Non-Gaussian features from the inverse volume corrections in loop quantum cosmology (postprint)

Authors: Li-Fang Li, Rong-Gen Cai, Zong-Kuan Guo, Bin Hu

Date: 2017-09-27T00:00:00+00:00

Abstract

In this paper we study the non-Gaussian features of the primordial fluctuations in loop quan-tum cosmology with the inverse volume corrections. The detailed analysis is performed in the single field slow-roll inflationary models. However, our results reflect the universal characteristics of bispectrum in loop quantum cosmology. The main corrections to the scalar bispectrum come from two aspects: one is the modifications to the standard Bunch-Davies vacuum, the other is the corrections to the background dependent variables, such as slow-roll parameters. Our calculations show that the loop quantum corrections make fNL of the inflationary models increase 0.1%. More-over, we find that two new shapes of non-Gaussian signal arise, which we name F1 and F2. The former gives a unique loop quantum feature which is less correlated with the local, equilateral and single types, while the latter is highly correlated with the local one.

Full Text

Preamble

Non-Gaussian Features from Inverse Volume Corrections in Loop Quantum Cosmology

Li-Fang Li[1,*], Rong-Gen Cai[1,†], Zong-Kuan Guo[1,‡], and Bin Hu[2,§]

[1]State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China

[2][Affiliation not provided in original]

∗ Email: lilf@itp.ac.cn
† Email: cairg@itp.ac.cn

‡ Email: guozk@itp.ac.cn
§ Email: hubinitp@gmail.com

Abstract

In this paper we study the non-Gaussian features of primordial fluctuations in loop quantum cosmology with inverse volume corrections. We perform a detailed analysis within single-field slow-roll inflationary models, though our results reflect universal characteristics of the bispectrum in loop quantum cosmology. The main corrections to the scalar bispectrum arise from two sources: modifications to the standard Bunch-Davies vacuum and corrections to background-dependent variables such as slow-roll parameters. Our calculations show that loop quantum corrections increase $f_{\rm NL}$ of inflationary models by approximately 0.1%. Moreover, we find that two new shapes of non-Gaussian signal emerge, which we denote as F_1 and F_2. The former provides a unique loop quantum feature that is weakly correlated with the local, equilateral, and single shapes, while the latter is strongly correlated with the local shape.

PACS numbers: 04.60.Pp, 98.80.Bp, 98.80.Jk

Introduction

As a non-perturbative and background-independent theory, Loop Quantum Gravity (LQG) has achieved remarkable successes in recent years, including the derivation of quantized area and volume operators, calculations of black hole entropy, and the development of Loop Quantum Cosmology (LQC). The non-perturbative quantization procedure of LQG is also applicable to a broader class of four-dimensional metric theories of gravity. As a specific application of LQG, LQC provides a quantization framework for symmetry-reduced models of the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker universe. The discrete spacetime geometry in LQC scenarios predicts a non-singular bouncing universe in simplified models that satisfies most astronomical and cosmological observational constraints. Although quantum correction effects become diluted as the universe expands, they persist in a weaker form, particularly on or near super-horizon scales.

Recently, gauge-invariant cosmological perturbation theory has been systematically constructed for inverse volume corrections and for holonomy corrections. Inverse volume corrections and holonomy corrections represent the two primary quantum corrections in LQC. Inverse volume corrections arise from the quantization of the inverse volume operator in LQG. Inverse volume terms appear in the Hamiltonian constraint of gravity and the matter Hamiltonian, particularly in kinetic terms. Since the volume operator can take zero values, well-defined inverses do not exist. Fortunately, using the Thiemann trick, we can construct well-defined inverse volume operators that introduce quantum corrections. Holonomy corrections, meanwhile, emerge from loop quantization based on holonomies rather than direct connections and become important when the energy scale of the universe approaches the Planck scale. Both modifications to scalar and tensor primordial power spectra from inverse volume corrections have been carefully investigated, showing that these corrections could enhance power spectra on large scales, producing a red tilt. However, similar features could arise from other mechanisms such as non-commutative geometry or string theory. Therefore, investigating non-Gaussian features in loop quantum cosmology is essential for seeking signatures unique to LQC.

Primordial non-Gaussianities are powerful tools for distinguishing inflationary models, and numerous studies have been devoted to examining non-Gaussianities in various inflation models. Inspired by previous work, this paper focuses on non-Gaussianities from inverse volume corrections in LQC. We consider only inverse volume corrections for the following reason: Let $\delta_{\rm inv}$ denote the correction term from the inverse volume operator and $\delta_{\rm hol}$ denote the correction from holonomy effects. We can estimate the inverse volume correction as $\delta_{\rm inv} \sim (\rho_{\rm Pl}/\rho)^{1/2}$, where $\rho_{\rm Pl}$ is the Planck density assumed to be the quantum gravity scale. This expression reveals that inverse volume corrections behave differently from typical quantum gravity expectations. At low densities, holonomy corrections are small, but inverse volume corrections may remain significant because they are amplified by the inverse of $\delta_{\rm hol}$. For example, small holonomy corrections of size $\delta_{\rm hol} < 10^{-6}$ require inverse volume corrections larger than $\delta_{\rm inv} > 10^{-6}$ even at densities $\rho \sim 10^{-9}\rho_{\rm Pl}$. These novel features make investigations of inverse volume corrections more interesting than holonomy corrections at sub-Planckian inflationary scales, which is why we focus exclusively on inverse volume corrections in this work.

Explicitly, in LQC perturbation theory, the inverse volume operator can be captured by a correction function of the form $\bar{\alpha} \simeq 1 + \alpha_0\delta_{\rm inv} = 1 + \alpha_0(a_{\rm inv}/a)^\sigma$, where a is the scale factor of the FLRW universe and $a_{\rm inv}$ describes the characteristic scale of inverse volume corrections, which is not generally the Planck scale. When $a_{\rm inv}/a \ll 1$, we can ignore the correction term. However, if $a_{\rm inv}/a \gtrsim 1$ during inflation, inverse volume corrections cannot be neglected. In this case, the correction approximates $\alpha_0\delta_{\rm inv}(k) \simeq \delta(k_0)(k_0/k)^\sigma$, where k and k_0 are respectively the perturbation wavenumber and a characteristic wavenumber involved in the inverse volume correction. Additionally, many LQC studies imply that $\sigma \in [0, 6]$. From the form of the inverse volume correction, we see that a small σ corresponds to small inverse volume corrections and vice versa. As an example, following previous work, we take $\sigma = 2$ in this paper; other σ values yield similar behavior. Furthermore, in terms of spherical multipoles the wavenumber can be expressed as $k \sim 10^{-4}hl$, with h representing the reduced Hubble parameter $h = 0.7$ and l the spherical multipoles. In the typical linear regime of Cosmic Microwave Background (CMB), multipoles l range from $2 < l < 1000$ or higher. Given these considerations, we expect new features to emerge in non-Gaussianities. The purpose of this work is to investigate the characteristic sizes and shapes of bispectra in LQC scenarios.

Note that bispectra for single-field slow-roll inflationary models have been calculated in several papers. For simplicity and comparison, we study the simplest single-field slow-roll inflationary model in LQC. Our results show that quantum corrections mainly arise from the third-order interaction Hamiltonian, the corrected vacuum state, and corrections to slow-roll parameters. The paper is organized as follows: In Section II, we briefly review the canonical formalism and slow-roll inflationary models in LQC scenarios. In Section III, we study the power spectrum in LQC and recover previous results. The effect of inverse volume corrections on non-Gaussianity in LQC is investigated in Section IV, with detailed analysis of sizes, shapes, and shape correlations presented in Sections IV and V. Throughout this paper we set $8\pi\gamma G = 1$ and adopt Einstein' s summation convention.

II. Review of Loop Quantum Cosmology

We briefly present the framework of LQG/LQC in this section. First, we discuss the canonical formalism in LQG, then introduce the dynamics of slow-roll inflationary models in LQC scenarios.

A. The canonical formalism in loop quantum gravity

In the LQG framework, the spatial metric as a canonical field is replaced by the densitized triad E^a_i, defined as $E^a_i = \det(e^b_j)e^a_i$, where e^a_i is the inverse of the cotriad e^b_j. The canonically conjugate variable to the densitized triad is the Ashtekar-Barbero connection $A^i_a = \Gamma^i_a + \gamma K^i_a$, where K^i_a is the extrinsic curvature and $\gamma \approx 0.274$ is the Barbero-Immirzi parameter. The densitized triad E^a_i and Ashtekar-Barbero connection A^i_a satisfy the commutator relation $\{A^i_a(x), E^b_j(y)\} = \gamma\delta^i_j\delta^3(x,y)$. The spin connection Γ^i_a is defined such that it leaves the triad covariantly constant and has the explicit form $\Gamma^i_a = -\epsilon^{ijk}e^b_j(\partial_{[a}e^k_{b]} + e^c_k\partial_{[c}e^m_{b]}\delta_{lm})$.

In the new Ashtekar variables, the Einstein-Hilbert action can be expressed in canonical form as $S_{\rm EH} = \int d^3x \dot{K}^i_a E^a_i - \int dt(D_{\rm grav}[N^a] + H_{\rm grav}[N] + G_{\rm grav}[\Lambda^i])$, where the diffeomorphism constraint is $D_{\rm grav}[N^a] = \int d^3x N^a(\partial_a A^i_b)E^b_i$ and the Hamiltonian constraint can be expressed as $H_{\rm grav}[N] = \int d^3x N\epsilon^{jk}_i(\partial_a A^i_b + \epsilon^i_{mn}A^m_a A^n_b)E^a_j E^b_k/\sqrt{\det E}$. The Gaussian constraint is $G_{\rm grav}[\Lambda^i] = \int d^3x \Lambda^i(\partial_a E^a_i + \epsilon^k_{ij}A^j_a E^a_k)$, which can be solved through standard procedures. Thus, solutions for scalar mode perturbations are completely determined by the Hamiltonian constraint and the diffeomorphism constraint.

B. Slow-roll inflationary models

In this subsection, we briefly review the inflationary dynamics of the Friedmann-Lemaître-Robertson-Walker universe in LQC scenarios. The modified Friedmann equation, Raychaudhuri equation, and Klein-Gordon equation are respectively given by:

[Equations would be preserved here as in original]

where $\bar{p} = a^2$ and a prime denotes derivative with respect to conformal time. $\bar{\alpha}$ and $\bar{\nu}$ are correction functions for inverse volumes and read $\bar{\alpha} \simeq 1 + \alpha_0 \delta_{\rm inv}$, $\bar{\nu} \simeq 1 + \nu_0 \delta_{\rm inv}$.

Following previous work, the slow-roll parameter ϵ can be straightforwardly calculated as $\epsilon = \frac{1}{2\bar{\nu}} \frac{d\bar{\nu}}{d\bar{p}} = \epsilon_0(1 + \gamma_\epsilon \delta_{\rm inv})$, where ϵ_0 denotes the usual slow-roll parameter. The explicit form of coupling constant γ_ϵ is given by [equation]. Typically we set $\alpha_0 = 0.06$, $\nu_0 = 0.17$, and $\epsilon_0 = 0.01$ in this paper. δ_0 is determined by quantum corrections and analysis after Eq.(43) implies it takes values of order 10^{-3}. As mentioned in the Introduction, we set $\sigma \in [0, 6]$ in this work. From the above expression we can easily estimate that γ_ϵ is of order 10^{-3}. Formally we can also express the other slow-roll parameter η as $\eta = \frac{1}{\bar{\nu}} \frac{d^2\bar{\nu}}{d\bar{p}^2} = \eta_0(1 + \gamma_\eta \delta_{\rm inv})$, where η_0 denotes the usual slow-roll parameter similar to ϵ_0.

As in the usual single-field inflation model, we can assume the two slow-roll parameters ϵ_0 and η_0 are both of order 10^{-2}. Thus the typical values of coupling constants γ_η and γ_ϵ are of order 10^{-3}. The terms proportional to $\delta_{\rm inv}$ represent inverse volume corrections. Here we emphasize that the subscript "inv" is introduced to avoid confusion with perturbations such as $\delta\phi$.

III. Power Spectrum

In this section, we first review the formalism of scalar perturbations in LQC, then derive the second-order Hamiltonian, and finally calculate the primordial power spectrum in the spatially flat gauge.

A. Formalism on the scalar modes

Considering only scalar perturbations, the general form of a perturbed metric around the isotropic FRW background is:

[Metric perturbation equations]

where the scale factor a is a function of conformal time τ, and (ϕ, ψ, E, B) are the four scalar metric perturbations. In perturbation theory, the triad can be described by $E^a_i = \bar{E}^a_i + \delta E^a_i$, where $\bar{E}^a_i = \bar{p}\delta^a_i$ and $\delta E^a_i = -2\bar{p}\psi\delta^a_i + \bar{p}(\delta^a_i \Delta - \partial_a \partial_i)E$.

The perturbed triad is described by the spatial part of the perturbed metric through ψ and E. Here Δ is the Laplace operator in flat space. Similarly, the perturbed lapse function and shift vector can be described by the other two scalar metric perturbations ϕ and B respectively: $\delta N = \bar{N}\phi$, $\delta N^a = \partial^a B$.

The extrinsic curvature can be perturbed as $K^i_a = \bar{K}^i_a + \delta K^i_a = \bar{k}\delta^i_a + \delta K^i_a$. For a general triad, the linearized spin connection becomes [equation]. As described above, the symplectic structure also splits into two parts: one for background variables and one for perturbations, with the background variables defined by [equations]. Here V_0 is some artificial finite volume.

In this paper, the matter sector is represented by a scalar field ϕ. Similarly, we split the field ϕ and its conjugate momentum π into homogeneous and inhomogeneous parts: $\phi = \bar{\phi} + \delta\phi$, $\pi = \bar{\pi} + \delta\pi$. The basic Poisson brackets then reduce to [equations].

For simplicity, we introduce the LQC formalism with finite cell V_0 rather than the whole $\mathbb{R}^3$ region in the above description. However, the unphysical feature of V_0 can be remedied by the lattice refinement model. Since our subsequent calculations only involve δ_{inv}, we adopt the lattice refinement parametrization procedure from previous work to eliminate the effect of the artificial volume V_0.

B. The second order Hamiltonian

According to previous work, the quantum-corrected second-order Hamiltonian constraint can be conveniently written as:

[Hamiltonian constraint equations]

where the definitions of counterterms can be found in the cited references. The perturbed second-order diffeomorphism constraint can be expressed as [equation].

Based on this corrected Hamiltonian, we obtain the homogeneous and inhomogeneous parts of the matter field as [equations], where f_1 and g_1 are counterterms.

C. Power spectrum

Calculations are greatly simplified in the spatially flat gauge ($\psi = 0$, $E = 0$), because the perturbed triad vanishes ($\delta E^a_i = 0$) in this gauge. We fix the gauge after having incorporated quantum corrections into the Hamiltonian and verified consistency. In contrast, some references fix the gauge beforehand, but we believe our treatment is more consistent.

By solving the constraint equations, the perturbed lapse function and shift vector are given by [equations]. The extrinsic curvature is [equation], where $\bar{\alpha}\bar{k} = \mathcal{H}$.

In the spatially flat gauge, the total second-order Hamiltonian becomes [equation], where $\bar{\theta}$ is another correction function for inverse volume and $\bar{\alpha}^2 = \bar{\nu}\bar{\theta}$. In this gauge, the dynamical inflaton perturbation $\delta\phi$ coincides with the Sasaki-Mukhanov variable $u = z\zeta$, where z is defined by [equation].

One can then derive the Mukhanov equation [equation], where c_s is the propagation speed of perturbations. The solution of this equation is given by [equation], where $\chi = \sigma\nu_0(1 + \sigma/6)/3 + \alpha_0(5 - \sigma/3)/2$, $C = \chi/(\sigma + 1)$, and $\delta_0 = \delta(k_0)/\alpha_0$. The variable $\delta(k_0)$ is constrained by cosmic observational data such as CMB and large-scale structure. For specific inflationary models with quadratic potential and $\sigma = 2$, δ_0 is of order 10^{-5}. In this paper we take $\alpha_0 \sim \mathcal{O}(10^{-2})$, i.e., the variable δ_0 is of order 10^{-3}.

Moreover, Eq.(42) tells us that, on the one hand, inverse volume corrections become important for long-wavelength modes with $k \ll k_0$; on the other hand, long-wavelength modes cross the horizon earlier than short ones. This means inverse volume corrections leave stronger imprints on large scales than small ones. These features differ significantly from those of inflationary models with higher-derivative terms such as K-inflation in Einstein gravity.

Using canonical quantization, we have $\zeta(\mathbf{k},\tau) = \zeta^+ + \zeta^- = \zeta(\mathbf{k},\tau)a_{\mathbf{k}} + \zeta^*(\mathbf{k},\tau)a^\dagger_{\mathbf{k}}$, where $\zeta(\mathbf{k},\tau) = u(\mathbf{k},\tau)/z(\mathbf{k})$. The two-point correlation functions of curvature perturbations can then be calculated straightforwardly as [equation], where the first and second terms come from the $z(\mathbf{k})$ factor in gauge transformations, and the third term attributes to modifications of the vacuum state (42).

Finally, we obtain the primordial power spectrum of curvature perturbations as [equation]. This result agrees with that in previous work. When all corrections vanish, this reduces to the standard single-field inflationary model in Einstein gravity. The primordial power spectrum and angular power spectrum are plotted in [Figure 1: see original paper], where the dotted (purple), dashed (deep blue), and solid (light blue) curves correspond to different values of the parameter C ($C = 0, 4 \times 10^{-4}, 3 \times 10^{-3}$) defined after Eq.(43).

IV. Bispectrum

In this section, we first derive the third-order Hamiltonian in the spatially flat gauge, then calculate the three-point functions of primordial curvature perturbations, and finally determine the sizes and shapes of the bispectrum.

A. The third order Hamiltonian and In-In formalism

We obtain the corrected third-order Hamiltonian [equation], where the expressions for $H^{(3)}_{\text{grav}}$, $H^{(3)}_{\pi}$, $H^{(3)}_{\nabla}$, and $H^{(3)}_{\phi}$ are complicated and listed in Appendix A.

The perturbed third-order diffeomorphism constraint is [equation]. We calculate the non-Gaussianity in the interaction picture using [equation]. Up to first order, we have [equation].

B. Sizes and shapes

Based on the second-order anomaly-free perturbative LQC theory, we combine Eq.(48) with Eq.(49) to obtain the third-order interaction Hamiltonian with counterterms:

[Interaction Hamiltonian equation]

where the first three terms are leading order under the slow-roll approximation. In our calculations, we consider only these terms. Comparing Eq.(52) with standard results, we can attribute two kinds of modifications in the interaction Hamiltonian to inverse volume corrections: one from modifications to the vacuum state (42), and the other from background-dependent coefficients such as $(\bar{\nu}, \bar{\theta}, \bar{\alpha}, g_1, ...)$. However, non-Gaussian signatures from the latter are heavily contaminated by cosmic variance. Hence, we ignore modifications in $\bar{\nu}$ etc. when calculating parts of three-point functions directly from the In-In formalism.

In other words, we ignore quantum anomaly behavior for the Hamiltonian in this work. To demonstrate our reasoning more clearly, we take the first term in (52) as an example. When we substitute these terms into the formalism (51) and perform the time integral, we obtain corrections of the form [equation], where the dots denote higher-order corrections. This term peaks at $\bar{k} \sim K$. Since larger k gives smaller $\delta_{\rm Pl}$ and the quadrupole is the lowest detectable mode in CMB, $\bar{k}$ corresponds to $\bar{l} = 2$. Thus Eq.(54) peaks in the very low $l \sim 2$ region where cosmic variance dominates over signals (see [Figure 1: see original paper] and [Figure 2: see original paper] in previous work). Although non-Gaussian features appear on both large and small scales, this analysis implies we can ignore effects on small scales. Therefore, we neglect such terms in our calculations.

Once we ignore corrections to background-dependent coefficients in Eq.(52), the third-order interaction Hamiltonian reduces to the usual form. To eliminate terms proportional to linear equations, we perform the field redefinition [equation]. Then the interaction Hamiltonian can be reduced to the simple form [equation].

According to standard results, after a field redefinition of the schematic form $\zeta = \zeta_c + \lambda\zeta_c^2$, the correlation function contains two terms: one computed by the In-In formalism and the other from the field redefinition itself.

Let us first calculate the first term [equation]. We emphasize again that corrections from background-dependent coefficients are neglected in these expressions. However, in subsequent calculations of contributions from field redefinition, such corrections must be included for consistency. Because background-dependent coefficients $(\bar{\phi}, ...)$ take values when the corresponding mode crosses the horizon $(\tau_* \sim \mathcal{H}_*^{-1}, k^{-1})$, they contain corrections such as $(k_0/k_i)^\sigma$. These terms become important, particularly in the squeezed triangle limit $(k_1 \ll k_2, k_3)$.

Contributions from field redefinition can be decomposed into two parts: [equations]. In summary, the forms of interaction Hamiltonian are exactly the same as usual, but inverse volume corrections $\delta_{\rm inv}$ contribute to the bispectrum. There are two main sources: modifications to the standard Bunch-Davies vacuum and the $z(k)$ factor in the gauge transformation $\zeta(k) = u(k)/z(k)$. Expanding these results in terms of $\delta_{\rm inv} = \delta_0(k_0/k_i)^\sigma$ yields [equation], where $F_{\rm single}$ is the usual leading term and F_1, F_2 are inverse volume correction terms.

The coefficients are given by [equations]. Notably, corrections from terms in the denominators of Eqs.(58), (59), and (60) are absorbed into F_2. The F_1 shape provides a unique signal from the LQC mechanism and, importantly, is independent of inflationary models because $\prod_i z^2(k_i)$ terms always appear in gauge transformations. Thus it is a universal signal in LQC scenarios.

The single shape is the usual one, while F_1 and F_2 arise only in LQC scenarios. The sizes of these two new parts are proportional to parameters $(\omega_1, ...)$, which are of order $(10^{-3}, 10^{-4})$. This means these new non-Gaussian features from LQC are smaller than those in standard Einstein gravity inflationary models by at least a factor of 10^{-3}. Although this is a tiny number, considering these features originate from quantum effects, this factor is not as small as initially expected. Furthermore, since inverse volume corrections in the interaction Hamiltonian can be neglected (allowing us to use the usual Hamiltonian directly in the In-In formalism by substituting the Bunch-Davies vacuum state with Eq.(42)), we argue that expectations for bispectrum sizes should hold for any inflationary models in LQC scenarios.

The difference between shapes is illustrated in [Figure 2: see original paper] and [Figure 3: see original paper]. All three shapes peak at the squeezed limit $(x_2 = 1, x_3 = 0)$, but substructures differ. Compared to the single shape, the F_1 shape upraises at another corner $(x_2 = 0, x_3 = 0.5)$, while F_2 flattens at the same point. From Figure 3: see original paper, we also see that F_1 and F_2 peak more dramatically in the squeezed corner.

V. Shape Correlations

In Figures 2 and 3, all three shapes appear similar. To quantify differences among shapes more precisely, we calculate correlations between them.

First, we define a 3D shape function [equation], where N is a normalization factor that does not affect subsequent calculations. Then we construct the product of two shape functions [equation], where $\omega(k_1, k_2, k_3)$ is a weight function and V_k is the integration domain constrained by the triangle inequality. Finally, we obtain the 3D shape function correlator [equation], which describes cross-correlations between two different shapes.

For a large class of well-motivated shapes, these descriptions can be simplified. One can define the distance from the origin of (k_1, k_2, k_3)-momentum space to a particular triangle slice perpendicular to the $(1, 1, 1)$ direction as [equation]. Then we introduce two new variables [equations]. In the domain constrained by the triangle inequality, $0 \leq \alpha, \beta \leq 1$. For models with homogeneous shape, meaning the powers of wavenumber in shapes are homogeneous, the k dependence in the 3D shape function can be separated as $S(k_1, k_2, k_3) = f(k)S(\alpha, \beta)$, with $V_k = dk_1 dk_2 dk_3 = k^2 dk d\alpha d\beta$. In fact, for the models considered here, $f(k) = \text{const}$. Hence we can focus on the 2D shape function $S(\alpha, \beta)$, and the integral over k cancels when calculating shape function correlators.

Further, one can introduce variables to square the integration regime. Using these variables, the integral measure becomes $d\alpha d\beta = xdxdy$. From this we read the weight function $w(x, y) = x$. This choice works well for all shapes mentioned in previous work. However, for our new shapes F_1 and F_2, the correlation matrices c_{mn} defined in (84) suffer from divergence. We therefore use new variables to eliminate such divergence: $d\alpha d\beta = \xi^3 d\xi dy$.

Using variables (ξ, y), we can decompose the shape $S(\xi, y)$ on any triangle slice with analogous radial polynomials $R_m(\xi)$ and shifted Legendre polynomials $\bar{P}_n(y)$: [equation]. The first few eigenfunctions are [equations]. Thus one can define the correlation matrix [equation].

The c_{mn} matrices for local, equilateral, single-field, F_1, and F_2 shapes are listed in [equation] and [Figure 4: see original paper]. From [Figure 4: see original paper], we see that the F_1 term differs explicitly from others, while F_2 is almost indistinguishable from the local form visually.

Armed with these results, one can calculate the 2D shape correlator [equation], where the product is defined through [equation]. The numerical results of the 2D correlators are listed in [TABLE:I]. We find that F_2 possesses high correlations with the local (particularly), equilateral, and single forms, while F_1 has low correlations with all other four shapes. This means the latter can provide a new window for probing loop quantum mechanisms using cosmic primordial bispectrum information. As argued previously, F_1 is a universal shape in LQC scenarios, so we can identify this shape as a new signature of LQC.

Finally, let us estimate the parameter $f_{\rm NL}$. Focusing on the two new shapes $F_{1,2}$, from Table I we see that F_2 is highly correlated with the local form, while F_1 is less correlated. The contributions to $f_{\rm NL}^{\rm local}$ from these shapes can be estimated as [equation], where ω represents parameters $(\omega_1, \omega_2', \omega_3')$ in (63) and (64), with typical values of order 10^{-3}. Since $f_{\rm NL}^{\rm local}$(single) in the usual case is of order ϵ_0, contributions from inverse volume corrections are completely negligible. However, as argued before, our results should be robust for other inflationary models in LQC scenarios, especially those with large non-Gaussianities such as K-inflation and DBI inflation. Thus one can anticipate that in models with large non-Gaussianities, features from inverse volume corrections in LQC scenarios might become observable.

VI. Conclusions

In this paper we investigated contributions to the cosmic primordial scalar bispectrum from inverse volume corrections in LQC scenarios. We derived the interaction Hamiltonian but found that new interactions contribute significantly only to modes with $k_1 + k_2 + k_3 \ll \bar{k}$. Because the scales corresponding to $\bar{k}$ are very large (we take $\bar{k} \sim 0.00014\text{Mpc}^{-1}$ corresponding to quadrupole mode $\bar{l} = 2$), it means the three wavelengths in the bispectrum are all on super-horizon scales. On such large scales cosmic variance usually dominates over signals, so we neglected these new interactions in our calculations. That is, the interaction Hamiltonian we used has the same form as for single-field inflation models in Einstein gravity. This greatly simplifies our calculations and, more importantly, makes our results robust—they should hold for other inflationary models in LQC scenarios.

Although the Hamiltonian shares the same form as in Einstein gravity, inverse volume corrections $\delta_{\rm inv} \propto (k_0/k)^\sigma$ still contribute to the bispectrum. Roughly speaking, there are two aspects: one from deviations from the standard Bunch-Davies vacuum, and the other from non-trivial gauge transformations $\zeta(k) = u(k)/z(k)$ from the spatially flat gauge to observable curvature perturbations. Consequently, we obtained three-point functions of gauge-invariant curvature perturbations. We found that, besides the usual single component in slow-roll inflation models, two new shapes arise from corrections, namely F_1 and F_2. Furthermore, we performed careful analysis of the new shapes. The whole profiles for all three shapes (single, F_1, F_2) are visually similar—they peak at the squeezed limit. However, substructures differ: compared to the single shape, the F_1 shape upraises at another corner, while F_2 flattens at the same point, and F_2 peaks more dramatically in the squeezed corner.

Additionally, we investigated correlations among five shapes, including local, equilateral, single, F_1, and F_2. The results show that F_2 is highly correlated with the local type, while F_1 is less correlated. This means the latter can provide a new window for probing loop quantum mechanisms using cosmic primordial bispectrum information. Finally, we estimated the observable parameter $\Delta f_{\rm NL}^{\rm local} \sim \mathcal{O}(10^{-3}) f_{\rm NL}^{\rm local}(\text{inflation})$ from inverse volume corrections.

The non-Gaussianity from inverse volume corrections in LQC scenarios is tiny and currently undetectable. However, considering these features are generated by quantum effects (naively expected to be of order $(\text{GUT/Planck})^\sigma \sim (10^{-5})^\sigma$), our findings become non-trivial. Especially, the results obtained here should generalize directly to other inflationary models with large non-Gaussianities, where inverse volume corrections would also become large. In addition, this paper investigated only non-Gaussianities from inverse volume corrections while ignoring those from holonomy corrections, which typically change the sound speed of scalar perturbations. Furthermore, we studied only the bispectrum from scalar modes, leaving tensor modes unexplored. These topics warrant further investigation.

Acknowledgments

We thank Qing-Guo Huang and Yun-Song Piao for useful discussions. BH thanks the hospitality of ITP-CAS/KITPC during his visit. This work is partially supported by the Knowledge Innovation Program of the Chinese Academy of Sciences, the National Basic Research Program of China under Grant Nos. 2010CB832805 and 2010CB833004, and the National Natural Science Foundation of China (Nos. 10821504, 10975168, 11175225, and 11035008).

Appendix A: Interacting Hamiltonian

In this appendix, we derive the perturbed Hamiltonian density and perturbed diffeomorphism constraint up to third order. According to previous work, the classical Hamiltonian includes two parts: $H[N] = H_{\mathrm{grav}}[N] + H_{\mathrm{matter}}[N]$. The gravitational Hamiltonian can be expressed in terms of extrinsic curvature [equations], and the matter part is [equations]. The diffeomorphism constraint is [equations].

To obtain the perturbed Hamiltonian density and perturbed diffeomorphism constraint up to third order, we need the following relations. We expand $(\det E)^{1/2}$ and $(\det E)^{-1/2}$ to third order as [equations].

Thus the third-order gravitational Hamiltonian density can be written as [equations]. The matter Hamiltonian can be expressed as [equations]. Combined with previous results, we obtain the perturbed Hamiltonian [equations]. Here we have ignored high-order correction terms caused by inverse volume, such as $\alpha^{(2)}H^{(2)}$, because in the in-in formalism these terms do not contribute to non-Gaussianity.

The diffeomorphism constraint up to third order is [equations].

Appendix B: Definitions of shape functions

For all shapes considered in this paper, $f(k) = \mathrm{const}$, so we have $S(k_1, k_2, k_3) = S(\xi, y)$. The explicit forms are:

For the local shape: [equation]
For the equilateral shape: [equation]
For the single shape: [equation]
For the F_1 shape: [equation]
For the F_2 shape: [equation]

[Bibliographic references would be preserved exactly as in original]

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.