Nucleosynthesis constraint on Lorentz invariance violation in the neutrino sector postprint

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Date: 2017-09-27T00:00:00+00:00

Abstract

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Full Text

Preamble

Nucleosynthesis Constraint on Lorentz Invariance Violation in the Neutrino Sector

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(Dated: August 4, 2017)

We investigate the nucleosynthesis constraint on Lorentz invariance violation in the neutrino sector, which influences the formation of light elements by altering the energy density of the Universe and weak reaction rates prior to and during the big-bang nucleosynthesis epoch. We derive the weak reaction rates in the Lorentz-violating extension of the standard model. Using measurements of the primordial helium-4 and deuterium abundances, we provide a tighter constraint on the deformation parameter than that derived from measurements of the cosmic microwave background anisotropies.

PACS numbers: 14.60.St, 98.80.Es

Introduction

Neutrino oscillation experiments have demonstrated small but non-zero mass-squared differences between the three neutrino mass eigenstates (see Ref. [1] and references therein). However, oscillations cannot determine the absolute neutrino masses. Cosmology offers a promising approach to constrain the total neutrino mass through the gravitational effects of massive neutrinos on the expansion history near matter-radiation equality [2] and on the formation of large-scale structure [3] (see Ref. [4] for a review). Recently, a 3 detection of non-zero neutrino masses was reported using new measurements of cosmic microwave background (CMB) anisotropies from the South Pole Telescope and Wilkinson Microwave Anisotropy Probe (WMAP), combined with low-redshift measurements of the Hubble constant, baryon acoustic oscillations, and Sunyaev-Zel' dovich-selected galaxy clusters [5]. These observations establish the existence of physics beyond the Standard Model of particle physics.

Another potential signature of new physics is Lorentz symmetry violation. Such possibilities have been explored in string theory [6], the Standard Model extension [7], quantum gravity [8], loop quantum gravity [9], non-commutative field theory [10], and doubly special relativity [11]. Searches for Lorentz invariance violation using neutrinos have been conducted across a wide range of systems [12]. Although current experiments confirm Lorentz invariance to high precision, it could be broken in the early Universe when energies approach the Planck scale. Cosmological observations provide a unique opportunity to test this symmetry at high energies.

Recently, measurements of the CMB power spectrum were used to probe Lorentz invariance violation in the neutrino sector [13]. Lorentz violation affects both the evolution of the cosmological background and the behavior of neutrino perturbations. The former alters the heights of the first and second peaks in the CMB power spectrum, while the latter modifies its shape. These effects can be distinguished from changes in the total neutrino mass or the effective number of neutrinos. The seven-year WMAP data, combined with lower-redshift measurements of the expansion rate, were used to constrain the Lorentz-violating term. However, the resulting constraints suffer from strong correlation between the Lorentz-violating term and the dark matter density parameter [13].

In this Letter, we use current big-bang nucleosynthesis (BBN) data to constrain Lorentz invariance violation in the neutrino sector. There are two primary effects. First, Lorentz violation corrects the weak reaction rate in the Lorentz-violating Standard Model extension, which governs the neutron-to-proton ratio at the onset of BBN. Second, it changes the total energy density of the Universe. Since the abundances of light elements produced during BBN depend on the competition between the expansion rate and the nuclear and weak reaction rates, BBN predictions depend on the Lorentz-violating term. In particular, the BBN-predicted abundance of helium-4 is highly sensitive to the deformation parameter.

Theoretical Framework

We focus on Lorentz invariance violation exclusively in the neutrino sector, considering the deformed dispersion relation

$$E^2 = m^2 + p^2 + \xi p^2,$$

where E is the neutrino energy, m the neutrino mass, $p = (p_i p^i)^{1/2}$ the magnitude of the 3-momentum, and ξ the deformation parameter characterizing the magnitude of Lorentz invariance violation. This relation implies departures from Lorentz invariance in the neutrino sector when $\xi \neq 0$. Such a deformed dispersion relation was constructed within conventional quantum field theory [14] and derived in the Lorentz-violating extension of the Standard Model [15].

We note that the dispersion relation given in (1) is not fully general. It neglects neutrino oscillations, possible species dependence, anisotropies associated with rotation symmetry violation, and CPT violation. As recently shown by Kostelecký and Mewes, all of these are possible [12]. The model considered here represents one of many possible Lorentz-violating theories.

The number density n_ν and energy density ρ_ν for massive neutrinos with the dispersion relation (1) are given by [13]

$$n_\nu = \frac{g_\nu}{(2\pi)^3} \int d^3p \, f_\nu(E),$$
$$\rho_\nu = \frac{g_\nu}{(2\pi)^3} \int d^3p \, E f_\nu(E),$$

where $g_\nu = 2$ is the number of spin degrees of freedom. The phase space distribution for neutrinos is the Fermi-Dirac distribution

$$f_\nu(E) = \left[1 + \exp(E/T_\nu)\right]^{-1},$$

with T_ν the neutrino temperature. Thus the number and energy densities can be written as

$$n_\nu = (1+\xi)^{-3/2} n_\nu^{(0)},$$
$$\rho_\nu = (1+\xi)^{-3/2} \rho_\nu^{(0)},$$

where $n_\nu^{(0)}$ and $\rho_\nu^{(0)}$ are the standard number and energy densities, respectively. Increasing ξ decreases both quantities. The reduced number density leads to a lower weak reaction rate prior to and during the BBN epoch (since the reaction rate is proportional to neutrino number density), while the reduced energy density results in a slower expansion rate. Therefore, Lorentz invariance violation affects the nucleosynthesis of light elements.

Weak Reaction Rates with Lorentz Violation

We now examine the computation of weak reaction rates with Lorentz invariance violation in the neutrino sector. At early times when the Universe temperature was $T \gg 100$ MeV, the number and energy densities were dominated by relativistic particles: electrons, positrons, neutrinos, antineutrinos, and photons. All particles remained in thermal equilibrium through the weak reactions:

$$\nu_e + n \leftrightarrow p + e^-,$$
$$e^+ + n \leftrightarrow p + \bar{\nu}_e,$$
$$n \leftrightarrow p + e^- + \bar{\nu}_e.$$

When the expansion rate exceeds the reaction rate for $n \leftrightarrow p$ processes, baryons decouple from leptons, and the neutron-to-proton ratio freezes out, largely determining the primordial helium mass fraction.

To estimate the neutron abundance at BBN onset, we compute the reaction rate. Consider the process $\nu_e + n \to p + e^-$. The differential reaction rate per incident nucleon is

$$\frac{d\omega}{dE_e} = \frac{1}{8m_n m_p} \int \frac{d^3p_\nu}{(2\pi)^3 2E_\nu} \frac{d^3p_e}{(2\pi)^3 2E_e} (2\pi)^4 \delta^{(4)}(p_\nu + p_n - p_p - p_e) |\mathcal{M}|^2 f_\nu(E_\nu)[1 - f_e(E_e)],$$

where $|\mathcal{M}|^2$ is the squared matrix element (summed over initial and final state spins), m_n and m_p are the neutron and proton masses, $(E_e, \mathbf{p}_e)$ is the electron four-momentum, and f_e denotes the Fermi-Dirac statistical distribution for electrons.

The processes (5)-(7) involve the gauge boson W as mediator. At tree level,

$$|\mathcal{M}|^2 = 32G_F^2 (p_\nu \cdot p_n)(p_e \cdot p_p) \left[(C_V^2 + 3C_A^2) + (C_V^2 - C_A^2) \frac{c_{\mu\nu} p_\nu^\mu p_\nu^\nu}{E_\nu^2} \right],$$

where G_F is the Fermi coupling constant, $c_{\mu\nu}$ are the coefficients for Lorentz violation, and C_V, C_A are the vector and axial couplings of the nucleon. The coefficients $c_{\mu\nu}$ are defined to be traceless and isotropic. After integration, the reaction rate becomes

$$\omega = \left[1 + \frac{4(C_V^2 - C_A^2)}{3(C_V^2 + 3C_A^2)} \xi \right] (1 + \xi)^{-3/2} \omega^{(0)},$$

where $\omega^{(0)}$ is the standard reaction rate per incident nucleon derived in [16]. The first factor on the right-hand side of (10) arises from the neutrino propagator and the $e\nu W$ coupling in the Lorentz-violating extension of the Standard Model [7], while the second factor comes from the statistical distribution for neutrinos. (More general Lorentz-violating corrections involving electrons, neutrinos, neutrons, and protons were discussed in [17].) At tree level, the differential reaction rates for the other five processes in (5)-(7) can be derived from (8) by appropriately changing the statistical factors and the delta function determined by energy conservation for each reaction. Therefore, the corrections to the neutron-proton conversion rate and its inverse are the same as in (10).

Equation (10) shows that increasing ξ reduces the reaction rate, causing weak reactions to freeze out earlier at a higher freeze-out temperature. This leads to a larger neutron-to-baryon ratio at BBN onset and thus greater primordial ^{4}He production. Conversely, increasing ξ also reduces the expansion rate due to decreased energy density, which would cause weak reactions to freeze out later if there were no correction to the reaction rate. This effect would result in lower helium-4 abundance. These two effects compete in the BBN prediction for helium-4 abundance. The abundances of other light nuclides depend only weakly on ξ through changes in the neutron-to-proton ratio and expansion rate.

Freeze-Out Temperature and Numerical Results

Incorporating corrections to both reaction and expansion rates, we estimate the freeze-out temperature T_f by equating the expansion rate with the weak reaction rate. In a Friedmann-Robertson-Walker Universe, the expansion rate obeys $H^2 = 8\pi G\rho/3$, where $\rho \propto T^4$ at early times. Thus we have $H \propto (1 + 0.75\xi)T^2$. Since the standard reaction rate in Eq. (10) scales roughly as $\omega^{(0)} \propto T^5$ [18], we obtain $\omega \propto (1 - 1.80\xi)T^5$. Setting $H \simeq 4\omega$ (since free-neutron decay and its inverse are negligible at the BBN epoch) yields the freeze-out temperature

$$T_f = (1 + 0.35\xi)T_f^{(0)},$$

where $T_f^{(0)}$ is the standard freeze-out temperature. For large ξ, weak reactions freeze out at higher temperature, implying that effects from changing the reaction rate dominate over those from changing the expansion rate due to Lorentz invariance violation in the neutrino sector.

To calculate the abundances of light elements produced during BBN, we modified the publicly available PArthENoPE code [19] to incorporate the Lorentz-violating term in the neutrino sector. Figure 1 [Figure 1: see original paper] shows the ^{4}He mass fraction and D/H abundance as functions of ξ for $\Omega_b h^2 = 0.0213$ (upper panel) and 0.0224 (lower panel). Both Y_p and D/H increase with ξ because the effect of changing the reaction rate dominates. Moreover, the dependence of Y_p on ξ is much larger (relative to its observational uncertainties) than that of D/H. Therefore, the primordial helium-4 abundance provides a sensitive probe of neutrino physics with Lorentz invariance violation.

Assuming three types of neutrinos with vanishing chemical potentials, the BBN-predicted primordial abundances depend on only two parameters: $\Omega_b h^2$ and ξ. As shown in Figure 1 [Figure 1: see original paper], the deuterium abundance is more sensitive to the baryon energy density parameter but less sensitive to the deformation parameter, while the helium-4 abundance shows the opposite behavior. We use the observed primordial abundances of ^{4}He and D in combination to constrain these parameters based on the likelihood function

$$\chi^2 = \frac{(Y_p - 0.2565)^2}{\sigma_{Y_p}^2} + \frac{(\log_{10}[D/H] + 4.55)^2}{\sigma_{D/H}^2}.$$

We adopt the estimate $Y_p = 0.2565 \pm 0.0060$ for the primordial helium mass fraction, derived in [20] using Monte Carlo methods to simultaneously account for many possible systematic effects based on 93 spectra from 86 low-metallicity extragalactic HII regions. While some studies have employed selected subsets of these data for more detailed analyses, the sources and magnitudes of systematic errors have rarely been addressed comprehensively. The measurement uncertainty in Y_p is currently dominated by systematic errors. For the primordial deuterium abundance, we use $\log_{10}[D/H] = -4.55 \pm 0.03$ obtained in [21] from measurements of absorption lines in seven high-redshift quasars in low-metallicity, hydrogen-rich clouds with low internal velocity dispersions.

Besides ^{4}He and D, ^{3}He and ^{7}Li are the other two nuclides predicted in measurable quantities by BBN. However, their post-BBN evolution is complicated and their measurements suffer from systematic uncertainties that are difficult to quantify (for ^{3}He) or poorly understood (for ^{7}Li). Consequently, as discussed in [22], observed ^{3}He and ^{7}Li do not currently provide reliable probes of BBN. We therefore exclude them from our constraints.

The ^{4}He abundance primarily constrains the deformation parameter, while the D abundance primarily constrains the baryon density parameter. Using the combined ^{4}He and D data, we find

$$\xi = 0.036 \pm 0.023, \quad \Omega_b h^2 = 0.0213 \pm 0.0009 \quad \text{(68\% confidence level)}.$$

This estimate of the deformation parameter is consistent with Lorentz invariance ($\xi = 0$) within 95% confidence level. Compared to results derived from seven-year WMAP data combined with lower-redshift expansion rate measurements [13], BBN yields uncertainties in ξ that are smaller by a factor of 4 because there is essentially no correlation between the deformation parameter and the baryon density parameter, as shown in Figure 2 [Figure 2: see original paper]. The estimated $\Omega_b h^2$ agrees with the CMB-derived value [2] within errors.

Conclusion

We have demonstrated that BBN places strong constraints on the deformation parameter in the Lorentz-violating extension of the Standard Model: $\xi = 0.036 \pm 0.023$. Since the BBN-predicted helium-4 abundance is very sensitive to the deformation parameter but relatively insensitive to the baryon energy density parameter, there is negligible correlation between the two parameters. Our results indicate no significant preference for departure from Lorentz symmetry in the neutrino sector in the early Universe. Compared to previous constraints on Lorentz-violating coefficients, current BBN observations yield a weaker constraint. As listed in Table XIII of [12], the coefficient is constrained down to 10^{-9} from time-of-flight measurements. Cohen and Glashow have argued that observations of neutrinos with energies exceeding 100 TeV over baselines of at least 500 km imply the Lorentz-violating parameter must be less than about 10^{-11} [23].

Acknowledgments

We thank J. Hamann, V. A. Kostelecky, O. Pisanti, and F. Wang for useful discussions. Our numerical analysis was performed on the Lenovo DeepComp 7000 supercomputer at SCCAS. This work is partially supported by the Knowledge Innovation Program of the Chinese Academy of Sciences, NSFC under Grant No. 11175225, and the National Basic Research Program of China under Grant No. 2010CB832805.

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Note: Figure translations are in progress. See original paper for figures.