

## Primordial power spectrum versus extension parameters beyond the standard model Postprint

**Authors:** Zong-Kuan Guo, Yuan-Zhong Zhang

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### Abstract

We reconstruct the shape of the primordial power spectrum of curvature perturbations in extended cosmological models, including addition of massive neutrinos, extra relativistic species or varying primordial helium abundance, from the latest cosmic microwave background data from the Wilkinson Microwave Anisotropy Probe, the Atacama Cosmology Telescope and the South Pole Telescope. We find that a scale-invariant primordial spectrum is disfavored by the data at 95% confidence level even in the presence of massive neutrinos, however it can lie within the 95% confidence region if the effective number of relativistic species or the primordial helium abundance is allowed to vary freely. The constraints on the extension parameters from WMAP7+ACT+H0+BAO, are the total mass of neutrinos  $M < 0.48$  eV (95% CL), the effective number of relativistic species  $N_{\text{eff}} = 4.50 \pm 0.81$  and the primordial helium abundance  $Y_p = 0.303 \pm 0.075$ . The constraints from WMAP7+SPT+H0+BAO, are  $M < 0.45$  eV (95% CL),  $N_{\text{eff}} = 3.86 \pm 0.63$  and  $Y_p = 0.277 \pm 0.050$ .

### Full Text

#### Preamble

Primordial power spectrum versus extension parameters beyond the standard model

Zong-Kuan Guo\* and Yuan-Zhong Zhang†

State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China

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Anisotropy Probe, the Atacama Cosmology Telescope, and the South Pole Telescope. We find that a scale-invariant primordial spectrum is disfavored by the data at 95% confidence level even in the presence of massive neutrinos; however, it can lie within the 95% confidence region if the effective number of relativistic species or the primordial helium abundance is allowed to vary freely. The constraints on the extension parameters are:

From WMAP7+ACT+H0+BAO:

- Total mass of neutrinos:  $\Sigma m < 0.48$  eV (95% CL)
- Effective number of relativistic species:  $N_{\text{eff}} = 4.50 \pm 0.81$
- Primordial helium abundance:  $Y_{\text{p}} = 0.303 \pm 0.075$

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## Introduction

Measurements of temperature anisotropy in the cosmic microwave background (CMB) provide us with a wealth of cosmological information. Since 2001, the Wilkinson Microwave Anisotropy Probe (WMAP) satellite has measured temperature anisotropy of the CMB up to  $\ell \approx 1200$  over the full sky [1, 2]. Ground-based telescopes such as the Atacama Cosmology Telescope (ACT) [3] and the South Pole Telescope (SPT) [4] have extended temperature measurements to smaller angular scales  $\ell \approx 3000$ . Measurements of the small-scale anisotropy of the CMB, in combination with WMAP's measurements of the degree-scale anisotropy, provide a powerful probe of early-universe physics. Increasing the angular dynamic range of CMB measurements can improve constraints on the shape of the primordial power spectrum of curvature perturbations [5-7]. Moreover, high-resolution measurements of the small-scale CMB can limit the total mass of neutrinos, the effective number of relativistic species, and the primordial helium abundance [3, 4].

Slow-roll inflationary models generically produce a nearly scale-invariant primordial power spectrum of curvature perturbations. Hence, measuring deviations from scale invariance is a critical test of cosmological inflation. For a power-law parameterization motivated by inflationary models with featureless inflaton potentials, the exact scale-invariant spectrum is excluded at more than 99% confidence level by the 7-year WMAP data [2]. Various other parameterizations have been considered: for example, a broken spectrum [8] caused by an interruption of the inflaton potential [9], a cutoff at large scales [10, 11] motivated by suppression of the lower multipoles in CMB anisotropies [12, 13], and more complicated shapes caused by features in the inflaton potential [14]. However, a strong theory prior on the form of the primordial power spectrum could lead to misinterpretation and biases in parameter determination. Recently, some

model-independent approaches have been proposed to reconstruct the shape of the primordial spectrum, based on linear interpolation [15], cubic spline interpolation in log-log space [6, 7], a minimally-parametric reconstruction [16], wavelet expansions [17], principle component analysis [18], and direct reconstruction via deconvolution methods [19-21]. In these works, the primordial power spectrum has been constrained within the context of the minimal  $\Lambda$ CDM model.

On the other hand, recent CMB measurements by ACT and SPT have revealed somewhat less fluctuation power at small angular scales than expected in the standard  $\Lambda$ CDM model. Except with the help of a primordial power spectrum with a large negative running spectral index, such small-scale suppression in the CMB angular power spectrum can also be explained by extra relativistic degrees of freedom or additional primordial helium abundance [3, 4]. Moreover, the addition of massive neutrinos can affect CMB anisotropies by changing the matter-to-radiation ratio around the photon decoupling epoch [1].

In this work, we reconstruct the shape of the primordial power spectrum in extended  $\Lambda$ CDM models, including the addition of massive neutrinos, extra relativistic species, or varying primordial helium abundance. To reduce degeneracy between the primordial power spectrum and the extension parameters, we use the seven-year WMAP data in combination with small-scale CMB data from ACT during its 2008 season and SPT during 2008 and 2009, and adopt two main astrophysical priors: on the Hubble constant ( $H_0$ ) measured from the magnitude-redshift relation of low- $z$  Type Ia supernovae, and on the distance ratios of the comoving sound horizon to angular diameter distances from Baryon Acoustic Oscillations (BAO) in the distribution of galaxies.

This paper is organized as follows. In Section II we describe the reconstruction method for the primordial power spectrum and the extended models in which the power spectrum is reconstructed. In Section III we describe the data used in our analysis and present our results. Section IV contains our conclusions.

## II. Beyond the Standard Model

In the standard  $\Lambda$ CDM model, the primordial power spectrum is commonly parameterized as a power law, there are three light neutrino species in the early Universe, and Big Bang Nucleosynthesis (BBN) took place with specific predictions for primordial element abundances. In this work, we consider simple extensions of the standard  $\Lambda$ CDM model including: (i) a general form of the primordial power spectrum, (ii) addition of massive neutrinos, (iii) extra relativistic species, and (iv) varying primordial helium abundance. Each type of model is described in detail below.

**Primordial power spectrum:** The primordial power spectrum,  $P(k)$ , is related to the angular power spectrum of the CMB by a radiative transfer function  $\Delta^X(k)$  via  $\frac{d \ln k P(k)}{d \ln k} = \Delta^X(k) \Delta^Y(k)$ , where  $X, Y$  denote the various temperature and polarization modes. Given a specific ansatz for the primordial spectrum motivated by theoretical models, one can fit it to the data. Alterna-

tively, given our complete ignorance of the underlying physics of the very early Universe, one can reconstruct the shape of the primordial power spectrum from existing data. In this paper, we use the method proposed in [6] to perform a reconstruction of the primordial power spectrum in extended  $\Lambda$ CDM models. We divide the spectrum into three bins equally spaced in logarithmic wavenumber between  $0.0002 \text{ Mpc}^{-1}$  and  $0.2 \text{ Mpc}^{-1}$ . We use cubic spline interpolation to determine the logarithmic value of the spectrum between knots. Outside this wavenumber range, we fix the slope of the spectrum at the boundaries since CMB data place only weak constraints on them.

**Massive neutrinos:** If neutrinos are non-relativistic at the present epoch and their mass eigenstates are degenerate, then the energy density parameter,  $\Omega_{\nu} h^2$ , can be written in terms of the total mass of neutrinos as  $\Sigma m_{\nu} / 94 \text{ eV}$ . This contributes as a dark matter component in the Universe that cannot be negligible even for a neutrino mass as small as  $m_{\nu} = 0.07 \text{ eV}$ , as established by solar and atmospheric neutrino oscillation experiments [22]. Here,  $h$  is the dimensionless Hubble parameter such that  $H = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ . However, at the photon decoupling epoch neutrinos were still relativistic as long as  $m_{\nu} < 1.8 \text{ eV}$ , and thus the matter density at decoupling was actually smaller than a naive extrapolation from the present value. This leads to accelerated decay of gravitational potential around the decoupling epoch. Therefore, by changing the matter-to-radiation ratio, massive neutrinos modify the CMB power spectrum [1]. Detecting this effect on CMB anisotropies allows us to establish the absolute neutrino mass scale [23].

**Extra relativistic species:** In the standard model of particle physics there are three light neutrino species. The total energy density of relativistic neutrinos is related to the photon energy density,  $\rho_{\gamma}$ , via  $(7/8)(11/4)^{4/3} N_{\text{eff}} \rho_{\gamma}$ , where  $N_{\text{eff}}$  is the effective number of neutrino species used to parameterize the relativistic degree of freedom. Three light neutrino species correspond to  $N_{\text{eff}} = 3.046$ , which slightly exceeds 3 because electron-positron annihilation at neutrino decoupling provides residual neutrino heating [24].

The addition of extra relativistic species increases the expansion rate prior to and during the photon decoupling epoch and thereby increases the angular scales of photon diffusion, as explained in [25]. Thus, increasing  $N_{\text{eff}}$  reduces fluctuation power at small angular scales. The effect from extra relativistic species not only produces a nearly constant decrease in the angular power spectrum at  $\ell > 200$  but also alters its shape at  $\ell > 200$  [25].

**Primordial helium abundance:** In the standard BBN model, the predicted value of the primordial helium abundance,  $Y_{\text{p}}$ , is generally related to the baryon density and the effective number of relativistic species. For the  $\Lambda$ CDM model, uncertainty in the baryon density leads to 0.02% uncertainty in the helium abundance, as  $N_{\text{eff}}$  is fixed to its standard value of 3.046. Therefore, for CMB analysis the primordial helium abundance is usually assumed to be  $Y_{\text{p}} = 0.24$  [1-3].

Since helium recombination happens much earlier than hydrogen recombination, the number density of free electrons at hydrogen recombination is given by  $n_e = n_b(1 - Y_p)$  where  $n_b$  is the baryon number density [26]. A large  $Y_p$  decreases the value of  $n_e$  and thereby increases the mean free path of photons. This leads to lower fluctuation power in the damping tail of the CMB. We can extend the BBN prior by promoting  $Y_p$  to a free parameter and then use CMB data to probe the helium abundance.

To summarize, we consider extended  $\Lambda$ CDM models described by the parameters  $\{\Omega_{bh}^2, \Omega_{ch}^2, \Theta_s, A_s, A_l, A_{l+1}, X\}$ , where  $\Omega_{bh}^2$  and  $\Omega_{ch}^2$  are the physical baryon and cold dark matter densities relative to the critical density,  $\Theta_s$  is the ratio of the sound horizon to the angular diameter distance at decoupling,  $A_s$  is the reionization optical depth,  $A_i = \ln[10^{10} P(k_i)]$  ( $i = 1, 2, 3$ ) are the logarithmic values of the primordial power spectrum at knots  $k_i$ , and  $X = N_{eff}$  or  $Y_p$  is the extension parameter.

### III. Cosmological Constraints

In this section, we first describe the CMB data from the WMAP satellite, the ACT and SPT experiments, and the astrophysical priors used in our analysis. We then combine these data sets to constrain the extended  $\Lambda$ CDM models and present our results.

#### Data

We use the 7-year WMAP (WMAP7) data including the high- $l$  TT power spectrum for  $l > 1200$  and TE power spectrum for  $l > 800$ , and the low- $l$  temperature ( $2 < l < 32$ ) and polarization ( $2 < l < 23$ ) data. We consider the Sunyaev-Zel'dovich (SZ) effect, in which CMB photons scatter off hot electrons in clusters of galaxies. Given an SZ template, the effect is described by a SZ template amplitude  $A_{SZ}$  as in the WMAP papers [1, 2].

We use the 148 GHz ACT data from its 2008 season, focusing on band powers in the multipole range  $1000 < l < 3000$  to improve constraints on primary cosmological parameters. Following [3], for computational efficiency we set the CMB contribution to zero above  $l = 4000$  where it is subdominant (less than 5% of total power). To use the ACT likelihood described in [3], aside from  $A_{SZ}$  there are two additional secondary parameters:  $A_p$  and  $A_c$ . The former is the total Poisson power from radio and infrared point sources, while the latter is the template amplitude of clustered power from infrared point sources. We impose positivity priors on these three secondary parameters, use the SZ template and clustered source template provided by the ACT likelihood package, and marginalize over these secondary parameters to account for SZ and point source contamination.

We also use the 150 GHz SPT data from 2008 and 2009, focusing on the temperature power spectrum binned in 40 band powers in the multipole range  $1000 < l < 3000$ . Following [4], we adopt Gaussian priors on three secondary parameters

in our analysis:  $A_p = 19.3 \pm 3.5 \text{ K}^2$ ,  $A_c = 5.0 \pm 2.5 \text{ K}^2$ , and  $A_{SZ} = 5.5 \pm 3.0 \text{ K}^2$ . Moreover, we require these secondary parameters to be positive. As discussed in [4], constraints on cosmological parameters do not depend strongly on these priors. The foreground terms are used only when calculating the SPT likelihood; they are not used when calculating the WMAP likelihood.

To improve constraints on cosmological parameters, we follow the methodology described in [1, 2] to include distance measurements from astrophysical observations: angular diameter distances measured from BAO in the distribution of galaxies, and the Hubble constant measured from the magnitude-redshift relation of low- $z$  Type Ia supernovae. The Gaussian priors on the distance ratios are  $r_s/D_V(z = 0.2) = 0.1905 \pm 0.0061$  and  $r_s/D_V(z = 0.35) = 0.1097 \pm 0.0036$ , derived from the Two-Degree Field Galaxy Redshift Survey and Sloan Digital Sky Survey data [27]. Here  $r_s$  is the comoving sound horizon size at the baryon drag epoch and  $D_V$  is the effective distance measure for angular diameter distance. The Gaussian prior on the Hubble constant is  $H = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , derived from the magnitude-redshift relation of 240 low- $z$  Type Ia supernovae at  $z < 0.1$  [28]. The error includes both statistical and systematic uncertainties.

## Results

We use a modified version of the publicly available CosmoMC package to sample the parameter space using Monte Carlo Markov Chain techniques [29]. We also use the WMAP likelihood, ACT likelihood, and SPT likelihood code for parameter estimation.

Figure 1 [Figure 1: see original paper] shows the reconstructed primordial power spectrum of curvature perturbations in the standard  $\Lambda$ CDM model (top-left panel) and non-standard models with massive neutrinos (top-right panel), with varying relativistic species (bottom-left panel), and with varying primordial helium abundance (bottom-right panel), derived from WMAP7+ACT+H0+BAO. The four panels correspond to: (top-left)  $\Sigma m_\nu = 0$ ,  $N_{\text{eff}} = 3.046$ ,  $Y_p = 0.24$ ; (top-right)  $\Sigma m_\nu$  free,  $N_{\text{eff}} = 3.046$ ,  $Y_p = 0.24$ ; (bottom-left)  $\Sigma m_\nu = 0$ ,  $N_{\text{eff}}$  free,  $Y_p = 0.24$ ; (bottom-right)  $\Sigma m_\nu = 0$ ,  $N_{\text{eff}} = 3.046$ ,  $Y_p$  free.

We can see that in the standard model, the scale-invariant spectrum is disfavored by the data at 95% confidence level. Including massive neutrinos enhances the primordial spectrum at low- $k$  but suppresses it at intermediate- $k$  due to enhancement of the early integrated Sachs-Wolfe effect, making the spectrum flatter at large scales. However, there is no significant degeneracy between massive neutrinos and the primordial power spectrum at small scales. Therefore, the scale-invariant spectrum remains marginally disfavored in the presence of massive neutrinos. In extended models with a free number of relativistic species (or free primordial helium abundance), the scale-invariant spectrum can lie well within the 95% confidence region because considerable degeneracy between rel-

ativistic species (or helium abundance) and the spectrum results in large errors and enhancement of the spectrum at high- $k$ . Compared to varying relativistic species, varying helium abundance leads to larger uncertainties in the primordial spectrum at both low- $k$  and high- $k$ . Varying helium abundance affects the primordial spectrum at small scales by modifying the recombination process, and indirectly affects it at large scales due to correlation between  $A$  and  $A$  as shown in Figure 2 [Figure 2: see original paper].

Table I lists the estimated values of the primordial power spectrum parameters and the extension parameters beyond the standard model. For a flat  $\Lambda$ CDM model with a power-law spectrum, the WMAP7+H0+BAO limit on the total neutrino mass is  $\Sigma m_\nu < 0.58$  eV (95% CL) [2]. In our model we find  $\Sigma m_\nu < 0.48$  eV (95% CL) from WMAP7+ACT+H0+BAO. The upper limit on the total neutrino mass is improved by adding small-scale ACT data. With the WMAP7+ACT+H0+BAO data combination, the effective number of neutrino species is estimated to be  $N_{\text{eff}} = 4.50 \pm 0.81$  (68% CL), which indicates that a Universe with no relativistic neutrinos is excluded at  $5.6\sigma$  and the standard model with  $N_{\text{eff}} = 3.046$  is consistent at  $1.8\sigma$ . This estimated value is consistent with the effective number of relativistic species estimated in the power-law  $\Lambda$ CDM model from WMAP7+H0+BAO data,  $N_{\text{eff}} = 4.34 \pm 0.88$  (68% CL) [2], and from WMAP7+ACT+H0+BAO data,  $N_{\text{eff}} = 4.56 \pm 0.75$  (68% CL) [3]. Therefore, constraints on the effective number of relativistic species do not depend strongly on additional degrees of freedom in the power spectrum.

For CMB analysis, the primordial helium abundance is usually assumed to be  $Y_p = 0.24$ . When it is allowed to vary, we find  $Y_p = 0.303 \pm 0.075$  (68% CL) using WMAP7+ACT+H0+BAO. This mean value is lower than that predicted in the power-law  $\Lambda$ CDM model from WMAP7+ACT data,  $Y_p = 0.313 \pm 0.044$  (68% CL), because a suppressed power spectrum of primordial curvature perturbations at small scales contributes to enhanced Silk damping of the CMB angular power spectrum. However, degeneracy between the power spectrum and helium abundance leads to larger uncertainties in  $Y_p$  compared to the power-law case.

Figure 3 [Figure 3: see original paper] and Table II show constraints on the power spectrum and extension parameters from WMAP7+SPT+H0+BAO. Compared to ACT data, adding SPT data reduces uncertainties in the power spectrum at small scales. The scale-invariant spectrum is disfavored at  $2\sigma$  even in the extended model with massive neutrinos. The conclusion that the scale-invariant spectrum is not excluded at  $2\sigma$  confidence level if either the effective number of relativistic species or the helium abundance is allowed to vary is confirmed using WMAP7+SPT+H0+BAO data. This data combination constrains the total neutrino mass to  $\Sigma m_\nu < 0.45$  eV (95% CL), which is almost consistent with the result in Table I.

We find that the WMAP7+SPT+H0+BAO limit on the effective number of relativistic species is  $N_{\text{eff}} = 3.86 \pm 0.63$  (68% CL). In the power-law model, the constraint improves to  $N_{\text{eff}} = 3.86 \pm 0.42$  (68% CL) [4]. Compared to

ACT data, SPT data prefers a smaller effective number of relativistic species. If the helium abundance is allowed to vary, we find  $Y_p = 0.277 \pm 0.050$  (68% CL) from WMAP7+SPT+H0+BAO. This mean value is lower than the result in the power-law model presented in [4],  $Y_p = 0.300 \pm 0.030$  (68% CL). Moreover, compared to ACT data, SPT data prefers a low helium abundance.

## IV. Conclusions

It is known that adding massive neutrinos can influence the CMB spectrum by changing the matter-to-radiation ratio around photon decoupling, that extra relativistic species can suppress the CMB damping tail by increasing the expansion rate prior to and during decoupling, and that excessive helium abundance can also suppress CMB power at small angular scales by decreasing the number density of free electrons at hydrogen recombination. Therefore, extension parameters can mimic the shape of the primordial power spectrum, while the general form of the primordial spectrum can affect constraints on extension parameters.

In such extended  $\Lambda$ CDM models, we have reconstructed the shape of the primordial power spectrum of curvature perturbations using cubic spline interpolation in log-log space. To reduce degeneracy between the primordial power spectrum and extension parameters, we use CMB data from WMAP7+ACT and WMAP7+SPT in combination with measurements of the Hubble constant and BAO features. Even when massive neutrinos are included, the scale-invariant spectrum is disfavored by the combined data at 95% confidence level. If the effective number of relativistic species (or the primordial helium abundance) is allowed to vary, the scale-invariant spectrum can lie within the  $2\sigma$  bound because of considerable degeneracy between  $A$  and  $N_{\text{eff}}$  (or  $Y_p$ ). Moreover, we find that the two data sets yield nearly the same limit on the total neutrino mass.

Compared to SPT data, ACT data mildly prefers a larger effective number of relativistic species or a larger primordial helium abundance.

The contribution from tensor modes was considered when testing the shape of the primordial power spectrum in [6]. Under the assumption of a scale-invariant tensor spectrum, including tensor modes can suppress the spectrum at large scales. In this case, the reconstructed spectrum deviates from scale invariance at 95% confidence level.

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data.

- [1] E. Komatsu, et al., *Astrophys. J. Suppl.* 180, 330 (2009) [arXiv:0803.0547].
- [2] E. Komatsu, et al., *Astrophys. J. Suppl.* 192, 18 (2011) [arXiv:1001.4538].
- [3] J. Dunkley, et al., arXiv:1009.0866;
- [4] R. Keisler, et al., arXiv:1105.3182.
- [5] R. Hlozek, et al., arXiv:1105.4887.
- [6] Z. K. Guo, D. J. Schwarz and Y. Z. Zhang, *JCAP* 08, 031 (2011) [arXiv:1105.5916].
- [7] Z. K. Guo and Y. Z. Zhang, *JCAP* 11, 032 (2011) [arXiv:1109.0067].
- [8] A. Blanchard, M. Douspis, M. Rowan-Robinson and S. Sarkar, *Astron. Astrophys.* 412, 35 (2003) [arXiv:astro-ph/0304237].
- [9] J. Barriga, E. Gaztanaga, M. Santos and S. Sarkar *Mon. Not. Roy. Astron. Soc.* 324, 977 (2001) [arXiv:astro-ph/0011398].
- [10] G. Efstathiou, *Mon. Not. Roy. Astron. Soc.* 343, L95 (2003) [arXiv:astro-ph/0303127].
- [11] M. Bridges, A. N. Lasenby and M. P. Hobson, *Mon. Not. Roy. Astron. Soc.* 369, 1123 (2006) [arXiv:astro-ph/0511573]; M. Bridges, A. N. Lasenby and M. P. Hobson, *Mon. Not. Roy. Astron. Soc.* 381, 68 (2007) [arXiv:astro-ph/0607404].
- [12] M. R.olta, et al., *Astrophys. J. Suppl.* 180, 296 (2009) [arXiv:0803.0593].
- [13] C. J. Copi, D. Huterer, D. J. Schwarz and G. D. Starkman, *Adv. Astron.* 2010, 847541 (2010) [arXiv:1004.5602].
- [14] A. A. Starobinsky, *JETP Lett.* 55, 489 (1992); J. A. Adams, G. G. Ross and S. Sarkar, *Nucl. Phys. B* 503, 405 (1997) [arXiv:hep-ph/9704286]; M. Aich, et al., arXiv:1106.2798; T. Bringmann, P. Scott and Y. Akrami, arXiv:1110.2484.
- [15] S. L. Bridle, A. M. Lewis, J. Weller and G. Efstathiou *Mon. Not. Roy. Astron. Soc.* 342, L72 (2003) [arXiv:astro-ph/0302306]; S. Hannestad, *JCAP* 0404, 002 (2004) [arXiv:astro-ph/0311491].
- [16] C. Sealfon, L. Verde and R. Jimenez, *Phys. Rev. D* 72, 103520 (2005) [arXiv:astro-ph/0506707]; L. Verde and H. V. Peiris, *JCAP* 0807, 009 (2008) [arXiv:0802.1219]; H. V. Peiris and L. Verde, *Phys. Rev. D* 81, 021302 (2010) [arXiv:0912.0268].
- [17] P. Mukherjee and Y. Wang, *Astrophys. J.* 598, 779 (2003) [arXiv:astro-ph/0301562 ]; P. Mukherjee and Y. Wang, *Astrophys. J.* 599, 1 (2003) [arXiv:astro-ph/0303211]; P. Mukherjee and Y. Wang, *JCAP* 0512, 007 (2005) [arXiv:astro-ph/0502136].
- [18] W. Hu and T. Okamoto, *Phys. Rev. D* 69, 043004 (2004) [arXiv:astro-ph/0308049]; S. Leach, *Mon. Not. Roy. Astron. Soc.* 372, 646 (2006) [arXiv:astro-ph/0506390].
- [19] N. Kogo, M. Matsumiya, M. Sasaki and J. Yokoyama, *Astrophys. J.* 607, 32 (2004) [arXiv:astro-ph/0309662]; R. Nagata and J. Yokoyama, *Phys. Rev. D* 78, 123002 (2008) [arXiv:0809.4537]; R. Nagata and J. Yokoyama, *Phys. Rev. D* 79, 043010 (2009) [arXiv:0812.4585]; K. Ichiki and R. Nagata, *Phys. Rev. D* 80, 083002 (2009); K. Ichiki, R. Nagata and J. Yokoyama, *Phys. Rev. D* 81,

- 083010 (2010) [arXiv:0911.5108].
- [20] A. Shafieloo and T. Souradeep, Phys. Rev. D 70, 043523 (2004) [arXiv:astro-ph/0312174]; A. Shafieloo and T. Souradeep, Phys. Rev. D 78, 023511 (2008) [arXiv:0709.1944]; J. Hamann, A. Shafieloo and T. Souradeep, JCAP 1004, 010 (2010) [arXiv:0912.2728].
- [21] D. Tocchini-Valentini, M. Douspis and J. Silk, Mon. Not. Roy. Astron. Soc. 359, 31 (2005) [arXiv:astro-ph/0402583]; D. Tocchini-Valentini, Y. Hoffman and J. Silk, Mon. Not. Roy. Astron. Soc. 367, 1095 (2006) [arXiv:astro-ph/0509478].
- [22] M. C. Gonzalez-Garcia, M. Maltoni and J. Salvado, JHEP 1004, 056 (2010) [arXiv:1001.4524].
- [23] C. P. Ma and E. Bertschinger, Astrophys. J. 455, 7 (1995) [arXiv:astro-ph/9506072]; J. Hamann, et al., Phys. Rev. Lett. 105, 181301 (2010) [arXiv:1006.5276]; Y. Y. Y. Wong, Ann. Rev. Nucl. Part. Sci. 61, 69 (2011) [arXiv:1111.1436]; A. G. Doroshkevich and M. Yu. Khlopov, Sov. Astron. Lett. 9, 171 (1983).
- [24] G. Mangano, et al., Nucl. Phys. B 729, 221 (2005) [arXiv:hep-ph/0506164].
- [25] Z. Hou, et al., arXiv:1104.2333; M. Archidiacono, E. Calabrese and A. Melchiorri, arXiv:1109.2767; M. Benetti, et al., Phys. Rev. D 84, 063509 (2011) [arXiv:1107.4992].
- [26] W. Hu, D. Scott, N. Sugiyama and M. White, Phys. Rev. D 52, 5498 (1995) [arXiv:astro-ph/9505043].
- [27] W. J. Percival, et al., Mon. Not. Roy. Astron. Soc. 401, 2148 (2010) [arXiv:0907.1660].
- [28] A. G. Riess, et al., Astrophys. J. 699, 539 (2009) [arXiv:0905.0695].
- [29] A. Lewis and S. Bridle, Phys. Rev. D 66, 103511 (2002) [arXiv:astro-ph/0205436].

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