**]. Measurements at higher resolution made with the Atacama Cosmology Telescope (ACT) [**?**] and the South Pole Telescope (SPT) [**?**] can provide complementary information about the early Universe on scales smaller than those probed by WMAP. The ACT experiment now measures fluctuations on scales from 0.4° to an arcminute, and a combination of WMAP and ACT data improves constraints on cosmological parameters.

Inflationary models with featureless potentials generically predict a primordial power spectrum of curvature perturbations close to scale invariant. Such models are usually parameterized by an amplitude of spectrum A_s, a spectral index n_s, and its running index α_s:

$$\ln P_R(k) = \ln A_s + (n_s - 1)\ln\left(\frac{k}{k_0}\right) + \alpha_s \ln^2\left(\frac{k}{k_0}\right)$$

where k_0 is a pivot scale. This parameterization represents a Taylor expansion in logarithmic amplitude and logarithmic wavenumber space around the pivot point. The special case with $n_s = 1$ and $\alpha_s = 0$ yields the Harrison-Zel' dovich (scale-invariant) spectrum. In slow-roll inflationary models, the spectral index and running index are first and second order in the slow-roll parameters respectively, and thus are expected to be small. For a power-law parameterization ($\alpha_s = 0$), 99.5% of the preferred models are incompatible with the scale-invariant spectrum when using the 7-year WMAP data if tensor modes are ignored [**?**]. Even when adding a running index, a slightly tilted power-law primordial spectrum without tensor modes remains an excellent fit to the data [**?**]. Although combining WMAP data with ACT data shows that the running index prefers a negative value at 1.8σ, indicating enhanced damping at small scales, there is no statistically significant deviation from a power-law spectrum [**?**]. Moreover, before concluding that a power-law spectrum is excluded, one should investigate extensions of the minimal ΛCDM model that could produce similar effects in the CMB spectrum (but not necessarily in the large-scale structure power spectrum). For instance, the marginal indication for enhanced damping in the small-scale CMB spectrum could also be explained by extra relativistic degrees of freedom [?]. Other simple extensions of the minimal ΛCDM model include small neutrino masses, spatial curvature, and a free primordial helium fraction. Here we do not consider such alternatives, but instead stick to the minimal ΛCDM paradigm and investigate only the issue of the primordial spectrum beyond the power-law assumption.

Indeed, motivated by theoretical models or features of the observed data, various other parameterizations of the primordial power spectrum have been considered: for example, a broken power spectrum [?] due perhaps to an interruption of the inflaton potential [?], a cutoff at large scales [?, ?] motivated by suppression of the lower multipoles in the CMB anisotropies [?, ?], and more complicated shapes of the spectrum caused by features in the inflaton potential [?].

Measuring deviations from scale invariance of the primordial power spectrum is a critical test of cosmological inflation. Either exact scale invariance or a strong deviation from scale invariance could falsify the inflationary paradigm. However, a strong theory prior on the form of the primordial power spectrum could lead to misinterpretation and biases in parameter determination. Several more general approaches have been proposed to reconstruct the shape of the primordial power spectrum from existing data, based on linear interpolation [?], cubic spline interpolation in log-log space [?], minimally-parametric reconstruction [?], wavelet expansions [?], principal component analysis [?], and direct reconstruction via deconvolution methods [?, ?, ?]. The first three approaches are sensitive to the overall shape of the spectrum while the last three reconstruction methods are sensitive to local features in the spectrum. Therefore they are complementary and needed to cross-check each other.

In this work we focus on uncorrelated band-power estimates of the primordial power spectrum to measure deviations from scale invariance, based on the localized principal component analysis introduced to study dark energy in [?]. We apply this method to the 7-year WMAP data [?] in combination with small-scale CMB data from the ACT experiment [?]. In our analysis we adopt two main astrophysical priors: the Hubble constant (H_0) measured from the magnitude-redshift relation of 240 low-z Type Ia supernovae at $z < 0.1$ [?], and the distance ratios of the comoving sound horizon to the angular diameter distances from Baryon Acoustic Oscillations (BAO) in the distribution of galaxies [?]. Moreover, we generate mock data for the Planck experiment and make forecasts using a Monte Carlo simulation approach. As expected, Planck could significantly improve constraints on the primordial power spectrum.

This paper is organized as follows. In Section 2 we describe the method and data used in our analysis, apply the method to the seven-year WMAP data and the combined WMAP+ACT data, and present our results. In Section 3 we analyze the sensitivity of the Planck experiment to the primordial power spectrum using a Monte Carlo approach. Section 4 presents our conclusions.

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2. Uncorrelated Constraints from Current Observations

We consider a spatially flat ΛCDM Universe described by the following cosmological parameters:

$$\Omega_b h^2, \Omega_c h^2, \Theta_s, \tau, A_1, A_2, ..., A_5$$

where $\Omega_b h^2$ and $\Omega_c h^2$ are the physical baryon and cold dark matter densities relative to the critical density, h is the dimensionless Hubble parameter such that $H_0 = 100h$ km s^{-1}Mpc^{-1}, Θ_s is the ratio of the sound horizon to the angular diameter distance at decoupling, and τ is the reionization optical depth. Since we do not consider extensions of the minimal flat ΛCDM model in this analysis, we fix the primordial helium fraction and effective neutrino number to their standard values, and do not introduce neutrino masses or tensor modes.

The primordial power spectrum parameters, $A_i \equiv \ln[10^{10} P_R(k_i)]$ for $i = 1, 2, ..., 5$, are the logarithmic values of the primordial power spectrum of curvature perturbations $P_R(k)$ at five knots k_i, equally spaced in logarithmic wavenumber between 0.0002 Mpc^{-1} and 0.2 Mpc^{-1}. To reconstruct a smooth spectrum with continuous first and second derivatives with respect to $\ln k$, we use cubic spline interpolation to determine logarithmic values of the primordial power spectrum between these nodes. Outside this wavenumber range we fix the slope of the primordial power spectrum at the boundaries since CMB data place only weak constraints on them. Using $\ln P_R(k)$ instead of $P_R(k)$ for splines ensures the positive definiteness of the primordial power spectrum at the expense of making the primordial power spectrum non-linear in the parameters. Otherwise, we must discard steps in the Markov chain if the interpolating spline between the knots goes negative due to steep slopes [**?**]. As discussed in [**?**], the method is insensitive to local features in the primordial power spectrum but sensitive to the overall shape.

The primordial spectrum parameters A_i are correlated due to geometric projection from the primordial power spectrum to the angular power spectrum and gravitational lensing. These correlations are encapsulated in the covariance matrix of the primordial spectrum parameters:

$$C = \langle (A_i - \langle A_i \rangle)(A_j - \langle A_j \rangle)^T \rangle$$

which can be obtained by taking the average of the Markov chain and marginalizing over other cosmological parameters. The diagonal elements of the covariance matrix are the variances of A_i and the non-diagonal elements represent correlations between the A_i bins that slowly decrease with increasing bin separation. To eliminate these correlations, we employ localized principal component analysis to construct a new basis where the new parameters $\tilde{A}_i$ are uncorrelated [?]. This variant of principal component analysis has recently been applied to probe the dynamics of dark energy [?, ?, ?]. We diagonalize the Fisher matrix $F \equiv C^{-1}$ such that:

$$F = O^T D O$$

where O is an orthogonal matrix and D is the diagonalized inverse covariance of the transformed bins. The localized principal component analysis corresponds to the weight matrix $W = O^T D^{1/2} O$, which is usually normalized so that its rows sum to unity. The weights are fairly localized in wavenumber since $D^{1/2}$ is absorbed into O. With this choice, the uncorrelated parameters can be obtained by changing the basis through the weight matrix rotation: $\tilde{A} = WA$. When discussing our results, we will generally refer to these uncorrelated estimates.

Data. We use the 7-year WMAP data (WMAP7) alone and in combination with the 148 GHz ACT data from its 2008 season. For WMAP, we use the low-l and high-l temperature and polarization power spectra. We also consider the Sunyaev-Zel' dovich (SZ) effect, in which CMB photons scatter off hot electrons in galaxy clusters. Given an SZ template, this is described by an SZ template amplitude A_{SZ} as in the WMAP papers [?, ?].

For the ACT data, we focus on band powers in the multipole range $1000 \leq l \leq 3000$. Following Ref. [?], for computational efficiency we set the CMB to zero above $l = 4000$ where the contribution is subdominant (less than 5% of the total power). To use the ACT likelihood described in [?], aside from A_{SZ} there are two additional secondary parameters, A_p and A_c. The former is the total Poisson power at $l = 3000$ from radio and infrared point sources, while the latter is the template amplitude of the clustered power from infrared point sources. We impose positivity priors on these three secondary parameters, use the SZ template and clustered source template provided by the ACT likelihood package, and marginalize over these secondary parameters to account for SZ and point source contamination. We adopt two main astrophysical priors: the present-day Hubble constant $H_0 = 74.2 \pm 3.6$ km s^{-1}Mpc^{-1} measured from the magnitude-redshift relation of 240 low-z Type Ia supernovae at $z < 0.1$ [?], and the distance ratios $r_s/D_V(z = 0.2) = 0.1905 \pm 0.0061$ and $r_s/D_V(z = 0.35) = 0.1097 \pm 0.0036$ measured from the two-degree field galaxy redshift survey and the Sloan Digital Sky Survey data [?]. Here r_s is the comoving sound horizon size at the baryon drag epoch and D_V is the effective distance measure for angular diameter distance.

[Figure 1: see original paper]

Results. Our analysis is carried out using a modified version of the publicly available CosmoMC package, which explores parameter space via Monte Carlo Markov Chains [?]. Figure 1 shows the uncorrelated constraints on the primordial power spectrum of curvature perturbations (68% and 95% CL) and the corresponding weight functions that describe the transformation from correlated parameters A_i to uncorrelated $\widetilde{A}_i$, derived from WMAP7+H_0+BAO (top panels) and from WMAP7+ACT+H_0+BAO (bottom panels). The power spectrum is best determined around $k \sim 0.007$ Mpc^{-1}, and less accurately determined at much lower and higher wavenumber due to cosmic variance and dominant noise respectively. As shown in the top-left panel of figure 1, 95% of the preferred models are incompatible with the assumption of scale invariance but remain compatible with a power-law primordial spectrum. Adding ACT data reveals more deviation from a simple scale-invariant spectrum due to reduced errors and a suppressed spectrum at high-k, though this is weaker than the corresponding result from WMAP adopting an inflation-motivated power-law spectrum prior [?]. The weights are fairly localized in k, as found in the context of dark energy measurements [?, ?, ?], and the weight functions in the top-right panel are similar to those in the bottom-right panel.

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3. Planck Forecast Constraints

In this section we apply Monte Carlo Markov Chain methods to assess the accuracy with which the primordial power spectrum can be constrained by the Planck experiment. Following the approach described in Ref. [?], we generate synthetic data for Planck and perform a systematic analysis on the simulated data. Assuming a fiducial ΛCDM model with a scale-invariant power spectrum, one can use a Boltzmann code such as CAMB [?] to calculate the angular power spectra C_l^{TT} for temperature, C_l^{TE} for cross temperature-polarization, C_l^{EE} for polarization, C_l^{dd} for the deflection field, and C_l^{Td} for cross temperature-deflection. We assume that beam uncertainties are small and that uncertainties due to foreground removal are smaller than statistical errors. For an experiment with known beam width and detector sensitivity, the noise power spectrum N_l^{TT} can be estimated. Here we use the FuturCMB package to calculate N_l^{dd} based on the quadratic estimator method proposed in [?], which provides an algorithm for estimating the noise spectrum of the deflection field from the observed CMB primary anisotropy and noise power spectra. For Planck we combine only the 100, 143, and 217 GHz HFI channels, with beam widths $\theta_{\rm FWHM} = (9.6', 7.0', 4.6')$ in arcminutes, temperature noise per pixel $\sigma_T = (8.2, 6.0, 13.1)$ in μK, and polarization noise per pixel $\sigma_E = (13.1, 11.2, 24.5)$ in μK (see Ref. [?] for Planck instrumental specifications). Given the fiducial spectra C_l and noise spectra N_l, one can generate mock data $\hat{C}_l$. We perform a Monte Carlo analysis through the likelihood function defined as:

$$-2\ln L = (2l+1)f_{\rm sky}\left|\bar{C}\right| + \ln\left|\hat{C}\right| - 3\ln\left|\bar{C}\right|$$

where

$$D = \hat{C}_l^{TT}\bar{C}_l^{EE} - (\hat{C}_l^{TE})^2 + \hat{C}_l^{TT}\bar{C}_l^{dd} - (\hat{C}_l^{Td})^2 + \hat{C}_l^{EE}\bar{C}_l^{dd}$$

$$\left|\bar{C}\right| = \bar{C}_l^{TT}\bar{C}_l^{EE}\bar{C}_l^{dd} - \bar{C}_l^{TT}(\bar{C}_l^{Td})^2 - \bar{C}_l^{EE}(\bar{C}_l^{Td})^2$$

$$\left|\hat{C}\right| = \hat{C}_l^{TT}\hat{C}_l^{EE}\hat{C}_l^{dd} - \hat{C}_l^{TT}(\hat{C}_l^{Td})^2 - \hat{C}_l^{EE}(\hat{C}_l^{Td})^2$$

Here $\bar{C}_l = C_l + N_l$ is the theoretical spectrum plus noise and $f_{\rm sky}$ is the sky fraction after foreground removal. For Planck we choose $f_{\rm sky} = 0.65$, corresponding to a $\pm 20^\circ$ galactic cut, and consider data up to $l = 2000$.

[Figure 2: see original paper]

Our results are presented in figure 2 and table 1 for Planck simulated data. As shown in table 1, Planck will reduce the uncertainties in $\tilde{A}_i$, particularly in $\tilde{A}_4$ by a factor of 3.9 and in $\tilde{A}_5$ by a factor of 4.7. Since large uncertainties in the power spectrum at low-k mainly arise from cosmic variance, measurement of $\tilde{A}_1$ is limited. The weight functions in the right panel of figure 2 are somewhat better localized in wavenumber than those in figure 1 because the weak lensing effect is extracted from CMB maps provided by Planck. Furthermore, we have verified that the weight functions depend only weakly on the fiducial cosmological model.

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4. Conclusions

Most inflationary models predict small deviations from a scale-invariant power spectrum. Therefore, measurements of deviations from an exact scale-invariant spectrum would provide a firm probe of the dynamics of the inflationary phase in the early Universe. The local principal component technique is a powerful tool for measuring deviations from the scale-invariant spectrum, complementary to other approaches for reconstructing the primordial power spectrum or directly testing slow-roll inflation [**?**, **?**, **?**]. In this paper we have used localized principal component analysis to produce uncorrelated estimates of the primordial power spectrum of curvature perturbations. Within the framework of a minimal ΛCDM model, we found that more than 95% of the preferred models are incompatible with the scale-invariant spectrum but remain compatible with a power-law primordial spectrum when using the 7-year WMAP data in combination with ACT data. This conclusion is slightly stronger than the corresponding result in Ref. [**?**] but weaker than when an inflation-motivated power-law prior is adopted. We have performed a systematic analysis of future constraints on the primordial power spectrum achievable with the Planck experiment. We found that Planck would be able to shrink the error bars on the spectrum bins, especially at small scales, by roughly a factor of 4, which is promising for definitively detecting these deviations.

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Note: Figure translations are in progress. See original paper for figures.