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Abstract

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Full Text

Preamble

Cosmological Constraints on Lorentz Invariance Violation in the Neutrino Sector

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We derive the Boltzmann equation in the synchronous gauge for massive neutrinos with a deformed dispersion relation. Combining the 7-year WMAP data with lower-redshift measurements of the expansion rate, we constrain the deformation parameter and find that it is strongly degenerate with the physical dark matter density rather than the neutrino mass. Our results show no evidence for Lorentz invariance violation in the neutrino sector. The ongoing Planck experiment could provide improved constraints on the deformation parameter.

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Introduction

Neutrino oscillations imply that at least two of the three neutrino types have non-zero mass (see Ref. [1] and references therein). Unfortunately, neutrino oscillation experiments only provide two mass-squared differences, not the overall mass scale. Cosmology offers a promising approach to this problem through the gravitational effects of massive neutrinos on the expansion history near matter-radiation equality and on the growth of large-scale structures at late times. Precision measurements of cosmic microwave background (CMB) anisotropies and large-scale structure have enabled us to determine or constrain the absolute neutrino mass scale [2–5] (see also Ref. [6] for a recent review).

Interestingly, neutrino oscillations can be explained by small Lorentz invariance violation even without introducing neutrino mass, as shown in [7, 8, 10–12]. Observed neutrino oscillations may originate from a combination of effects involving neutrino masses and Lorentz invariance violation. Possibilities for Lorentz invariance violation have been explored in quantum gravity [13], loop quantum gravity [14], non-commutative field theory [15], and doubly special relativity [16].

Lorentz symmetry is a fundamental feature of modern descriptions of nature, including both Einstein' s general relativity and the Standard Model of particle physics. One might expect that breaking Lorentz symmetry could leave imprints in astrophysical observations such as CMB anisotropies and the large-scale structure of our universe. In this paper, we consider cosmological tests of Lorentz invariance violation in the neutrino sector, focusing on the Coleman-Glashow model where the energy-momentum relation is modified by a Lorentz-violating interaction within conventional quantum field theory [8]. We construct a Lagrangian, derive the Boltzmann equation in synchronous gauge for massive neutrinos with deformed dispersion relations, and constrain Lorentz invariance violation by combining 7-year WMAP data [3] with recent distance measurements from Baryon Acoustic Oscillations (BAO) in galaxy distributions [17] and Hubble constant (H_0) measurements [18].

This paper is organized as follows. In Sec. II we write down the Lagrangian for neutrinos with deformed dispersion relation. In Sec. III we derive the Boltzmann equation for neutrinos in synchronous gauge. In Sec. IV we constrain the deformation parameters using CMB data combined with H_0 and BAO measurements. Section V presents our conclusions.

II. Deformed Dispersion Relation

At a phenomenological level, the deformed dispersion relation for massive neutrinos can be generally parameterized by

$$E^2 = m^2 + p^2 + \sum_n \alpha_n \frac{p^n}{M^{n-2}},$$

where E is neutrino energy, m is neutrino mass, $p = (p_i p^i)^{1/2}$ is the magnitude of 3-momentum, α_n are dimensionless coefficients, and M denotes the energy scale corresponding to Lorentz symmetry violation (typically taken to be the Planck mass). Such a deformed dispersion relation implies departures from Lorentz invariance in the neutrino sector if $\alpha_n \neq 0$. The $n = 1$ term would produce huge effects at low energy and has been strongly constrained. The p^n term with $n > 1$ is suppressed by $1/M^{n-2}$. In this work we therefore consider the case $n = 2$:

$$E^2 = m^2 + p^2 + \xi \frac{p^2}{M^2},$$

where ξ is the deformation parameter characterizing the size of Lorentz symmetry violation. This deformed dispersion relation was constructed by Coleman and Glashow within conventional quantum field theory [8].

We note that the dispersion relations given in (1) and (2) are not fully general, as they neglect oscillations, possible species dependence, anisotropies associated with rotation symmetry violation, and CPT violation. As shown recently by Kostelecky and Mewes, all of these are possible [9]. For example, odd values of n in Eq. (1) correspond to CPT violation and produce sign differences for neutrinos and antineutrinos [9]. We emphasize that the model considered here is one of many possible Lorentz-violating theories.

The scalar perturbations of the Friedmann-Lemaître-Robertson-Walker metric in synchronous gauge can be written as

$$ds^2 = a^2(\tau) \left[d\tau^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right],$$

where $a(\tau)$ is the scale factor, τ is conformal time, and the scalar mode of h_{ij} is represented by two functions h and η defined by

$$h_{ij}(\mathbf{x}, \tau) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left[\hat{k}_i \hat{k}_j h(\mathbf{k}, \tau) + (\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij}) 6\eta(\mathbf{k}, \tau) \right],$$

where $q_i = q n_i$ is the comoving 3-momentum written in terms of its magnitude and direction with $n_i n^i = 1$, and $\epsilon = \sqrt{m^2 a^2 + (1+\xi) q^2}$ is the comoving energy of neutrinos.

III. Boltzmann Equation

The number density n_ν, energy density ρ_ν, and pressure P_ν for massive neutrinos with dispersion relation (2) are given by

$$n_\nu = \int \frac{d^3q}{(2\pi)^3} f_0(q),$$

$$\rho_\nu = \int \frac{d^3q}{(2\pi)^3} \epsilon f_0(q),$$

$$P_\nu = \int \frac{d^3q}{(2\pi)^3} \frac{(1+\xi)q^2}{3\epsilon} f_0(q).$$

Here the zeroth-order phase space distribution is well approximated by the relativistic Fermi-Dirac distribution

$$f_0(q) = g_s \left[1 + \exp\left(\frac{\sqrt{1+\xi}\, q}{T_0}\right)\right]^{-1},$$

where T_0 is the neutrino temperature today and $g_s = 2$ is the number of spin degrees of freedom. If $m \ll T_0$, the total mass of neutrinos is

$$\Sigma m = 94(1+\xi)^{3/2}\Omega_\nu h^2 \text{ eV}.$$

The perturbed energy density, pressure, energy flux, and shear stress in Fourier space $\mathbf{k}$ are respectively given by

$$\delta\rho_\nu = \int \frac{d^3q}{(2\pi)^3} \epsilon f_0(q)\Psi_0,$$

$$\delta P_\nu = \int \frac{d^3q}{(2\pi)^3} \frac{(1+\xi)q^2}{3\epsilon} f_0(q)\Psi_0,$$

$$(\rho_\nu + P_\nu)\theta_\nu = \int \frac{d^3q}{(2\pi)^3} (1+\xi) q f_0(q)\Psi_1,$$

$$(\rho_\nu + P_\nu)\sigma_\nu = \int \frac{d^3q}{(2\pi)^3} \frac{(1+\xi)q^2}{\epsilon} f_0(q)\Psi_2,$$

where the perturbations Ψ_l satisfy the Boltzmann equation

$$\dot{\Psi}_l + (1+\xi)\frac{qk}{\epsilon}\left[\frac{l+1}{2l+1}\Psi_{l+1} - \frac{l}{2l+1}\Psi_{l-1}\right] - \left[\dot{h} + 2\dot{\eta}\frac{d\ln f_0}{d\ln q}\right]\delta_{l0} + \frac{k}{\epsilon}\frac{d\ln f_0}{d\ln q}\left[\frac{l+1}{2l+1}\Psi_{l+1} + \frac{l}{2l+1}\Psi_{l-1}\right] = 0,$$

in synchronous gauge. This Boltzmann hierarchy is effectively truncated by adopting the scheme [20]

$$\Psi_{l_{\max}+1} = \frac{2l_{\max}+1}{(2l_{\max}+1)\epsilon}(1+\xi)qk\tau\Psi_{l_{\max}} - \Psi_{l_{\max}-1}.$$

The initial conditions for the perturbation Ψ_l and η on super-horizon scales ($k\tau \ll 1$) are given by

$$\Psi_0 = -C\frac{d\ln f_0}{d\ln q},$$

$$\Psi_1 = \frac{C}{3\sqrt{1+\xi}}\frac{qk}{\epsilon}\frac{d\ln f_0}{d\ln q},$$

$$\Psi_2 = \frac{2C}{5+9\sqrt{1+\xi}R_\nu}\frac{d\ln f_0}{d\ln q},$$

$$\eta = 2C - \frac{5+4\sqrt{1+\xi}R_\nu}{6(15+4\sqrt{1+\xi}R_\nu)}C(k\tau)^2,$$

where C is a dimensionless constant determined by the amplitude of fluctuations from inflation and $R_\nu = \rho_\nu/(\rho_\nu+\rho_\gamma)$ during radiation domination.

To compute the theoretical CMB power spectrum, we modified the Boltzmann CAMB code [27] to incorporate the Lorentz-violating term in the neutrino sector. This term affects both the evolution of the cosmological background and the behavior of neutrino perturbations. From Eqs. (11)-(13), we see that increasing ξ decreases the number density, energy density, and pressure of neutrinos, thereby increasing the redshift of matter-radiation equality and reducing the expansion rate prior to and during photon-baryon decoupling. This leads to reduced heights of the first and second CMB peaks while producing a nearly constant increase in acoustic oscillation amplitudes at $\ell > 600$. Additionally, the coefficient of the second term in the Boltzmann equation (18) actively alters the CMB power spectrum shape by changing neutrino propagation. Decreasing ξ increases fluctuation power both at $\ell < 10$ and $\ell > 100$.

Moreover, the CMB is more sensitive to negative values of ξ than positive ones. These effects can be distinguished from changes in the total neutrino mass or the effective number of relativistic species [21-26], as shown in Figure 1.

[Figure 1: see original paper]

Figure 1: Theoretical angular power spectrum of the CMB for $\xi = -0.1, 0, 0.1$. Here $\Sigma m = 3 \times 0.3$ eV is fixed.

IV. Cosmological Constraints

In our analysis we use a modified version of the publicly available CosmoMC package to explore parameter space via Monte Carlo Markov Chain techniques [28]. We consider flat ΛCDM models with three Lorentz-violating neutrino species, described by cosmological parameters $\Omega_b h^2$, $\Omega_c h^2$, Θ_s, τ, n_s, A_s, Σm, and ξ, where h is the dimensionless Hubble parameter such that $H_0 = 100h$ km

s^{-1} Mpc^{-1}, $\Omega_b h^2$ and $\Omega_c h^2$ are physical baryon and dark matter densities relative to the critical density, Θ_s is the ratio of sound horizon to angular diameter distance at photon decoupling, τ is reionization optical depth, n_s and A_s are the spectral index and amplitude of primordial curvature perturbations at pivot scale $k_0 = 0.002$ Mpc^{-1}, Σm is the total neutrino mass assuming approximately degenerate masses, and ξ is the deformation parameter.

We use seven-year WMAP (WMAP7) data with the likelihood code supplied by the WMAP team. We include the Sunyaev-Zel' dovich (SZ) effect, where CMB photons scatter off hot electrons in galaxy clusters. Given an SZ template, the effect is described by amplitude A_{SZ} as in the WMAP paper [3]. We also impose Gaussian priors on the Hubble constant, $H_0 = 73.8 \pm 2.4$ km s^{-1} Mpc^{-1}, measured from low-z Type Ia supernova magnitude-redshift relations [18], and on distance ratios $r_s/D_V(z = 0.2) = 0.1905 \pm 0.0061$ and $r_s/D_V(z = 0.35) = 0.1097 \pm 0.0036$, measured from BAO in galaxy distributions [17]. Here r_s is the comoving sound horizon at baryon drag epoch and D_V is the effective distance measure for angular diameter distance.

Table I summarizes our results. With WMAP7+H_0+BAO data, the deformation parameter is estimated as $\xi = -0.077 \pm 0.089$, implying a null detection of Lorentz invariance violation within error limits. The large uncertainties in ξ mainly arise from strong correlation between the deformation parameter and physical dark matter density, as shown in Figure 2 [Figure 2: see original paper]. In this case, uncertainties in $\Omega_c h^2$ are about three times larger than those in the standard ΛCDM model [3]. Since the deformation parameter is nearly uncorrelated with total neutrino mass (Figure 2), our constraints on ξ are not significantly changed if neutrinos are massless, as seen in Table I.

Compared to particle physics experiments, cosmological observations yield much weaker constraints on Lorentz-violation parameters in the neutrino sector. As listed in Table XIV of Ref. [29], previous constraints range from parts in 10^5 to parts in 10^{15} from time-of-flight measurements and various threshold analyses.

We also present constraints from the ongoing Planck experiment [30] in Table I. Following the MCMC method described in Ref. [31], we generate synthetic Planck data and perform systematic analysis. As Table I shows, Planck data plus H_0 and BAO measurements reduce uncertainties in ξ by a factor of 2.4. Therefore, Planck CMB measurements could detect Lorentz invariance violation signatures at 2σ confidence level if $|\xi| > 0.074$.

[Figure 2: see original paper]

Figure 2: Two-dimensional joint marginalized constraints (68% and 95% confidence level) on deformation parameter ξ, physical dark matter density $\Omega_c h^2$ (left), and total neutrino mass Σm (right), derived from WMAP7+H_0+BAO data. The dashed line corresponds to Lorentz invariance.

V. Conclusions

We studied cosmological consequences of Lorentz-violating neutrinos. We obtained the generalized Lagrangian for neutrinos with Coleman-Glashow type dispersion relation, derived its Boltzmann equation and initial conditions in linear cosmological perturbation theory. Using 7-year WMAP data combined with H_0 and BAO measurements, we found no evidence for Lorentz invariance violation in the neutrino sector. The ongoing Planck experiment is expected to provide more stringent constraints on the deformation parameter.

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Note: Figure translations are in progress. See original paper for figures.