CMB anomalies from an inflationary model in string theory (Postprint)

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Abstract

Recent Planck measurements show some CMB anomalies on large angular scales, which confirms the early observations by WMAP. We show that an inflationary model, in which before the slow-roll inflation the Universe is in a superinflationary phase, can generate a large-scale cutoff in the primordial power spectrum, which may account for not only the power suppression on large angular scales, but also a large dipole power asymmetry in the CMB. We discuss an implementation of our model in string theory.

Full Text

Preamble

CMB Anomalies from an Inflationary Model in String Theory

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Recent Planck measurements reveal several CMB anomalies on large angular scales, confirming earlier observations by WMAP. We demonstrate that an inflationary model in which the Universe undergoes a superinflationary phase prior to slow-roll inflation can generate a large-scale cutoff in the primordial power spectrum. This may account for both the observed power suppression on large angular scales and the large dipole power asymmetry in the CMB. We discuss an implementation of our model within string theory.

Introduction

The Planck collaboration has recently reported a hemispherical power asymmetry in the CMB [1], confirming the WMAP result with improved precision. This asymmetry has also been identified through power spectrum estimation in two hemispheres using the quadratic maximum likelihood method [2]. Additionally, Planck has reported a power deficit in the low-ℓ CMB power spectrum at $\ell < 40$ [1] at 3σ significance, which differs from the 2.5σ statistical significance of the Planck best-fit model, although the data points remain consistent with cosmic variance.

The Planck data exhibit greater statistical significance than WMAP, making it difficult to attribute these anomalies to foreground effects [3, 4]. Consequently, these anomalies likely share a common underlying physical origin that warrants serious consideration.

The CMB power asymmetry can be modeled as a dipole modulation of the power spectrum [5, 6] (see also [7]), arising from a superhorizon perturbation crossing the observable Universe [8, 9]. This modulation can be understood in terms of spatial variation of the primordial curvature perturbation spectrum:

$$\mathcal{P}(k, \mathbf{x}) = \mathcal{P}_0(k) \left[1 + A(k) \frac{\hat{\mathbf{p}} \cdot \mathbf{x}}{x_{\mathrm{ls}}} \right]$$

where $\hat{\mathbf{p}}$ is the unit vector of the dipole modulation direction, x_{ls} is the distance to the last scattering surface, $\mathcal{P}_0(k)$ is the unmodulated power spectrum, and $A(k)$ is the modulation amplitude given by [9, 10]:

$$A(k) = \left| \frac{\nabla P(k, \mathbf{x}) x_{\mathrm{ls}}}{P(k)} \right| \sim \frac{1}{\epsilon} \frac{1}{(k_L x_{\mathrm{ls}})}$$

where k_L is the modulating mode and $\epsilon = \dot{H}/H^2$. We have $A(k) \sim 0.1$ [8, 9, 11].

In single-field inflationary scenarios, the spectrum is nearly scale-invariant, making the modulation amplitude too small on large angular scales to fit observations [8, 9]. Furthermore, the near scale-invariance of the inflationary spectrum cannot explain the power deficit on large angular scales.

However, a large modulation amplitude consistent with observations requires breaking the scale invariance of the power spectrum on large angular scales, which simultaneously helps explain the power suppression on corresponding scales [10]. From this perspective, the large-scale anomalies may hint at pre-inflationary physics potentially related to the initial singularity [12, 13].

Here we show that an inflationary model with a superinflationary phase preceding slow-roll inflation can generate a large-scale cutoff in the primordial power spectrum, accounting for both the power suppression and the large dipole power asymmetry in the CMB.

Pre-inflationary physics should be governed by a fundamental theory such as string theory. Embedding inflationary scenarios into string theory remains a significant issue that has been studied extensively [14]. It is therefore intriguing to expect that a stringy mechanism of inflation could produce the observed CMB anomalies on large angular scales, as explored in [4, 15] using the string landscape and in [16, 17] with a fast-roll phase in fiber inflation [18]. We discuss an implementation of our model in string theory based on Refs. [19, 20].

II. The Modulating Mode from a Superinflationary Phase

We first calculate the primordial perturbations generated in this inflationary model and identify the corresponding modulating mode from the superinflationary phase. The curvature perturbation equation in momentum space is:

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0$$

where $u_k \equiv z\mathcal{R}_k$ is defined, with $z \equiv a\sqrt{2\epsilon}/c_s$. The prime denotes derivative with respect to conformal time η, and we have $c_s = 1$ for a canonical scalar field.

The Universe initially resides in a superinflationary phase with $\epsilon_{\mathrm{pre}} \sim -O(1)$, after which it transitions to an inflationary phase with $\epsilon_{\mathrm{inf}} \ll 1$. We neglect matching details for simplicity. Thus in conformal time, after adopting instantaneous matching, we have:

$$\frac{z''}{z} = \begin{cases} \frac{2}{\eta^2}\left(1 - \frac{3}{2}\epsilon_{\mathrm{pre}}\right) & \text{for superinflation} \\ \frac{2}{\eta^2}\left(1 - \frac{3}{2}\epsilon_{\mathrm{inf}}\right) & \text{for inflation} \end{cases}$$

where $\eta < 0$ in the superinflationary phase and $\eta > 0$ in the inflationary phase, respectively, and $a = a_0$ for $\eta = 0$. H_0 is the comoving Hubble length at the matching surface $\eta = 0$, which sets the inflationary energy scale by H_0/a_0. Here $\epsilon_{\mathrm{pre}} \sim -1$ is applied. In principle, other values with $|\epsilon_{\mathrm{pre}}| \gtrsim 1$ may also be used, which however hardly alter the result qualitatively.

The evolution of the superinflationary phase with arbitrary $\epsilon < 0$ and the generated primordial perturbations have been studied earlier in Ref. [21]. The case with $\epsilon \sim -1$ corresponds to the slow expansion scenario of the primordial universe, proposed earlier in Ref. [22] and investigated in detail in Ref. [23].

When $k^2 \gg z''/z$, i.e., $k\eta \gg 1$, the perturbation is deep inside its horizon and we have $u_k \sim e^{-ik\eta}$. In the superinflationary phase, $z''/z \simeq (H_0\eta)^2$. When $k^2 \ll z''/z$, the solution of Eq. (3) is:

$$u_k = \sqrt{-\eta}\left[C_1 H^{(1)}_{3/2}(-k\eta) + C_2 H^{(2)}_{3/2}(-k\eta)\right]$$

where $H_{3/2}^{(1)}$ is the 3/2th-order Hankel function of the first kind, $H_{3/2}^{(2)}$ is the 3/2th-order Hankel function of the second kind, and C_1 and C_2 are constants dependent only on k.

We require that all physical quantities continuously pass through the matching surface. The continuity of the curvature perturbation gives:

$$C_1 = \frac{\pi e^{ik/H_0}}{2\sqrt{2}H_0^{3/2}}\left[H_0^{(1)}\left(\frac{k}{H_0}\right) + iH_2^{(1)}\left(\frac{k}{H_0}\right)\right]$$

$$C_2 = \frac{\pi e^{ik/H_0}}{2\sqrt{2}H_0^{3/2}}\left[H_0^{(1)}\left(\frac{k}{H_0}\right) - iH_2^{(1)}\left(\frac{k}{H_0}\right)\right]$$

where $H_0^{(1)}$ is the zeroth-order Hankel function of the first kind and $H_2^{(1)}$ is the second-order Hankel function of the first kind.

Thus the power spectrum of curvature perturbation is:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2 = \mathcal{P}_{\mathcal{R}}^{\text{inf}}(k)\left|C_1 - C_2\right|^2$$

where $\mathcal{P}_{\mathcal{R}}^{\text{inf}}(k)$ is that of standard slow-roll inflation, which may have a slight red spectrum consistent with observations, and C_1 and C_2 are determined by Eqs. (9) and (10), respectively.

The spectral index is:

$$n_s = n_{\text{inf}} + \frac{d\ln|C_1 - C_2|^2}{d\ln k}$$

Here, H_0 is the comoving Hubble length at the matching surface $\eta = 0$. The modulating mode corresponds to large scales $k \ll H_0$, generated during the superinflationary evolution. We may expand the Hankel functions for $k \ll H_0$ and obtain:

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \mathcal{P}_{\mathcal{R}}^{\text{inf}}(k)\left(\frac{k}{H_0}\right)^{-2\epsilon_{\text{pre}}}$$

Thus the spectrum is strongly blue-tilted. It is precisely the superinflationary evolution that brings the modulating mode with $k \ll H_0$ to large angular scales. As shown in Eq. (2), the corresponding mode contributes a large modulation to the power spectrum. Therefore, this model may produce dipole power asymmetry on the relevant scales, consistent with the observation $A(k) \sim 0.07$. We plot Eq. (11) in Fig. 1, which agrees with our analytical result.

In the inflationary phase, $z''/z \simeq 2/\eta^2$. When $k^2 \gg z''/z$, i.e., $k\eta \gg 1$, the solution of Eq. (3) is:

$$u_k = \sqrt{-\eta}\left[C_1 H_{3/2}^{(1)}(-k\eta) + C_2 H_{3/2}^{(2)}(-k\eta)\right]$$

In Ref. [10], a similar spectrum was found for a bouncing inflation model, where before slow-roll inflation the Universe is in a contracting phase [12].

While at intermediate and small angular scales, i.e., $k \gg H_0$, we have:

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \mathcal{P}_{\mathcal{R}}^{\text{inf}}(k)\left[1 + \mathcal{O}\left(\frac{H_0}{k}\right)^2\right]$$

Thus the spectrum is nearly scale-invariant but modulated by small oscillations, which is the standard result of slow-roll inflationary evolution. Consequently, the dipole power asymmetry is small on these scales, consistent with constraints from the SDSS quasar sample [25] and also [26].

III. The CMB Angular Power Spectrum with Planck

We demonstrate the fit of our model to the CMB TT spectrum and the corresponding signals in TE and EE power spectra.

The slow-roll inflationary spectrum in Eq. (11) may be parameterized as a power law with $A_{\text{inf}}(k/k_0)^{n_{\text{inf}}-1}$, where A_{inf} is the perturbation amplitude. We follow Ref. [1] and choose the pivot scale $k_0 = 0.05\,\text{Mpc}^{-1}$, roughly in the middle of the logarithmic range probed by Planck.

We assume a standard flat ΛCDM cosmology described by four free parameters: $\Omega_b h^2$, $\Omega_c h^2$, Θ_s, and τ. Here h is the dimensionless Hubble parameter such that $H_0 = 100h\,\text{km s}^{-1}\text{Mpc}^{-1}$ (note this H_0 is unrelated to the cutoff scale), $\Omega_b h^2$ and $\Omega_c h^2$ are the physical baryon and dark matter densities relative to the critical density today, Θ_s is the ratio of the sound horizon to the angular diameter distance at photon decoupling, and τ is the Thomson scattering optical depth due to reionization.

We modify the numerical Boltzmann code CAMB [29] to calculate the lensed TT, TE, EE power spectra and the 2-point correlation function, shown in Fig. 2. The blue dashed curves show the pure power law, while the black solid curves show our model (11) with the best-fit value $\ln(H_0/\text{Mpc}^{-1}) = 7.47$. We see that the TT, TE, and EE spectra for our model are suppressed for $\ell < 6$ compared to the pure power law. Since corresponding signals appear in the TE and EE spectra, ongoing Planck polarization data are expected to improve constraints on the model parameter H_0. As shown in [30], polarization data can test the parity asymmetry of the CMB pattern. Note the small bump around $\ell \sim 6$ in the TT spectrum due to oscillations in the primordial power spectrum at large scales. The predicted 2-point correlation function at $\theta > 50^\circ$ fits the Planck data much better than the pure power-law spectrum [31].

We use the Planck CMB temperature likelihood [1] supplemented by the WMAP large-scale polarization likelihood [32] (Planck+WP). The Planck temperature likelihood includes high-ℓ TT data ($50 \leq \ell \leq 2500$) and low-ℓ TT data ($2 \leq \ell \leq 49$). Due to contributions to multi-frequency spectra from unresolved radio point sources, cosmic infrared background, Sunyaev-Zeldovich effects, calibration and beam uncertainties, the Planck high-ℓ likelihood includes 14 nuisance parameters that must be marginalized in the analysis. As discussed in [1], large-scale E-mode polarization data are important for constraining reionization. Hence we also use the 9-year WMAP large-scale polarization likelihood including TE, EE, and BB spectra.

We employ the Markov Chain Monte Carlo sampler implemented in the CosmoMC package [33] to construct posterior parameter probabilities. Since the Planck high-ℓ likelihood includes many nuisance parameters that are fast parameters, we adopt a new sampling method for decorrelating fast and slow parameters to efficiently scan the parameter space [34]. We impose a flat prior on $\ln(H_0/\mathrm{Mpc}^{-1})$ in the range [4, 10]. For other cosmological parameters, prior ranges are chosen to be much larger than the posterior.

For the Planck+WP likelihood we find the best-fit value $\ln(H_0/\mathrm{Mpc}^{-1}) = 7.47$ with $\mathcal{L}_{\mathrm{max}} = 9803.0$. This means our model improves the fit to the data by $\Delta \ln \mathcal{L} = 4.8$ relative to the standard power-law model. In contrast, a two-parameter exponential cutoff of the primordial power spectrum improves the fit by only $\Delta \ln \mathcal{L} = 2.9$ as reported in [35]. The reason is that the small bump in the temperature spectrum induced by primordial power spectrum oscillations improves the fit. Fig. 3 shows the marginalized posterior distributions for H_0 from Planck+WP data, illustrating the asymmetric shape of the likelihood functions.

Recently, several explanations have attempted to address the anomalies [9, 11, 15, 36, 37, 38, 39, 40]. However, most involve only the dipole power asymmetry in CMB, not the power deficit on large angular scales. By contrast, our model generates both the power asymmetry and suppression on large angular scales, similar to the bouncing inflationary model in [10].

Power suppression on large angular scales has also been implemented in fiber inflation [16, 17, 18], brane SUSY breaking models [13], and punctuated inflation [42]. However, these studies did not explain the dipole power asymmetry in CMB.

IV. An Implementation in String Theory

Embedding such an inflationary model into string theory is interesting. We discuss an implementation based on warped compactifications with brane/flux annihilation [43], where the effective potential may support cosmological inflation [19, 20]. We find that a superinflationary phase may precede slow-roll inflation.

In a 10-dimensional CY manifold with a warped KS throat, the throat metric is:

$$ds^2 = f(r)^{-1/2} dx_{1,3}^2 + f(r)^{1/2}(dr^2 + r^2 ds_{X_5}^2)$$

where r is the proper distance to the tip of the throat, $ds_{X_5}^2$ is the angular part of the internal metric, and $f(r)$ is the warp factor with minimal value at r_0 determined by:

$$R^4 \equiv \frac{27\pi}{4} g_s N \alpha'^2$$

where N equals the product of fluxes for RR and NSNS 3-forms, g_s is the string coupling, and α' is set by the string scale.

When $r > r_0$, this metric can be glued to the bulk compact space metric, usually taken to be a CY manifold. When $r_0 < r < r_*$, $f(r) \approx (R^4/r^4)$.

Following Ref. [43], when p D3-branes sit at the KS throat tip, the system forms a nonsupersymmetric NS5-brane "giant graviton" configuration, where the NS5-brane wraps an S^2 in S^3 and carries p units of flux, inducing D3-charge. The S^2 tends to expand as a spherical shell in S^3, parameterized by an angle $0 \le \psi \le \pi$, where $\psi = 0$ corresponds to the north pole of S^3 and $\psi = \pi$ to the south pole. The angular position acts as a scalar in the worldvolume action, describing NS5-brane motion across S^3. The effective potential controlling the evolution is:

$$V_{\text{eff}}(\psi) = \beta^4 T_3 \left[\frac{p}{\mathcal{N}} \sin^4 \psi + \tilde{V}_2(\psi) + \tilde{V}_0(\psi) \right]$$

where $\tilde{V}_2(\psi) = \frac{1}{2\pi} \sin(2\psi)$, $b_0 \simeq 0.9$, and T_3 is the D3-brane tension. This potential is plotted in Fig. 4.

For $p/\mathcal{N} < 0.08$, a metastable bound state forms corresponding to a static NS5-brane wrapping an S^2 in S^3. This metastable state corresponds to $\psi = 0$ with $V_{\text{eff}}(0) = 2p\beta^4 T_3$. The true minimum is at $\psi = \pi$ with zero potential energy.

For $p/\mathcal{N} \gtrsim 0.08$, this metastable state disappears, implying that the nonsupersymmetric configuration of p D3-branes becomes classically unstable and relaxes to the supersymmetric minimum through classical rolling of ψ along its potential. This rolling can lead to slow-roll inflation, studied in detail in Ref. [20]. Inflation ends when $\psi = \pi$ and the potential energy vanishes. The result is $p \to p - \Delta p$ D3-branes instead of the original p branes, while the 3-form flux changes to $\mathcal{N} \to \mathcal{N} - 1$, i.e., brane/flux annihilation [43].

During the period before slow-roll inflation, when $p/\mathcal{N} < 0.08$, the Hubble expansion is:

$$H^2 = \frac{2p\beta^4 T_3}{3M_P^2}$$

where $8\pi/M_P^2 = 1$. When D3-branes are continuously pulled into the throat, the metastable minimum rises [44], implying that H increases rapidly. Thus:

$$\epsilon = -\frac{\dot{H}}{H^2} \sim -\frac{\Delta p}{2p}$$

where Δp is the change in p per unit $1/H$. We assume $\Delta p/2p \gtrsim 1$, consistent with $\epsilon_{\rm pre} \sim -O(1)$, where p_I is the initial number of D3-branes at the KS throat tip. Here all moduli are assumed fixed, and interactions between D3-branes are neglected for simplicity.

Thus, in this model the Universe initially undergoes a superinflationary phase with $\epsilon_{\rm pre} \sim -O(1)$, during which the number of D3-branes at the throat tip increases rapidly. After sufficient D3-branes enter the throat, making p reach its critical value, ψ slowly rolls to its true minimum at $\psi = \pi$, and the Universe enters a slow-roll inflationary phase. Therefore, the stringy physics before slow-roll inflation produces a large-scale cutoff in the primordial power spectrum.

We conclude that a stringy inflationary model with an initial superinflationary phase can generate a large-scale cutoff in the primordial power spectrum, accounting for both the power suppression on large angular scales and the large dipole power asymmetry in the CMB. This model also predicts distinct signals in the TE and EE power spectra that may be tested by CMB polarization observations.

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