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Authors: Rong-Gen Cai, Zong-Kuan Guo, Bo Tang

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Full Text

Constraints on the ΛCDM Model with Redshift Tomography

Rong-Gen Cai[1],*, **Zong-Kuan Guo**[1],†, and **Bo Tang**[1],‡

[1]State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China

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Abstract

Recently released Planck data favor a lower value of the Hubble constant and a higher value of the matter density fraction in the standard ΛCDM model, which are discrepant with some low-redshift measurements. Within the context of this cosmology, we examine the consistency of the estimated values for the Hubble constant and matter density fraction using redshift tomography. By dividing the data into three redshift bins and using SNe Ia, Hubble parameter, BAO, and reduced CMB data, we find no statistical evidence for any tension among the three redshift bins.

*Electronic address: cairg@itp.ac.cn
†Electronic address: guozk@itp.ac.cn
‡Electronic address: tangbo@itp.ac.cn

I. INTRODUCTION

More than a decade ago, observations of type Ia supernovae (SNe Ia) revealed that our universe is undergoing accelerated expansion [?, ?]. This observation is consistent with other astronomical measurements such as the Hubble parameter, large-scale structure, and cosmic microwave background (CMB) radiation. To explain this accelerated expansion, one must either introduce dark energy with negative pressure within the framework of general relativity or modify general relativity at cosmic scales. Despite suffering from some theoretical issues, the cosmological constant introduced by Einstein in 1917 [?, ?] represents the simplest and most economical candidate for dark energy. Indeed, the standard ΛCDM model proves consistent with several precise astronomical observations, including SNe Ia [?], Wilkinson Microwave Anisotropy Probe (WMAP) measurements of the CMB [?], and baryon acoustic oscillations. If the standard ΛCDM model properly describes our universe, the current Hubble constant H_0 and matter density fraction Ω_{m0} should be consistent across different observations made at different redshifts.

However, the recently released Planck data [?] favor a higher value of $\Omega_{m0} = 0.315$ and a lower value of $H_0 = (67.3 \pm 1.2)$ km s^{-1} Mpc^{-1} in the standard six-parameter ΛCDM cosmology, obtained using Planck+WP, where WP stands for WMAP polarization data. These values are in tension with the magnitude-redshift relation for SNe Ia and recent direct measurements of H_0, such as the Hubble Space Telescope observations of Cepheid variables yielding $H_0 = (73.8 \pm 2.4)$ km s^{-1} Mpc^{-1} using a mid-infrared calibration of the Cepheid distance scale [?] and $H_0 = [74.3 \pm 1.5(\text{stat.}) \pm 2.1(\text{sys.})]$ based on observations at 3.6 μm with the Spitzer Space Telescope [?]. Of course, if one relaxes the restrictions of the standard six-parameter ΛCDM model—for example, by considering dynamical dark energy models [?] or including dark radiation [?]—the tension might be alleviated. Hu et al. [?] found another way to alleviate this tension in modified gravity models. Furthermore, Li et al. [?] reported that the tension may also be reduced if one first calibrates the light-curve fitting parameters in the distance estimation from SNe Ia observations using the angular diameter distance data of galaxy clusters, with the help of the distance-duality relation. Very recently, Efstathiou [?] reanalyzed the Cepheid data and found $H_0 = (70.6 \pm 3.3)$ km s^{-1} Mpc^{-1} based on the NGC 4258 maser distance and $H_0 = (72.5 \pm 2.5)$ km s^{-1} Mpc^{-1} with three distance anchors combined, which alleviates the tension compared to the result obtained by Riess et al. [?], but the latter still differs by 1.9σ from the Planck value. In addition, by comparing eight ultra-low-redshift SNe Ia data ($z = 0.0043$ to 0.0072) [?] with low-redshift data ($z < 0.04$) from the Union2.1 compilation [?] and Planck data [?], Zhang and Ma found that the present expansion rate estimated from low-redshift measurements is higher than that estimated from high-redshift observations in the ΛCDM model [?]. In other words, higher-redshift measurements yield a lower value of h, the reduced Hubble constant.

These discrepancies seemingly imply that the standard ΛCDM model cannot adequately describe the properties of the universe at all redshifts if the major sources of systematic error in these observations have been controlled. In this paper, we investigate these discrepancies in the ΛCDM model using redshift tomography. We divide the redshift range under consideration into three bins and use observational data in each bin to separately constrain the Hubble constant and matter density fraction in the ΛCDM model. In the literature, redshift tomography is often employed to study the dynamical properties of dark energy through piecewise parametrization of the equation of state. Here, our goal is to examine the consistency of the ΛCDM model at different redshifts; therefore, we focus exclusively on the ΛCDM model. The datasets we use include the Union2.1 SNe Ia data [?], 19 Hubble parameter $H(z)$ measurements [?, ?, ?], baryon acoustic oscillation (BAO) data from the 6-degree Field Galaxy Survey (6dFGS), SDSS DR7, SDSS DR9, and WiggleZ surveys, reduced nine-year WMAP data (WMAP9), and reduced Planck data, all based on the flat ΛCDM model.

The paper is organized as follows. In Section II, we describe the redshift tomography method and observational data. In Section III, we present the results from different combinations of datasets constraining the base ΛCDM model based on SNe Ia data and redshift tomography analysis. We summarize our conclusions in Section IV.

II. METHOD AND DATA

In a spatially flat Friedmann-Robertson-Walker universe, the Hubble parameter for the ΛCDM model is given by the Friedmann equation

$$H^2(z) = H_0^2\left[\Omega_{r0}(1+z)^4 + \Omega_{dm0}(1+z)^3 + \Omega_{b0}(1+z)^3 + (1-\Omega_{m0})\right]$$

where the redshift z is defined by $(1+z) = 1/a$, and Ω_{r0}, Ω_{dm0}, and Ω_{b0} are the present values of the fractional energy densities for radiation, dark matter, and baryonic matter, respectively. The latter two are often combined as the total matter density $\Omega_{m0} = \Omega_{b0} + \Omega_{dm0}$. The radiation density is the sum of photons and relativistic neutrinos [?]:

$$\Omega_{r0} = \Omega_\gamma^{(0)}\left(1 + 0.2271 N_{\text{eff}}\right)$$

where $N_{\text{eff}} = 3.046$ is the effective number of neutrino species in the Standard Model [?], and $\Omega_\gamma^{(0)} = 2.469\times10^{-5}h^{-2}$ for $T_{\text{CMB}} = 2.725\text{K}$ ($h \equiv H_0/100$ km s^{-1} Mpc^{-1}).

We focus on constraining the Hubble constant and matter density fraction in the context of the ΛCDM cosmology using low-redshift observational data including the Union2.1 SNe Ia sample, Hubble parameter measurements, and BAO data, in combination with high-redshift CMB measurements. We adopt a redshift tomography method to examine the flat ΛCDM model. Since SNe Ia data alone cannot constrain the ΛCDM model very well, and this becomes even more problematic in each redshift bin due to the decreasing number of data points (as is also true for Hubble parameter data), we divide the redshift range into three bins so that the BAO data can be distributed uniformly in the first two bins, while the CMB data are placed in the third bin. Consequently, the data are divided into three combinations in the following redshift bins: $0 < z < 0.28$, $0.28 < z < 0.73$, and $z > 0.73$. The distribution of data is listed in Table 1 . For comparison, we use WMAP9 and Planck data separately.

The best-fit values of Ω_{m0} and h, along with their 68% and 95% confidence level (CL) errors, are obtained by performing Markov Chain Monte Carlo analysis in the multidimensional parameter space within a Bayesian framework. Since the Hubble constant is completely degenerate with the absolute magnitude of SNe Ia, SNe Ia data are not sensitive to the Hubble constant. Therefore, in our analysis we marginalize analytically over the Hubble constant when using SNe Ia data. Moreover, note that the fractional baryon energy density Ω_{b0} is involved in the likelihood for BAO and CMB data.

A. Type Ia Supernovae The SNe Ia dataset is an important tool for understanding the evolution of the universe. In this work, we adopt the Union2.1 compilation [**?**], containing 580 SNe Ia data over the redshift range $0.015 < z < 1.414$. The chi-square is defined as

$$\chi^2_{\mathrm{SN}} = \sum_{i=1}^{N} \frac{[\mu_{\mathrm{obs}}(z_i) - \mu_{\mathrm{th}}(z_i)]^2}{\sigma^2_{\mathrm{SN}}(z_i)}$$

where N is the number of data points in the redshift interval of interest, $\mu_{\mathrm{obs}}(z)$ is the measured distance modulus from the data, and $\mu_{\mathrm{th}}(z)$ is the theoretical distance modulus, defined as

$$\mu_{\mathrm{th}}(z) = 5\log_{10} d_L + \mu_0, \quad \mu_0 = 42.384 - 5\log_{10} h.$$

The luminosity distance is

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{E(z')}$$

where $E(z) \equiv H(z)/H_0$. We can eliminate the nuisance parameter μ_0 by expanding χ^2 with respect to μ_0 [**?**]:

$$\chi^2_{\mathrm{SN}} = A + 2B\mu_0 + C\mu_0^2$$

where

$$A = \sum_i \frac{[\mu_{\mathrm{th}}(z_i; \mu_0 = 0) - \mu_{\mathrm{obs}}(z_i)]^2}{\sigma^2_{\mathrm{SN}}(z_i)},$$

$$B = \sum_i \frac{\mu_{\mathrm{th}}(z_i; \mu_0 = 0) - \mu_{\mathrm{obs}}(z_i)}{\sigma^2_{\mathrm{SN}}(z_i)},$$

$$C = \sum_i \frac{1}{\sigma^2_{\mathrm{SN}}(z_i)}.$$

The χ^2_{SN} has a minimum at $\tilde{\chi}^2_{\mathrm{SN}} = A - B^2/C$, which is independent of μ_0. This technique is equivalent to performing a uniform marginalization over μ_0 [**?**]. We adopt $\tilde{\chi}^2_{\mathrm{SN}}$ as the goodness-of-fit statistic instead of χ^2_{SN}.

B. Observational Hubble Parameter (HUB) The observational Hubble parameter can be obtained using the differential ages of passively evolving galaxies as

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}.$$

We use 19 observational Hubble measurements over the redshift range $0.07 < z < 2.3$, which include 11 data points obtained from the differential ages of passively evolving galaxies [**?**, **?**] and 8 $H(z)$ measurements at different redshifts obtained from the differential spectroscopic evolution of early-type galaxies as a function of redshift [**?**]. The chi-square is defined as

$$\chi^2_{\mathrm{HUB}} = \sum_i \frac{[H_{\mathrm{th}}(z_i) - H_{\mathrm{obs}}(z_i)]^2}{\sigma^2_H(z_i)}$$

where $H_{\mathrm{th}}(z)$ and $H_{\mathrm{obs}}(z)$ are the theoretical and observed values of the Hubble parameter, and σ_H denotes the error bar of the observed data.

C. Baryon Acoustic Oscillation As a standard ruler for measuring the distance-redshift relation, baryon acoustic oscillations provide an efficient method for probing the expansion history of the universe using features in galaxy clustering from large-scale surveys. Here we use results from five BAO surveys: the 6dF Galaxy Survey, SDSS DR7, SDSS DR9, WiggleZ measurements, and radial BAO measurements.

1. 6dF Galaxy Survey

The 6dFGS BAO detection constrains the distance-redshift relation at $z_{\text{eff}} = 0.106$ [?]. The low effective redshift of 6dFGS makes it a competitive and independent alternative to Cepheids and low-redshift supernovae for constraining the Hubble constant. They measured the distance ratio

$$r_s(z_d)/D_V(z = 0.106) = 0.336 \pm 0.015,$$

where $r_s(z_d)$ is the comoving sound horizon at the baryon drag epoch when baryons became dynamically decoupled from photons. The redshift z_d is well approximated by [?]

$$z_d = \frac{1291(\Omega_{m0}h^2)^{0.251}}{1 + 0.659(\Omega_{m0}h^2)^{0.828}} \left[1 + b_1(\Omega_{b0}h^2)^{b_2}\right],$$

where

$$b_1 = 0.313(\Omega_{m0}h^2)^{-0.419} \left[1 + 0.607(\Omega_{m0}h^2)^{0.674}\right], \quad b_2 = 0.238(\Omega_{m0}h^2)^{0.223}.$$

The effective "volume" distance D_V is a combination of the angular-diameter distance $D_A(z)$ and the Hubble parameter $H(z)$:

$$D_V(z) = \left[\frac{cz(1+z)^2 D_A(z)^2}{H(z)}\right]^{1/3}.$$

The χ^2_{6dF} is given by

$$\chi^2_{6dF} = \left[\left(\frac{r_s(z_d)}{D_V(0.106)}\right)_{\text{th}} - 0.336\right]^2 / (0.015)^2.$$

2. SDSS DR7

The joint analysis of the 2-degree Field Galaxy Redshift Survey data and the Sloan Digital Sky Survey Data Release 7 provides distance ratios at $z = 0.2$ and $z = 0.35$ [?]:

$$\frac{r_s(z_d)}{D_V(z = 0.2)} = 0.1905 \pm 0.0061, \quad \frac{r_s(z_d)}{D_V(z = 0.35)} = 0.1097 \pm 0.0036.$$

When the two data points fall in the same redshift bin, we adopt the χ^2_{DR7} given by

$$\chi^2_{\rm DR7} = X^T V^{-1} X,$$

where

$$X = \begin{pmatrix} \left[\frac{r_s(z_d)}{D_V(0.2)}\right]_{\rm th} - 0.1905 \\ \left[\frac{r_s(z_d)}{D_V(0.35)}\right]_{\rm th} - 0.1097 \end{pmatrix},$$

and the inverse covariance matrix is

$$V^{-1} = \begin{pmatrix} 30124 & -17227 \\ -17227 & 86977 \end{pmatrix}.$$

When the two data points fall in different redshift bins, their chi-square values are respectively given by

$$\chi^2_{\rm DR7a} = \left[\left(\frac{r_s(z_d)}{D_V(0.2)}\right)_{\rm th} - 0.1905\right]^2 / (0.0061)^2,$$

$$\chi^2_{\rm DR7b} = \left[\left(\frac{r_s(z_d)}{D_V(0.35)}\right)_{\rm th} - 0.1097\right]^2 / (0.0036)^2.$$

3. SDSS DR7 Reanalysis

By applying reconstruction techniques [?] to the clustering of galaxies from the SDSS DR7 Luminous Red Galaxies sample to sharpen the BAO feature, Padmanabhan et al. obtained the distance ratio at $z = 0.35$ [?]:

$$\frac{r_s(z_d)}{D_V(z = 0.35)} = 0.1126 \pm 0.0022.$$

The $\chi^2_{\rm DR7\text{-}re}$ used in the Markov Chain Monte Carlo analysis is

$$\chi^2_{\rm DR7\text{-}re} = \left[\left(\frac{r_s(z_d)}{D_V(0.35)}\right)_{\rm th} - 0.1126\right]^2 / (0.0022)^2.$$

Since the SDSS DR7 and SDSS DR7 reanalysis results are based on the same survey and the latter provides higher precision, we include the SDSS DR7 reanalysis data when performing the full redshift analysis, but not both simultaneously. On the other hand, when conducting redshift tomography, we may refer to part of the SDSS DR7 data at $z = 0.2$, and when the redshift bin contains $z = 0.35$, we use the SDSS DR7 reanalysis data.

4. SDSS DR9

The SDSS DR9 measurement at $z = 0.57$ analyzed by Anderson et al. [?] gives

$$\frac{r_s(z_d)}{D_V(z = 0.57)} = 0.0732 \pm 0.0012,$$

which represents the most precise determination of the acoustic oscillation scale to date. The chi-square is defined as

$$\chi^2_{\text{DR9}} = \left[\left(\frac{r_s(z_d)}{D_V(0.57)}\right)_{\text{th}} - 0.0732\right]^2 / (0.0012)^2.$$

5. The WiggleZ Measurements

The WiggleZ team incorporates shape information from the power spectrum to measure the acoustic parameter [?]:

$$A(z) = \frac{D_V(z)\sqrt{\Omega_{m0} H_0^2}}{cz}.$$

Measurements of the baryon acoustic peak at redshifts $z = 0.44$, 0.6, and 0.73 in the galaxy correlation function of the final dataset from the WiggleZ Dark Energy Survey yield the acoustic parameter:

$$A(z = 0.44) = 0.474 \pm 0.034, \quad A(z = 0.60) = 0.442 \pm 0.020, \quad A(z = 0.73) = 0.424 \pm 0.020.$$

The chi-square is defined as

$$\chi^2_{\text{Wig}} = X^T V^{-1} X,$$

where

$$X = \begin{pmatrix} A(z = 0.44)_{\text{th}} - 0.474 \\ A(z = 0.60)_{\text{th}} - 0.442 \\ A(z = 0.73)_{\text{th}} - 0.424 \end{pmatrix},$$

and its inverse covariance matrix is

$$V^{-1} = \begin{pmatrix} 1040.3 & -807.5 & 336.8 \\ -807.5 & 3720.3 & -1551.9 \\ 336.8 & -1551.9 & 2914.9 \end{pmatrix}.$$

6. Radial BAO

The radial (line-of-sight) baryon acoustic scale can also be measured using SDSS data. This measurement is independent of the BAO measurements described above, which are averaged over all directions or measured in the transverse directions. The measured quantity is

$$\Delta z(z) = H(z) r_s(z_d),$$

with values given by [?]

$$\Delta z(0.24) = 0.0407 \pm 0.0011, \quad \Delta z(0.43) = 0.0442 \pm 0.0012.$$

D. Cosmic Microwave Background In CMB measurements, the distance to the last scattering surface can be accurately determined from the locations of peaks and troughs of acoustic oscillations. There are two key quantities: the "acoustic scale"

$$l_A = (1 + z_*) \frac{\pi D_A(z_*)}{r_s(z_*)}$$

and the "shift parameter"

$$R = \sqrt{\Omega_{m0} H_0^2} (1 + z_*) D_A(z_*).$$

These quantities can be used to constrain cosmological parameters without requiring the full WMAP9 dataset [?]. Here z_* is the redshift at the last scattering surface [?]:

$$z_* = 1048 \left[1 + 0.00124(\Omega_{b0}h^2)^{-0.738}\right] \left[1 + g_1(\Omega_{m0}h^2)^{g_2}\right],$$

where

$$g_1 = \frac{0.0783(\Omega_{b0}h^2)^{-0.238}}{1 + 39.5(\Omega_{b0}h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_{b0}h^2)^{1.81}}.$$

Wang and Wang [?] obtained the mean values and covariance matrix of $\{R, l_A, \Omega_{b0}h^2, n_s\}$ without assuming a flat universe. On the other hand, Shafer and Huterer [?] derived related results about $\{R, l_A, z_*\}$ from WMAP9 and Planck data, respectively, based on the flat wCDM model. For our purposes, following [?] and [?], we first extract the mean values and covariance matrix of $\{R, l_A, z_*\}$ from WMAP9 and Planck data, respectively, based on a flat ΛCDM model.

1. WMAP9

Using the WMAP9 data, we obtain the mean values

$$\{R, l_A, z_*\} = \{1.7257, 301.95, 1088.96\}.$$

Their inverse covariance matrix is

$$C^{-1}_{\mathrm{WMAP9}} = \begin{pmatrix} 0.11095 & -2.4517 & 0.92293 \\ -2.4517 & 11495 & -5359.9 \\ 0.92293 & -5359.9 & 11068 \end{pmatrix}.$$

The chi-square for the reduced WMAP9 data is defined by

$$\chi^2_{\mathrm{WMAP9}} = X^T C^{-1}_{\mathrm{WMAP9}} X,$$

where $X = \{R_{\mathrm{th}} - 1.7257, (l_A)_{\mathrm{th}} - 301.95, z_*^{\mathrm{th}} - 1088.96\}$.

2. Planck

Using the Planck data, we obtain the mean values

$$\{R, l_A, z_*\} = \{1.7500, 301.65, 1090.33\}.$$

Their inverse covariance matrix is

$$C^{-1}_{\mathrm{Planck}} = \begin{pmatrix} 0.11231 & -1.9154 & 0.88494 \\ -1.9154 & 8633.7 & -4261.1 \\ 0.88494 & -4261.1 & 13053 \end{pmatrix}.$$

The chi-square for the reduced Planck data is defined as

$$\chi^2_{\mathrm{Planck}} = X^T C^{-1}_{\mathrm{Planck}} X,$$

where $X = \{R_{\mathrm{th}} - 1.7500, (l_A)_{\mathrm{th}} - 301.65, z_*^{\mathrm{th}} - 1090.33\}$.

III. RESULTS

Using the Union2.1 sample in combination with the other measurements described in the previous section, we constrain the base ΛCDM model. The best-fit values of Ω_{m0} and h with 68% CL errors are summarized in Table 2 , and their likelihood distributions are shown in Figure 1 [Figure 1: see original paper].

From Table 2, we see that SNe Ia data alone favor a lower value of Ω_{m0} than when combined with other datasets. Including BAO and WMAP9/Planck data significantly improves the constraint on Ω_{m0}. Including the reduced Planck data yields the highest Ω_{m0} and the lowest h. We find these estimates of Ω_{m0} are consistent with each other within 1σ CL, but remain in tension with the results derived by Planck [?]. The estimates of h from HUB, BAO, and WMAP9 are compatible with those from Planck, but are discrepant with those from fitting the calibrated SNe magnitude-redshift relation [?].

Using the SN+HUB+BAO+WMAP9/Planck data distributed across three different redshift bins, we present constraints on Ω_{m0} and h for the ΛCDM model in Table 3 . The corresponding marginalized posterior distributions are shown in Figure 2 [Figure 2: see original paper].

Our analysis shows that low-redshift observations yield a higher value of Ω_{m0}, while high-redshift observations ($z > 0.73$) with WMAP9 data give a lower value. However, high-redshift observations with Planck data favor a relatively higher value of Ω_{m0}, which is inconsistent with the high-redshift value from WMAP9 at about 1.1σ CL. Additionally, there are large uncertainties in the estimation of Ω_{m0} from data in the redshift range $0 < z < 0.28$. From Table 3, we find that data in the mid-redshift range $0.28 < z < 0.73$ favor a lower Hubble constant with somewhat larger uncertainty than data at low and high redshifts. Figure 3 [Figure 3: see original paper] shows the best-fit values of Ω_{m0} and h with 1σ errors for the data in the three redshift bins.

In our analysis, the Hubble constant is marginalized as a nuisance parameter in the SNe Ia likelihood function. Therefore, constraints on h primarily come from the HUB, BAO, and WMAP9/Planck data. Reference [?] recently obtained a higher value of the Hubble constant from measurements of nearby SNe Ia using Cepheid variable measurements, compared to the Planck result. However, our estimates of h from data in the redshift ranges $z < 0.28$ and $0.28 < z < 0.73$ are lower than the result obtained in [?]. Moreover, high-redshift data ($z > 0.73$) including WMAP9 favor a higher value of h than data in the first two redshift bins.

IV. CONCLUSIONS

The estimates of Ω_{m0} and h in the base ΛCDM model should be consistent across measurements made in different redshift intervals if the simplest ΛCDM model completely describes the evolution of our universe and the unknown sources of systematic error in these measurements are negligible. Recent Planck observations of the CMB yield a Hubble constant of $h = 0.673 \pm 0.012$ and a matter density parameter of $\Omega_{m0} = 0.315 \pm 0.017$ [?], which differ from low-redshift measurements. In this work, we have studied the consistency of estimated values for the Hubble constant and matter density parameter from different redshift data.

We first obtained reduced CMB data for $\{R, l_A, z_*\}$ from WMAP9 and Planck data based on a flat ΛCDM model. We then placed constraints on the base ΛCDM model using astrophysical measurements of SNe Ia, Hubble parameters, and BAO, in combination with reduced WMAP9/Planck CMB data. We found that SNe Ia data alone favor a lower value of Ω_{m0}, and adding HUB, BAO, and reduced WMAP9/Planck data yields a higher value, but it remains in tension with the Planck result. Moreover, estimates of h from HUB, BAO, and WMAP9 are compatible with those from Planck but are discrepant with those from fitting the calibrated SNe magnitude-redshift relation [?]. There is no tension on h among the three redshift bins, as shown in Figure 3 [Figure 3: see original paper].

We have also implemented redshift tomography analysis in the context of the ΛCDM cosmology using SNe Ia, HUB, BAO, and CMB data. We found that low-redshift observations ($z < 0.28$) yield a higher value of Ω_{m0}, as estimated by Planck, while high-redshift observations ($z > 0.73$) with WMAP9 data give a lower value, which is inconsistent with that from SN+HUB+BAO data in the high-redshift range combined with Planck data at about 1.1σ CL. Additionally, data in the mid-redshift range $0.28 < z < 0.73$ favor a lower Hubble constant. The current data cannot provide statistically significant evidence for any tension among the different redshift bins.

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