Obtaining the CMB anomalies with a bounce from the contracting phase to inflation postprint

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Full Text

Preamble

Obtaining the CMB anomalies with a bounce from the contracting phase to inflation

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Abstract

Recent Planck data reveal anomalies in CMB fluctuations on large angular scales, confirming earlier observations by WMAP. We continue our study of an inflationary model in which the universe undergoes a contracting phase before slow-roll inflation, and we fit this model to Planck data. We show that this model can generate not only the power deficit at low multipoles but also a large hemispherical power asymmetry in the CMB. We also discuss the implications of our results for the eternal inflation scenario.

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Introduction

The inflation scenario is the current paradigm for the early universe. Inflation may be realized through various inflationary models, which can be distinguished by observations. Recently, the Planck collaboration has released data on the cosmic microwave background (CMB) power spectrum [?, ?], which favors single-field slow-roll inflationary models with a concave potential.

However, the Planck collaboration has also reported a power deficit in the low-ℓ CMB power spectrum at $\ell \lesssim 40$ [?], a feature previously found in WMAP data that is not concordant with the Planck best-fit model, though the data points remain consistent with cosmic variance. Its statistical significance is about 2.5–3σ. Meanwhile, the Planck collaboration has reported a hemispherical power asymmetry in the CMB [?], confirming a similar WMAP result [?, ?] but with better precision. The Planck data show larger statistical significance than WMAP, making it difficult to attribute this asymmetry to foreground contamination.

These anomalies are intriguing and have motivated numerous studies. The curvaton scenario may explain the power asymmetry [?, ?, ?, ?, ?], but we consider a different possibility: these anomalies might hint at pre-inflationary physics. In this case, inflation might last only for the minimum number of e-folds, with the Planck best-fit single-field inflationary model providing a good fit only for intermediate and small angular scales. Following the WMAP first-year data, the low-ℓ power deficit has been investigated along these lines in Refs.~[?, ?, ?, ?, ?, ?, ?, ?, ?, ?].

Bouncing models have a long history, such as the pre-big-bang (PBB) scenario [?] and the ekpyrotic scenario [?]. In bouncing models, the universe initially contracts and then bounces into an expanding phase, offering a solution to the cosmological singularity problem. In Refs.~[?, ?], a model in which the universe is in a contracting phase before slow-roll inflation and begins to inflate after the bounce has been studied; we call this the bouncing inflation model for simplicity. The contracting phase resembles that in the PBB scenario (see [?, ?] for reviews) and could in principle also resemble the ekpyrotic scenario. In the PBB scenario, the spectrum of adiabatic perturbations generated during kinetic contraction is highly blue, which is inconsistent with observations. However, here this blue spectrum is precisely what is needed for power suppression on large angular scales [?].

Slow-roll inflation generally begins at a high energy scale, required to ensure that the amplitude of primordial perturbations matches observations and that the reheating temperature is suitable for hot big bang evolution after inflation. Recently, in the eternal inflation scenario, it has been argued that if the scale of the eternally inflating background is very low, the onset of slow-roll inflation would require large upward tunneling, which is exponentially suppressed. However, introducing a bounce before slow-roll inflation might significantly alter this conclusion [**?**, **?**, **?**].

In Ref.~[**?**], it was shown that in different cycles of a cyclic universe, the universe may reside in different minima of a landscape, with bouncing inflation responsible for the emergence of the observable universe. In Ref.~[**?**], it was shown that inflation after the bounce causes cosmological hysteresis, leading to increased amplitude in subsequent cycles.

Thus, the study of bouncing inflation models is interesting both theoretically and observationally. We clarify the generation of primordial perturbations in this model in Sec.~II. In Sec.~III, we fit the model to Planck data and show that it can generate both the low-ℓ power deficit and the hemispherical power asymmetry in the CMB, consistent with Planck data. Sec.~IV presents our conclusions. We briefly illustrate model building and discuss implications for the eternal inflation scenario in the Appendices.

Note added: While this work was being completed, Ref.~[**?**] appeared, in which the authors discussed the effect of an instantaneous superinflationary phase after the bounce on the inflationary power spectrum.

II. The Primordial Perturbation in Bouncing Inflation Scenario

We clarify the results for primordial perturbations in the bouncing inflation scenario. Here we require that the bounce occurs at a higher energy scale than the inflationary scale and that all physical quantities pass continuously through the bounce. We will see that the result is insensitive to the detailed implementation of the bounce.

The quadratic action for curvature perturbation is

$$S = \int d\eta d^3x\, a^2 M_P^2 \epsilon \left(\mathcal{R}'^2 - (\nabla\mathcal{R})^2\right)$$

which is universal for single-field models [**?**]. The equation of motion in momentum space is [**?**, **?**]

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0,$$

where $z \equiv a\sqrt{2\epsilon}/c_s$ and the curvature perturbation is defined as $\mathcal{R}_k = u_k/z$. Here $\epsilon \equiv -\dot{H}/H^2$ and $c_s^2 = 1$ for a canonical scalar field.

When $k^2 \gg z''/z$, the perturbation mode is deep inside the horizon. When $k^2 \ll z''/z$, the mode is far outside the horizon and the solution of Eq.~(2) is given by

$$u_k = C_1 z + C_2 z \int \frac{d\eta}{z^2},$$

where C_1 is the constant mode and C_2 is the decaying mode. The evolution of the D mode depends on the evolution of z.

Before the bounce the universe is kinetic-dominated, while after the bounce it enters an inflationary phase with $\epsilon_{\rm inf} \ll 1$. See the Appendix for detailed models. In conformal time, after adopting instantaneous matching between both regimes, we have

$$\frac{z''}{z} \simeq \begin{cases} \frac{2}{\eta^2} & \text{for the contraction} \\ \frac{2}{(-H_0\eta)^2} & \text{for the inflation} \end{cases}$$

where $\eta < 0$ in the contracting phase and $\eta > 0$ in the inflationary phase, and $a = a_0(-H_0\eta)$ for $\eta < 0$ and $a = a_0/(1 + H_0\eta)$ for $\eta > 0$. Here H_0 is the comoving Hubble length at the matching time $\eta = 0$, which sets the inflationary energy scale by $H_{\rm inf} = H_0/a_0$.

When $k^2 \gg z''/z$, the perturbation is deep inside its horizon and u_k oscillates with constant amplitude, $u_k \simeq e^{-ik\eta}/\sqrt{2k}$. In the contracting phase before inflation, $z''/z \simeq 2/\eta^2$, which increases with time. When $k^2 \ll z''/z$, i.e., when the perturbation is far outside the horizon, the solution of Eq.~(2) is

$$u_k \simeq \tilde{C}_1 \sqrt{-k\eta} H_0^{(1)}(-k\eta) + \tilde{C}_2 \sqrt{-k\eta} H_0^{(2)}(-k\eta),$$

where $H_0^{(1)}$ is the Hankel function of the first kind and zeroth order.

In the inflationary phase, when $k^2 \ll z''/z$, the solution of Eq.~(2) is

$$u_k \simeq C_1 \sqrt{-k\eta} H_{3/2}^{(1)}(-k\eta) + C_2 \sqrt{-k\eta} H_{3/2}^{(2)}(-k\eta),$$

where $H_{3/2}^{(1)}$ and $H_{3/2}^{(2)}$ are Hankel functions of the first and second kind of order $3/2$, and C_1 and C_2 depend only on k.

When the bounce is nonsingular, all physical quantities should pass continuously through the bounce. The continuity of the curvature perturbation and its derivative yields

$$\mathcal{R}_{\rm cont} = \mathcal{R}_{\rm inf}, \quad \mathcal{R}'_{\rm cont} = \mathcal{R}'_{\rm inf}.$$

The spectrum of curvature perturbation is

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \frac{|\mathcal{R}_k|^2}{M_P^2} = \frac{k^3}{2\pi^2 M_P^2} \frac{|C_1|^2 + |C_2|^2}{\epsilon_{\rm inf}}.$$

Substituting the matching conditions into this expression, we obtain

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2 M_P^2 \epsilon_{\rm inf}} \left[\left| \frac{\sqrt{\pi}}{2} e^{i\pi/4} H_0^{(2)}\left(\frac{k}{H_0}\right) \right|^2 + \left| \frac{\sqrt{\pi}}{2} e^{-i\pi/4} H_0^{(2)}\left(\frac{k}{H_0}\right) \right|^2 \right],$$

where $H_0^{(2)}$ is the Hankel function of the second kind and zeroth order.

Here, $\mathcal{P}_{\mathcal{R}}^{\rm inf}(k) = H_{\rm inf}^2/(8\pi^2 M_P^2 \epsilon_{\rm inf})$ is the standard slow-roll inflation result, which may have a slight red tilt consistent with observations. The coefficients C_1 and C_2 are determined by the matching conditions. We have expanded $H_{3/2}^{(1)}$ and $H_{3/2}^{(2)}$ in terms of $k/H_0 \ll 1$ and $k/H_0 \gg 1$.

For $k \ll H_0$, the spectrum is strongly blue:

$$\mathcal{P}_{\mathcal{R}}(k \ll H_0) \simeq \frac{36\pi^4 M_P^2}{\epsilon_{\rm inf}} \left(2 + \ln\frac{k}{H_0}\right)^2 \frac{k^3}{H_0^3},$$

which is the usual result of the PBB scenario. For $k \gg H_0$, we have

$$\mathcal{P}_{\mathcal{R}}(k \gg H_0) \simeq \frac{4\pi^3 M_P^2}{\epsilon_{\rm inf}} \left(1 + \frac{H_0}{k}\right) \mathcal{P}_{\mathcal{R}}^{\rm inf}(k),$$

which is almost scale-invariant but modulated by small oscillations. The scale invariance is the result of inflationary evolution after the bounce, because when $k \gg H_0$ the perturbation mode is still inside the horizon and its evolution is determined by Eq.~(6), insensitive to the background. Only in the contracting phase, when the corresponding perturbation mode leaves the horizon, does the perturbation spectrum become determined by the background evolution.

We plot $\mathcal{P}_{\mathcal{R}}$ from Eq.~(14) as a function of k in Fig.~1. We see that for $k > H_0$, the spectrum is almost scale-invariant with a slight red tilt and oscillations with decaying amplitude, while for $k < H_0$ the amplitude decreases rapidly and exhibits a cutoff, consistent with our analytical results (16) and (17).

[Figure 1: see original paper]

III. The CMB Anomalies with Planck

A. The power deficit in low-ℓ

In Eq.~(14), $\mathcal{P}_{\mathcal{R}}$ may be parameterized as a power law with

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm inf} \left(\frac{k}{k_0}\right)^{n_{\rm inf}-1}.$$

We emphasize that with this definition, the spectral index of curvature perturbation defined in Eq.~(15) is $n_R = n_{\rm inf}$ for $k \gg H_0$ and $n_R = n_{\rm inf} + 3$ for $k \ll H_0$. Thus the primordial spectrum (14) is described by three free parameters: $A_{\rm inf}$, $n_{\rm inf}$, and H_0. The pivot scale is chosen as $k_0 = 0.05\,{\rm Mpc}^{-1}$, roughly in the middle of the logarithmic range probed by Planck. In addition, late-time cosmological evolution is characterized by four free parameters: $\Omega_b h^2$, $\Omega_c h^2$, Θ_s, and τ, where h is the dimensionless Hubble parameter such that $H_0 = 100h\,{\rm km\ s^{-1}Mpc^{-1}}$, $\Omega_b h^2$ and $\Omega_c h^2$ are the physical baryon and cold dark matter densities relative to the critical density, Θ_s is the ratio of the sound horizon to the angular diameter distance at photon decoupling, and τ is the reionization optical depth. We impose a uniform prior on $\ln(H_0/\mathrm{Mpc}^{-1})$ in the range $[-10, -4]$. For the other parameters, prior ranges are chosen to be much larger than the posterior. To compute the theoretical CMB power spectrum, we modify the Boltzmann CAMB code from [?].

In Fig.~2 we plot the angular power spectrum for the pure power law (dashed) and bouncing inflation with the best-fit value $\ln(H_0/\mathrm{Mpc}^{-1}) = -8.60$ (solid). Compared to the standard power-law model, the C_ℓ spectrum in the bouncing universe is suppressed in the quadrupole and octupole. Moreover, a small bump around $\ell = 6$ arises from oscillations in the primordial power spectrum at large scales.

[Figure 2: see original paper]

We use the combination of Planck CMB temperature power spectrum [?, ?] with WMAP large-scale polarization data [?] (denoted "Planck+WP"). The Planck temperature likelihood uses a hybrid approach, combining a pixel-based likelihood at low multipoles ($2 \leq \ell \leq 49$) with a Gaussian likelihood approximation at high multipoles ($\ell \leq 2500$). The Planck high-ℓ likelihood involves 14 nuisance parameters to describe unresolved small-scale foregrounds and CMB secondary anisotropies. Since Planck has not released polarization data, we use WMAP polarization data at low multipoles ($2 \leq \ell \leq 23$) to constrain the optical depth.

In our analysis we use a modified version of the publicly available CosmoMC package to explore parameter space via Markov Chain Monte Carlo techniques [?]. From Planck+WP data we find best-fit values $\ln(H_0/\mathrm{Mpc}^{-1}) = -8.60$, $\ln(10^{10} A_{\mathrm{inf}}) = 3.084$, and $n_{\mathrm{inf}} = 0.961$ with $\chi^2_{\mathrm{min}} = 4901.6$. This indicates that the bouncing inflation model improves the fit to data by $\Delta\chi^2_{\mathrm{eff}} \approx -4.6$ relative to the standard power-law model. However, a phenomenological exponential cutoff of the primordial power spectrum improves the fit by only $\Delta\chi^2_{\mathrm{eff}} \approx -2.9$ as reported in [?]. Moreover, the exponential cutoff in [?] is described by two parameters (cutoff steepness λ_c and cutoff scale k_c), while in the bouncing inflation model the cutoff is characterized by only H_0.

Since A_{inf} and n_{inf} characterize the global shape of the power spectrum while H_0 characterizes local features (see [?] for a general shape reconstructed from CMB data), there is almost no correlation between them, as shown in Fig.~3. The marginalized posterior distribution for H_0 is shown in Fig.~4, illustrating the asymmetric shape of the likelihood function.

[Figure 3: see original paper]

[Figure 4: see original paper]

B. The hemispherical power asymmetry

Recently, the Planck collaboration reported a hemispherical power asymmetry in the CMB [?], confirming a similar WMAP result [?] with better precision. This asymmetry could arise from a superhorizon perturbation crossing the observable universe. We estimate the hemispherical power asymmetry in bouncing inflation following the approach of Ref.~[?] by Erickcek et al. and Ref.~[?] by Lyth.

The CMB power asymmetry can be modeled as a dipole modulation of the power spectrum [?, ?, ?]:

$$\mathcal{P}_{\mathcal{R}}(k, \mathbf{x}) = \left[1 + A(k)\frac{\hat{\mathbf{p}} \cdot \mathbf{x}}{x_{\rm ls}}\right]\mathcal{P}_{\mathcal{R}}(k),$$

where $A(k)$ is the modulation amplitude, $\hat{\mathbf{p}}$ is the dipole modulation direction, $x_{\rm ls}$ is the distance to the last scattering surface, and $\mathcal{P}_{\mathcal{R}}(k)$ is given by Eq.~(14).

The asymmetry $A(k)$ can be calculated as

$$A(k) = \frac{1}{\mathcal{P}_{\mathcal{R}}(k)}\left|\frac{d\mathcal{P}_{\mathcal{R}}(k, \mathbf{x})}{d\ln k}\right|\frac{k_L x_{\rm ls}\mathcal{P}_{\mathcal{R}}(k_L)}{\epsilon_{\rm per}},$$

where $\mathcal{P}_{\mathcal{R}}(k_L)$ is the amplitude of the power spectrum of a single modulating mode k_L (i.e., $\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}(k_L)\delta(\ln k - \ln k_L)$), and $\epsilon_{\rm per} = \epsilon_{\rm C}$ for the contracting phase and $\epsilon_{\rm per} = \epsilon_{\rm inf}$ for the inflationary phase. The inflationary result is recovered for $\epsilon_{\rm per} \ll 1$ in Eq.~(15), since $k = aH$. The factor $\epsilon_{\rm per}$ arises from the dependence of H on time: $|\nabla\mathcal{R}_L| = k_L\mathcal{R}_L = k_L\sqrt{\mathcal{P}_{\mathcal{R}}(k_L)}$ [?, ?].

The maximum achievable value of $A(k)$ is limited by the contribution of the superhorizon mode with k_L to the CMB quadrupole via the Grishchuk-Zel' dovich effect. Using the Sachs-Wolfe approximation, Ref.~[?] computed this as

$$\mathcal{P}_{\mathcal{R}}(k_L) \lesssim \frac{1}{x_{\rm ls}^2(k_L x_{\rm ls})^2}\left(\frac{k}{k_L}\right)^2\mathcal{P}_{\mathcal{R}}(k).$$

Requiring that this not exceed the measured rms value of the quadrupole gives $(k_L x_{\rm ls})^2\mathcal{P}_{\mathcal{R}}(k_L) \lesssim 10^{-8}$, which yields $(k_L x_{\rm ls})\mathcal{P}_{\mathcal{R}}(k_L) \lesssim 1$ for perturbation theory to apply. Plugging this into Eq.~(20) gives an upper bound on the modulation amplitude:

$$A(k) \lesssim \frac{0.02}{\epsilon_{\rm per}}\left|\frac{d\ln\mathcal{P}_{\mathcal{R}}(k)}{d\ln k}\right|,$$

since $n_R(k) \equiv d\ln\mathcal{P}_{\mathcal{R}}(k)/d\ln k$ and the measured CMB power asymmetry is $A(k) \simeq 0.07$ from 5-year WMAP analyses [?], consistent with Planck results [?].

In single-field inflationary scenarios, $\epsilon_{\rm per} \ll 10^{-4}$, which is too small to fit the observation, as pointed out in Refs.~[?, ?]. The situation is altered in the curvaton scenario [?, ?].

In the bouncing inflation scenario discussed here, on large angular scales $1/k > 1/H_0$, the curvature perturbation originates from fluctuations of ϕ during contraction. We have $\epsilon_{\rm per} \sim 3$ and $n_R(k) - 1 \sim 3$, as calculated in Sec.~II. Thus the power asymmetry on large angular scales is

$$A_B(k) \sim 0.06,$$

which is consistent with Planck data. The power spectrum at intermediate and small angular scales is that of slow-roll inflation, so the corresponding power asymmetry is small, consistent with constraints from the SDSS quasar sample [**?**].

Here, Eq.~(25) applies only to scales $1/k \gtrsim 1/H_0$. Our best-fit value is $1/H_0 \simeq 1.5\,\mathrm{Gpc}$, which corresponds to $1/H_0 \simeq x_{\rm ls}/9$, while the required range is $x_{\rm ls}/60 \lesssim 1/k \lesssim x_{\rm ls}/3$, since the distance to the last scattering surface is estimated as $x_{\rm ls} \simeq 14\,\mathrm{Gpc}$. This result seems to imply tension with observations. However, as shown in Fig.~3, at 2σ confidence level $1/H_0$ may be as large as $x_{\rm ls}/30$, and at 3σ it may be $x_{\rm ls}/9$. Thus our model is consistent with Planck constraints, though the observation places strong constraints that make the model easily falsifiable by further Planck data.

IV. Conclusion

Recently, the Planck collaboration released CMB power spectrum data consistent with slow-roll inflationary models. However, Planck data also show a power deficit at $\ell \lesssim 40$ and a hemispherical power asymmetry in the CMB, confirming earlier WMAP observations. These results are intriguing as they might hint at physics in the pre-inflationary epoch.

We have continued studying the bouncing inflation model, in which the universe initially contracts and begins to inflate after the bounce. The contraction before the bounce leads to a primordial power spectrum on large scales $1/k > 1/H_0$ that is strongly blue, while H_0 is a new degree of freedom setting the cutoff scale. We find that this spectrum generates not only the low-ℓ power deficit but also the hemispherical power asymmetry in the CMB, potentially consistent with Planck data. Thus our model can explain the CMB anomalies and may be falsified by further Planck data.

The bouncing inflation model not only provides natural initial conditions for slow-roll inflation but also connects to pre-inflationary physics. We discussed model building in the Appendix. While embedding a bouncing model into a fundamental theory is interesting, and different implementations of the bounce may yield different models, these details do not qualitatively affect the primordial spectrum derived here.

We also showed that in the eternal inflation scenario, bouncing inflation might be a favored channel for realizing slow-roll inflation. Thus, a detailed study in a string landscape-motivated model would be worthwhile.

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Appendix A: The Models of Bouncing Inflation

We discuss some bouncing inflation models. The Lagrangian is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - V(\phi),$$

where the potential depends only on the field ϕ. We consider

$$V(\phi) = \lambda_2\phi^2 + \lambda_4\phi^4 + \lambda_6\frac{\phi^6}{M_\phi^2} + \lambda_0.$$

This is a Higgs-like potential for parameters $\lambda_2 < 0$, $\lambda_4 > 0$, and $\lambda_6 = 0$. For $\lambda_4 < 0$ and $\lambda_2, \lambda_6 > 0$, this potential corresponds to that from the minimal supersymmetric standard model [**?**]. Depending on parameter values, the potential may have two or three minima, as shown in Fig.~5.

Here ψ is a ghost field whose only role is to implement the bounce. In principle, the ghost instability may be dispelled by applying Galileon interactions [**?**]. The bounce may also be implemented as in Refs.~[**?**, **?**] for the PBB scenario or [**?**] for the ekpyrotic scenario. The Lagrangian for ψ is chosen for convenience as it yields an analytical solution for $a(t)$ around the bounce.

We plot the evolution of ϕ, H, and a in Fig.~6 for the potential in the upper panel of Fig.~5, and the evolutions of ϕ, kinetic energy, and potential energy in Figs.~7 and 8 for the potential in the lower panel of Fig.~5. The universe initially contracts, with the field ϕ in one of the potential minima. Before the bounce, the field climbs up its potential [**?**, **?**] and its kinetic energy $\dot{\phi}^2$ becomes dominant. After the bounce, the kinetic energy of ϕ is rapidly diluted, the universe enters an inflationary phase, and finally the field rolls down the potential to another minimum. The contracting phase provides a homogeneous patch for the beginning of slow-roll inflation, helping to relax the initial conditions problem discussed in Ref.~[**?**].

The Lagrangian for ψ implies $\rho_\psi = c_\psi/a^{12}$. When $\dot{\phi}^2$ dominates, we have $\rho_\phi = c_\phi/a^6$. The Friedmann equation is

$$H^2 = \frac{1}{3M_P^2}\left(\frac{c_\phi}{a^6} - \frac{c_\psi}{a^{12}}\right).$$

There is a bounce at $a_B^6 = c_\psi/c_\phi$, with $H_B = 0$, while $a \propto t^{1/3}$ when a deviates from a_B, consistent with Fig.~6.

The field ϕ travels a certain distance during the kinetic-dominated phase before finally "landing"in an inflationary region. Here "landing"means that the effective potential begins to dominate. The change in ϕ before landing is $\Delta\phi$. We have

$$\Delta\phi \simeq M_P \ln \frac{H_{\mathrm{kin}}}{H_B},$$

where $H = 1/(3t)$ for $a \propto t^{1/3}$ is applied, H_B is the Hubble parameter at the bounce, and H_{kin} is the Hubble parameter when the field' s kinetic energy begins to dominate. Generally $H_B > H_{\mathrm{kin}}$, implying $\Delta\phi \gtrsim M_P$.

[Figure 5: see original paper]

[Figure 6: see original paper]

[Figure 7: see original paper]

[Figure 8: see original paper]

Appendix B: The Implication for the Eternal Inflation Scenario

In the eternal inflation scenario [**?**], an infinite number of universes are spawned in an eternally inflating background. One might think that a phase of slow-roll inflation and reheating is required for a spawned universe to become our observable universe. Slow-roll inflation should start at a high scale to ensure that the amplitude of primordial perturbations matches observations and that the reheating temperature is suitable for hot big bang evolution. Thus, if the scale of the eternally inflating background is very low, spawning an observable universe would require large upward tunneling, which is exponentially suppressed.

However, introducing a nonsingular bounce might significantly alter this conclusion [**?**, **?**, **?**]. Here we briefly revisit this issue in bouncing inflation, showing that it provides a favored channel to slow-roll inflation in a given landscape.

Consider a landscape with an AdS minimum 'A' , a dS minimum 'B' with lower energy, a dS minimum 'C' with higher energy, and a slow-roll inflationary region 'I' . The possible transitions are 'B' $\rightarrow$ 'A' , 'B' $\rightarrow$ 'I' , and 'I' 'C' . Additionally, 'I' may classically roll into 'B' , the AdS crunch in 'A' is replaced by a bounce, and the corresponding probabilities of bouncing to 'B' , 'C' , and 'I' are Q_B, Q_C, and Q_I, respectively, with $\sum_i Q_i = 1$.

Following Ref.~[**?**], the rate equations describing the fractions f_j in corresponding regions are

$$\dot{f}_j = \sum_i (\kappa_{ij} f_i - \kappa_{ji} f_j) + S_j,$$

where κ_{ij} is the transition rate, Γ_{ij} is the bubble nucleation rate, and $\kappa_{ij} = 4\pi\Gamma_{ij}/3H_j^3$. The source term $S_{BI} \sim 1/t_{BI}$ accounts for classical rolling, where $t_{BI} = N/H_{\rm inf}$ is the duration of slow-roll inflation and N is the number of e-folds.

The distributions f_j become fixed at late times. Using $Q_{BA} \ll 1$ and $\kappa_{IB} \ll \kappa_{AB}$, we obtain

$$\frac{f_I}{f_B} \simeq \frac{Q_{IA}\kappa_{AB}}{\kappa_{IB}}(1 + \kappa_{AI}/S_{BI}),$$

consistent with Garriga and Vilenkin [**?**]: the ratio is not suppressed by the small upward tunneling rate.

We are interested in the ratio of probabilities for different channels to the slow-roll inflationary region. One channel is the AdS bounce from ‘A’ , while others are upward tunneling from ‘B’and tunneling from ‘C’. The incoming probability currents into the slow-roll region are

$$\dot{P}_{A\to I} = Q_{IA}f_A, \quad \dot{P}_{B\to I} = \kappa_{IB}f_B, \quad \dot{P}_{C\to I} = Q_{IC}f_C.$$

The ratio of $P_{A\to I}$ to $P_{B\to I}$ is

$$\frac{P_{A\to I}}{P_{B\to I}} = \frac{Q_{IA}\kappa_{AB}}{\kappa_{IB}}(1 + \kappa_{AI}/S_{BI}) \gg 1,$$

since generally $\kappa_{AB} \gg \kappa_{IB}$ (as κ_{IB} is the upward tunneling rate). Thus, compared to upward tunneling to slow-roll inflation, bouncing inflation is exponentially favored.

Similarly, the ratio of $P_{A\to I}$ to $P_{C\to I}$ is

$$\frac{P_{A\to I}}{P_{C\to I}} \simeq \frac{Q_{IA}}{Q_{CA}}.$$

Thus, in a given landscape, bouncing inflation and inflationary bubbles from ‘C’ have comparable probabilities. However, if Q_{CA} is negligible, we have $P_{A\to I}/P_{C\to I} \gg 1$, where Q_{CA} is the contribution from the AdS bounce.

[Figure 9: see original paper]

Note: Figure translations are in progress. See original paper for figures.

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