On asymmetric brane creation (postprint)

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Abstract

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Full Text

Preamble

On Asymmetric Brane Creation

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We exhaustively study brane instanton solutions—an Einstein brane inhabiting different positions in a 5-dimensional Einstein bulk with negative curvature. We construct a brane instanton model consisting of a brane with asymmetric bulk on its two sides, and analyze the junction condition of the resulting spacetime within the framework of induced gravity (DGP model). In the spirit of quantum gravity path integral formalism, we calculate the Euclidean actions on three canonical paths and compare the Euclidean actions of different instantons per unit 4-volume. We also compare the Euclidean actions per unit 4-volume of instantons consisting of a brane gluing to a fixed half with other Euclidean actions of halves possessing different cosmological constants.

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Introduction

Quantum cosmology deals with the creation of our universe where and when classical gravity theory may fail. In the formula Ψ[Σ, h] = D[g_E] exp(-S_E[M, g_E]), Ψ is the propagator from a 3-manifold (usually taken to be nothing) to (Σ, h), and S_E is the Euclidean action of the "path" M. The standard model of the quantum origin of the universe begins with a configuration consisting of a compact, path-connected, oriented Riemannian manifold M_R adhered to a Lorentzian manifold M_L by a totally geodesic spacelike hypersurface Σ, which serves as an initial Cauchy surface for the Lorentzian evolution on manifold M_L. Given this setup, we pass to the double 2M_R = M_R _Σ M_R by joining two copies of M_R across Σ.

This is a closed, path-connected, oriented Riemannian 4-manifold M = 2M_R with a mirror isometry that fixes the totally geodesic submanifold Σ. Here M is a very generic topological manifold. A smaller set is often considered: the gravitational instantons, where M is an Einstein manifold [1]. Here we generalize the concept of instanton—we permit non-mirror symmetry between M_R and M_R' . On the other hand, the concept of branes emerging in recent years is very important in high energy physics and cosmology. In the brane world scenario, standard model particles are confined to the 3-brane, while gravity can propagate in the whole space [2]. An inflating brane world with positive curvature and mirror symmetry created from "nothing" together with its Anti-de Sitter (AdS) bulk has been considered in [3]. Then the creation of the inflationary brane universe in 5D bulk Einstein and Einstein-Gauss-Bonnet gravity has been analyzed in [4]. Brane instantons in F-theory have been explored in [5]. The relation between brane instanton and Taub-NUT in M-theory is studied in [6]. Brane instanton intersecting at sine angle is presented in [7]. Many different types of brane world creation models have been investigated widely [8]. Brane world models without mirror symmetry have been investigated to some extent: different black hole masses on the two sides of the brane [9], zero black hole mass and different cosmological constants on the two sides [10], and allowing for both types of generalizations [11]. The quantum creation of closed branes by totally antisymmetric tensor and gravity was treated as an interesting solution to cosmological problems in [12]. In this paper, we shall investigate the creation of a brane world model with non-mirror-symmetric bulk.

We study all three types of solutions—negative curvature, positive curvature, and Ricci-flat Einstein branes inhabiting different positions in a 5D negative curvature Einstein manifold and investigate instantons including branes without mirror symmetry in bulk in Section II. Negative curvature Einstein manifold means R = kg, where R is the Ricci tensor, g denotes the metric, and k is a negative real number; while positive curvature is characterized by a positive k. In Section III we analyze the junction condition of the brane instanton solutions in the induced gravity framework. The main purpose of Section IV is to obtain the actions of the three different types of branes, with or without induced gravity term, so as to find clues for comparing their creation probabilities. Different from the symmetric case [3], we find the Gibbons-Hawking boundary term in the context of asymmetric instanton must be considered. Finally we present our conclusions and discussions in Section V.

Instanton Solutions

We consider a 4D brane embedded in a 5D Einstein bulk. In Gaussian normal coordinates the metric ansatz of a 5D Euclidean-Einstein space is written as

$$^{(5)}g_E = g_{ab}dx^a dx^b = dr^2 + b^2(r)ds_4^2$$

where Latin indices run from 0 to 4, and b(r) has the dimension of length. The induced 4-metric on the brane is $g_E = b^2(r)ds_4^2$. Then we introduce $h_E = ds_4^2$ for convenience.

There is an interesting relation between $^{(5)}g_E$ and g_E, as shown in the following lemma:

Lemma: The bulk with metric ansatz (2) is a negative curvature Einstein manifold only if it has an Einstein submanifold.

Proof: One selects an orthonormal basis $e_r = dr$, $e_\mu = b\tilde{e}_\mu$, where $\tilde{e}_\mu$ is an orthonormal basis of h_E (Greek indices label 0-3). Under this basis one has

$$^{(5)}R_{rr} = -4\frac{b''}{b}$$

where prime denotes derivative with respect to r. If $^{(5)}g_E$ is an Einstein manifold, one has $^{(5)}R_{rr} = Cg_{rr}$, where C is a constant. Set $C = -4/l^2$, then one further has

$$b'' = \frac{b}{l^2}$$

The general solution of the above equation is

$$b = c_1 e^{r/l} + c_2 e^{-r/l}$$

where c_1 and c_2 are two constants and l has the dimension of length. Along the directions of the submanifold we find

$$^{(5)}R_{\alpha\mu} = \frac{1}{b^2}\tilde{R}_{\alpha\mu} - \left[\frac{b''}{b} + 3\left(\frac{b'}{b}\right)^2\right]\delta_{\alpha\mu}$$

If $^{(5)}g_E$ is an Einstein manifold, i.e., $^{(5)}R_{\alpha\mu} = Cg_{\alpha\mu} = -\frac{4}{l^2}\delta_{\alpha\mu}$, substituting the solution for b into the equation above yields

$$\tilde{R}_{\alpha\mu} = \frac{4c_1c_2}{l^2}\delta_{\alpha\mu}$$

We see this Ricci tensor $\tilde{R}_{\alpha\mu}$ can be treated as a solution of the 4D vacuum Einstein equation with a cosmological constant $4c_1c_2/l^2$. For giving prominence to the key point of this paper, we set $4c_1c_2/l^2 = \epsilon$, where $\epsilon = -1, 0, 1$ denotes negative curvature, Ricci-flat, or positive curvature canonical submanifold respectively. In this setup we have put the dimension of g_E into b^2, so both the components of h_E and the coordinates x^μ are dimensionless. Hence $\tilde{R}_{\alpha\mu}$ is dimensionless while $R_{\alpha\mu}(g_E)$ has dimension of $[length]^{-2}$.

A simple calculation gives

$$R_{\alpha\mu}(g_E) = \epsilon b^{-2}\delta_{\alpha\mu}$$

where c_1 and c_2 have dimension of $[length]$.

Here we point out that the three types of hypersurfaces—positive, negative, and Ricci-flat—correspond to different slicings of the 5D Einstein manifold. To illuminate this point, we first suppress three dimensions of the original 5D manifold, taking a cross-section of the 5D manifold. Then we perform a Wick rotation to return to the Lorentzian manifold and choose $c_1 = c_2 = l/2$, $c_1 = c_2 = l/2$, and $c_1 = l, c_2 = 0$ to represent canonical positive curvature, negative curvature, and Ricci-flat submanifolds respectively. If the spacetime we consider is not maximally symmetric, it cannot be embedded in a 6D flat manifold, but we can always embed it into a Ricci-flat 6D manifold [13]. Whether embedded into a 6D flat or 6D Ricci-flat spacetime, the cross-section remains the same. It is obvious that the remaining 2D manifold is maximally symmetric. We embed this maximally symmetric manifold in the following 3D pseudo-Euclidean manifold where

$$ds^2 = dW^2 + dZ^2 - dT^2$$

In the parametrization

$$T = l\cosh(r_1/l)\sin(t_1), \quad W = l\cosh(r_1/l)\cos(t_1), \quad Z = l\sinh(r_1/l)$$

we have

$$g = dr_1^2 + l^2\cosh^2(r_1/l)dt_1^2$$

This chart covers the whole manifold except some singularities which form a zero-measure set. In this chart, negative curvature hypersurfaces stand at $r_1 =$ constant.

In the parametrization

$$T = \frac{l}{2}e^{r_2/l}t^2, \quad W = l\cosh(r_2/l) + \frac{l}{2}e^{r_2/l}t^2, \quad Z = l\sinh(r_2/l) + \frac{l}{2}e^{r_2/l}t^2$$

the metric becomes

$$g = dr_2^2 + l^2 e^{2r_2/l}dt^2$$

This chart covers half of the manifold, namely the region $Z + W > 0$. In this chart, Ricci-flat hypersurfaces stand at $r_2 =$ constant.

The third parametrization is

$$T = l\sinh(r_3/l)\sinh(t_3), \quad W = l\cosh(r_3/l), \quad Z = l\sinh(r_3/l)\cosh(t_3)$$

by which we derive the metric

$$g = dr_3^2 + l^2\sinh^2(r_3/l)dt_3^2$$

This chart covers half of the manifold, that is, the region $W > 0$. In this chart, positive curvature hypersurfaces stand at $r_3 =$ constant.

Now we present a simple conclusion which is not obvious a priori. We know there are three classes of parameterizations depending on the sign of c_1c_2. All metrics of hypersurfaces in the same class can be written in standard form, given by equations (14), (16), or (18), simply by a coordinate transformation. Here we prove this conclusion for the positive curvature class. For any $c_1c_2 < 0$, define new coordinates t_5, r_5:

$$T = \sqrt{-4c_1c_2}\left(c_1e^{r_5/l} + c_2e^{-r_5/l}\right)\sinh t_5, \quad W = \sqrt{-4c_1c_2}\left(c_1e^{r_5/l} + c_2e^{-r_5/l}\right)\cosh t_5, \quad Z = \sqrt{-4c_1c_2}\left(c_1e^{r_5/l}\right.$$

We find

$$g = dr_5^2 + b^2dt_5^2 = dr_5^2 - 4c_1c_2\left(c_1e^{r_5/l} + c_2e^{-r_5/l}\right)^2 dt_5^2$$

Considering the normalization condition (9), this is just equation (5). It is easy to find the transformation between (r_5, t_5) and (r_3, t_3):

$$t_3 = t_5, \quad \sinh(r_3/l) = \sqrt{-4c_1c_2}\left(c_1e^{r_5/l} + c_2e^{-r_5/l}\right)$$

Evidently, for any c_1, c_2 in the same family (such as positive slicing), with a general conformal factor, the only work needed to obtain the standard chart is to rescale the coordinate r_5. Similar conclusions hold for the cases of negative and Ricci-flat slicings. Therefore, we always suppose that the metric of the Einstein bulk together with its Einstein brane has been rescaled to the standard form (14), (16), or (18). This assumption provides many conveniences for discussing asymmetric instanton solutions.

If one only considers mirror-symmetric instantons, one can construct them by excising the spacetime region at $r > r_0$ and gluing two copies of the remaining spacetime along the 4-hypersurface at $r = r_0$, giving $M = M_R \cup_\Sigma M_R$. We consider the case where the bulk on the two sides of the brane has no mirror symmetry; that is, the cosmological constants are different on the two sides of the brane. Because the 4-metric induced by the 5-metric of the left bulk must be identical to the metric induced by the right bulk (i.e., the 4-metric on the brane is unique), we have

$$g_E^{\text{left}} = g_E^{\text{right}}$$

Obviously, if g_E^{left} belongs to the positive curvature class but g_E^{right} belongs to the negative curvature class, the bulk on the two sides cannot be glued together. So we consider gluing two halves of the bulk with sectional hypersurfaces in the same canonical class but with different cosmological constants. For example, for a brane in the positive curvature class, the junction condition (23) becomes

$$l_1 \sinh(r_1/l_1) = l_2 \sinh(r_2/l_2)$$

where $l_{1,2} = \sqrt{-6/^{(5)}\Lambda_{L,R}}$ is the characteristic length of the left (right) bulk and r_1 (r_2) is the position of the brane in the left (right) bulk. The outline of an asymmetric instanton is shown in Figure 1: see original paper.

Note that maximally symmetric manifolds–Minkowski, de Sitter, or anti-de Sitter–are certainly Einstein manifolds, but we have many other choices: Schwarzschild (Schwarzschild-(Anti-)de Sitter) solution, Kerr (Kerr-(Anti-)de Sitter) solution, etc. Here we note that the Randall-Sundrum Minkowski brane is the flat solution included in the Ricci-flat class with $c_1 c_2 = 0$. Furthermore, for convenience of later development we give the (Anti-)de Sitter manifold and Schwarzschild-((Anti-)de Sitter) manifold clearly in Euclidean form.

For (Anti-)de Sitter:

$$ds_4^2 = d\chi^2 + \sin^2\chi\, d\Omega_{(3)}^2 \quad \text{(de Sitter)}$$

$$ds_4^2 = d\chi^2 + \sinh^2\chi\, d\Omega_{(3)}^2 \quad \text{(anti-de Sitter)}$$

where $d\Omega_{(3)}^2$ is the 3-sphere metric.

For Schwarzschild-((Anti-)de Sitter):

$$ds_4^2 = \left(1 + \epsilon r'^2 - \frac{2m}{r'}\right) d\chi^2 + \frac{dr'^2}{\left(1 + \epsilon r'^2 - \frac{2m}{r'}\right)} + r'^2 d\Omega_{(2)}^2$$

where r' is the radial coordinate on the brane and $d\Omega_{(2)}^2$ is the 2-sphere metric. Kerr-(Kerr-(Anti-)de Sitter) solution and any other Einstein manifold can be written similarly.

III. Junction Condition

In this section we investigate the general junction condition in the framework of brane-induced gravity. The induced gravity brane model was proposed by Dvali et al. [14], and we work in the generalized Dvali-Gabadadze-Porrati model presented in [15].

We consider an asymmetric 5D gravitational instanton with a 4D brane on which an induced Ricci scalar term is confined:

$$S = S_{\text{bulk}}^L + S_{\text{brane}} + S_{\text{bulk}}^R$$

where

$$S_{\text{bulk}}^{L,R} = \int d^5x \sqrt{\det({}^{(5)}g_{L,R})} \left[\frac{{}^{(5)}R_{L,R}}{16\pi G_5} - \frac{{}^{(5)}\Lambda_{L,R}}{8\pi G_5} + {}^{(5)}\mathcal{L}_m^{L,R} \right]$$

and

$$S_{\text{brane}} = \int d^4x \sqrt{\det(g)} \left[\frac{R}{16\pi G} + \mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi) \right] + \int d^4x \sqrt{\det(g)}\, K^{\pm}$$

Here G_5 is the 5D gravitational constant, ${}^{(5)}R$ and ${}^{(5)}\mathcal{L}_m$ are the 5D scalar curvature and matter Lagrangian in the bulk respectively. A quantity with subscript L denotes it is valued in the left bulk, and R denotes the right bulk. ${}^{(5)}\Lambda$ is the cosmological constant in the bulk. x^μ ($\mu = 0, 1, 2, 3$) are the induced 4D coordinates on the brane, $K^{\pm}$ is the trace of extrinsic curvature on either side of the brane, and $\mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi)$ is the effective 4D Lagrangian, given by a generic functional of the brane metric $g_{\alpha\beta}$ and matter fields ψ.

Consider the brane Lagrangian

$$\mathcal{L}_{\text{brane}} = \frac{R}{16\pi G} - \lambda + \mathcal{L}_m$$

where λ is the cosmological constant on the brane and $\mathcal{L}_m$ denotes matter confined to the brane. This is a generalized version of the DGP model, which is obtained by setting $\lambda = 0$ and ${}^{(5)}\Lambda = 0$. The covariant equations for the case ${}^{(5)}\Lambda_L = {}^{(5)}\Lambda_R$ have been obtained in [15]; for ${}^{(5)}\Lambda_L \neq {}^{(5)}\Lambda_R$, the covariant equations were derived in [16].

A mirror-symmetric closed brane-world instanton M can be constructed by excising the spacetime region at $r > r_0$ and gluing two copies of the remaining spacetime along the 4-hypersurface at $r = r_0$, giving $M = M_R \cup_\Sigma M_R$. We consider a more general class of instantons without mirror symmetry. Certainly the junction condition (23) must be satisfied. Furthermore, the energy-momentum tensor on the brane is constrained by the second fundamental form of the brane relative to the two sides of the bulk. The relation is Israel' s junction condition:

$$[K_{\mu\nu} - K g_{\mu\nu}]_{\pm} = 8\pi G_5 \tau_{\mu\nu}$$

where $\tau_{\mu\nu}$ is the effective energy-momentum stress tensor on the brane, $K_{\mu\nu}$ is the second fundamental form of the brane, $K = g^{\mu\nu} K_{\mu\nu}$, and $[K_{\mu\nu} - K g_{\mu\nu}]_{\pm}$ denotes the discontinuity across the brane. It is easy to obtain from this:

$$\tau_{\mu\nu} = \lambda g_{\mu\nu} + \frac{1}{8\pi G} G_{\mu\nu} + \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$$

We omit the matter term $\mathcal{L}_m$ in this section for the sake of instanton solutions. From the lemma we have

$$R_{\mu\nu} = 3\epsilon b^{-2} g_{\mu\nu}$$

Since b depends only on the fifth dimension in the above equation, the induced term $G_{\mu\nu}$ acts as a cosmological constant from the brane perspective.

From Israel' s junction condition we arrive at

$$K_{\mu\nu} = g_{\mu\nu} x$$

where

$$x = \frac{1}{2}\left(\frac{1}{l_1}\coth(r_1/l_1) + \frac{1}{l_2}\coth(r_2/l_2)\right)$$

for positive curvature brane, and

$$x = \frac{1}{2}\left(\frac{1}{l_1}\tanh(r_1/l_1) + \frac{1}{l_2}\tanh(r_2/l_2)\right)$$

for negative curvature brane. For Ricci-flat brane:

$$x = \frac{1}{2}\left(\frac{1}{l_1} + \frac{1}{l_2}\right)$$

Without announcement for a quantity Q, we define $Q = \frac{1}{2}(Q^+ + Q^-)$ and $\Delta Q = Q^+ - Q^-$, where Q^+ or Q^- is the value of the quantity on one side of the brane respectively. On the other hand,

$$K_{\mu\nu} = \frac{1}{2}\mathcal{L}_n g_{\mu\nu} = \frac{b'}{b} g_{\mu\nu}$$

If the brane stands at $r = r_1$ relative to the left bulk, then the equations give

$$\lambda = \frac{3}{8\pi G_5}\left(\frac{b_1'}{b_1} + \frac{b_2'}{b_2}\right)$$

which yields, for positive curvature brane,

$$\lambda = \frac{3}{8\pi G_5}\left[\frac{1}{l_1}\coth(r_1/l_1) + \frac{1}{l_2}\coth(r_2/l_2)\right]$$

for negative curvature brane,

$$\lambda = \frac{3}{8\pi G_5}\left[\frac{1}{l_1}\tanh(r_1/l_1) + \frac{1}{l_2}\tanh(r_2/l_2)\right]$$

and for Ricci-flat brane

$$\lambda = \frac{3}{8\pi G_5}\left(\frac{1}{l_1} + \frac{1}{l_2}\right)$$

Under these conditions: (1) the brane Lagrangian does not contain an induced gravity term R; (2) the instanton is mirror-symmetric; and (3) the brane in the instanton is a positive curvature brane, the junction condition degenerates to the condition in [3].

In Figure 2: see original paper we plot the warping factor b(r) across the bulk. For Ricci-flat and negative curvature cases, the qualitative properties of the warping factors are the same as the positive case.

Instanton Action

In the spirit of sum-over-histories formalism, the path integral extends over all paths—differentiable and non-differentiable. We know that the measure is concentrated on non-differentiable paths in the ordinary path integral case. In the flat space case we can smooth the non-differentiable path by general analytical techniques. On the other hand, because of the singularity theorem, any reasonable spacetime must contain at least one singularity. The discovery of the cosmological constant, via breaking energy conditions, helps to escape the singularity theorem. But even in the case of singular spacetime, we can define the gravitational action in a meaningful way. In the case of black holes, the path integral should be taken over Euclidean, that is, positive definite metrics. This means that the singularities of black holes, like the Schwarzschild solution, do not appear on the Euclidean metrics which do not go inside the horizon. Instead, the horizon is like the origin of polar coordinates. The action of the Euclidean metric is therefore well-defined. Based on these considerations we permit singular paths, such as bulk which is Einstein space containing black strings.

In the following we calculate the actions of various paths. As a first step we omit the induced gravity term in the brane action for a while. The Euclidean action of the instanton becomes

$$S_E = \int_{M_L \cup M_R} d^5x \sqrt{\det({}^{(5)}g_E)}\left(\frac{{}^{(5)}R}{16\pi G_5} - \frac{{}^{(5)}\Lambda}{8\pi G_5}\right) + \int_{\text{brane}} d^4x \sqrt{\det(g_E)}(4K) + \int_{\text{brane}} d^4x \sqrt{\det(g_E)}(2\lambda)$$

Using the junction condition, for positive curvature brane in the negative curvature Einstein bulk we obtain

$$S_E^+ = W^+(l_1, l_2, r_1, r_2) \cdot V^+$$

where $V^+ = \int d^4x \sqrt{\det(h_E)}$, h_E takes the positive curvature metric given in (2), and thus V^+ is dimensionless and

$$W^+(l_1, l_2, r_1, r_2) = \frac{1}{16\pi G_5}\left[4l_2^3 \coth(r_2/l_2)\sinh^4(r_1/l_1) + \frac{3}{2}l_1^3 \sinh(2r_1/l_1) + \frac{1}{8}l_1^3 \sinh(4r_1/l_1) + \frac{3}{2}l_2^3 \sinh(2r_2/l_2)\right.$$

Using equation (24) one can eliminate a parameter, for example r_2, from the expression, obtaining

$$W^+(l_1, l_2, r_1) = \frac{1}{32\pi G_5}\left[12l_2^3 \operatorname{arcsinh}\left(\frac{l_1}{l_2}\sinh(r_1/l_1)\right) + 12l_1^3 \sinh^3(r_1/l_1)\cosh(r_1/l_1) + \frac{l_1^3}{2}\sinh(2r_1/l_1) + \frac{l_1^3}{8}\sinh\right.$$

One can check that the result in [3] is a special case of our result under the mirror symmetry condition $l_1 = l_2$.

By the same method we derive the action for instanton with negative curvature brane:

$$S_E^- = W^-(l_1, l_2, r_1) \cdot V^-$$

where $V^- = \int d^4x \sqrt{\det(h_E)}$, h_E takes the negative curvature metric given in (2), V^- is dimensionless and

$$W^-(l_1, l_2, r_1) = \frac{1}{32\pi G_5}\left[l_1^3 \cosh^3(r_1/l_1)\left(12r_1 + 8l_1 \sinh(2r_1/l_1) + l_1 \sinh(4r_1/l_1)\right) + l_2^3\left(12 \operatorname{arccosh}\left(\frac{l_1}{l_2}\cosh(r_1/\right.\right.\right.$$

The action of the instanton with Ricci-flat brane can be obtained by analogy:

$$S_E^{\text{flat}} = W^{\text{flat}}(l_1, l_2, r_1) \cdot V^{\text{flat}}$$

where $V^{\text{flat}} = \int d^4x \sqrt{\det(h_E)}$, h_E takes the Ricci-flat metric given in (2), V^{flat} is also dimensionless and

$$W^{\text{flat}}(l_1, l_2, r_1) = \frac{1}{16\pi G_5}\left[\frac{l_1^3}{2}\left(e^{4r_1/l_1} - 1\right) + \frac{l_2^3}{2}\left(1 - e^{-4r_1/l_1}\right)\right]$$

We know some Euclidean spaces whose 4-volumes are well-defined. First, for positive curvature Einstein branes there are two examples whose volumes we can calculate–de Sitter space and Nariai space. For a de Sitter brane:

$$V^{+}(\mathrm{dS}) = 2\pi^2$$

For a Nariai brane we have $m = 1$ in equation (27) and the topology of the brane becomes $S^2 \times S^2$. Hereby

$$V^{+}(\mathrm{Na}) = 4\pi^2$$

For Ricci-flat Einstein branes we also present some examples. The volume of the RS Euclidean (flat) brane is divergent without proper regularization. But as discussed above, one can introduce instantons containing singularities. We calculate the actions of 4D Schwarzschild and Kerr metrics here. Under these conditions we must add a Gibbons-Hawking boundary term of the brane (boundary of boundary) in the action. Considering this term, the action becomes

$$S_E^{\mathrm{flat}} = W^{\mathrm{flat}}(l_1, l_2, r_1) \cdot V^{\mathrm{flat}} + \int d^3x \sqrt{\gamma}\, I$$

where I is the trace of the second fundamental form of the boundary of the brane. For the Schwarzschild solution, the extra contribution to the action is $4\pi G m^2$, therefore

$$S_E^{\mathrm{flat}} = W^{\mathrm{flat}}(l_1, l_2, r_1) \cdot V^{\mathrm{flat}} + 4\pi G m^2$$

Here m is the mass of the black hole. Note that the dimension of m is $[mass]$ in the above equation, which is different from m in equation (27) where m is dimensionless.

For the Kerr solution, the extra contribution to the action is [17]:

$$\Delta S = \frac{\pi(r_+^2 + a^2)}{r_+ - r_-} + \frac{J^2}{2m(r_+ - r_-)}$$

where m is the black hole mass, r_+ is the radius of the outer horizon, r_- is the inner horizon, and J is the angular momentum of the black hole.

There is another point to explain. One knows the Schwarzschild black hole on the brane is an extensive object which is a black string in the 5D bulk. We only obtain the boundary term on the brane, but how does the boundary of the string act off the brane? Generally speaking, temperature makes no sense on an Einstein manifold with negative curvature. In fact, the total actions of the bulk have been included in the first term of the action. However, we still do not find the 4-volume of the brane appearing in the action, and thus the concrete value of the action S_E^{flat} is left open. Nor do we know the 4-volume of the negative curvature brane without any compactifications "by hand" . Whereas in a sense one can compare the actions per unit 4-volume all the same.

We plot actions per unit 4-volume W^+, W^{flat}, W^- in [Figure 4: see original paper] and [Figure 5: see original paper], where the two instantons have different asymmetric degrees. In order to contrast with the symmetric case, we also plot W^+, W^{flat}, W^- for mirror-symmetric instantons in [Figure 3: see original paper].

Generally, for a mirror-symmetric instanton the whole instanton is fixed if we fix the bulk on one side of the brane; that is, the bulk on the other side can be obtained by reflection and the energy-momentum tensor on the brane is just the difference between the second fundamental forms along the two sides of the brane. However, for the asymmetric case, fixing one half of the bulk is not enough to fix the whole instanton. We have many choices of other halves of bulk and corresponding branes. It is interesting to study the actions of such a sequence of instantons. We present our results in [Figure 6: see original paper], [Figure 7: see original paper], and [Figure 8: see original paper].

Now we turn to the action with an induced Ricci term. Integrating it straightforwardly gives:

$$S_E^{\text{induced}} = \frac{1}{16\pi G_5}\int_{M_L\cup M_R} d^5x\sqrt{\det({}^{(5)}g_E)}\,{}^{(5)}R + \frac{1}{16\pi G}\int_{\text{brane}} d^4x\sqrt{\det(g_E)}\,R + \int_{\text{brane}} d^4x\sqrt{\det(g_E)}\,\lambda$$

which evaluates to

$$S_E^{\text{induced}} = \frac{V}{8\pi G_5}\int dr\, b^4(r)\,{}^{(5)}\Lambda + \lambda b^4(r_1) - \frac{b^2(r_1)\epsilon}{4\pi G}$$

where ϵ is defined by equation (9), V is the volume of the 4D Euclidean-Einstein brane with metric $h_E = ds_4^2$ from equation (2), and V is a dimensionless number. The final integration is fairly easy while the result is rather messy. However, paying attention to equations (33) and (34), we see that on an Einstein brane the effect of the induced Ricci tensor acts as a cosmological constant from the brane perspective. We find the final result behaves similarly to the case without the induced term, so it sheds no further light on our understanding of this asymmetric brane creation problem.

V. Conclusions and Discussions

We present a quantum cosmological scenario of brane world creation with an induced gravity term on the brane. In this scenario, a brane is created together with bulk from nothing. The quantum creation is described by the brane instanton—a positive, negative, or flat Einstein 4-manifold, which separates two asymmetric patches of negative curvature Einstein 5-manifolds. We study all three classes of branes residing in a negative curvature Einstein manifold and find they dwell in different positions, related to one another up to a boost isometry for different classes. All branes in the same class can be written in standard form simply by a rescaling of the radial coordinate. Then we analyze the junction condition of the brane with asymmetric bulk in the induced gravity framework.

In Euclidean quantum gravity formalism, which is the application of quantum path integral formalism to gravity theory, we have $p \propto \exp(-2S_E)$, where p is the probability associated with the path. Thus the action of an instanton may offer clues about the creation probabilities of the instantons. As an example, from equations (53), (54), (43), and (58) one can immediately say that the de Sitter brane is more likely to be created than the Nariai brane. We investigate in detail the Euclidean action of three canonical types of instantons.

We find that the Gibbons-Hawking boundary term should be considered in the asymmetric case. We provide the analytical forms of the instanton action in terms of three parameters—the characteristic lengths of the bulk on the left and right, and the position of the brane in the instanton. We find that for most brane metrics we are not fortunate enough to obtain a definite action for an instanton. The total actions of open and flat branes are ill-defined without proper regularizations imposed "by hand" , as shown in Section IV. Even in the case of positive curvature brane, for instance as simple as Schwarzschild-de Sitter metric, we do not know its Euclidean action very well because it is a non-equilibrium system (i.e., its temperature is not definite). Therefore, we compare the actions of the three types of instantons per unit 4-volume. We also present the actions of instantons with a brane gluing to a fixed bulk but with different other bulk. All three canonical types of branes are studied.

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