# Reconstruction of the primordial power spectra with Planck and BICEP2 postprint

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## Abstract

By using the cubic spline interpolation method, we reconstruct the shape of the primordial scalar and tensor power spectra from the recently released Planck temperature and BICEP2 polarization cosmic microwave background data. We find that the vanishing scalar index running (dns/d ln k) model is strongly disfavored at more than 3  confidence level on the k = 0.0002 Mpc−1 scale. Furthermore, the power-law parameterization gives a blue-tilt tensor spectrum, no matter using only the first 5 bandpowers nt = 1.20+0.56 (95%CL) or the full 9 bandpowers nt = 1.24+0.51 (95%CL) of−0.64 −0.58BICEP2 data sets. Unlike the large tensor-to-scalar ratio value (r 0.20) under the scale-invariant tensor spectrum assumption, our interpolation approach gives r0.002 < 0.060 (95%CL) by using the first 5 bandpowers of BICEP2 data. After comparing the results with/without BICEP2 data, we find that Planck temperature with small tensor amplitude signals and BICEP2 polarization data with large tensor amplitude signals dominate the tensor spectrum reconstruction on the large and small scales, respectively. Hence, the resulting blue tensor tilt actually reflects the tension between Planck and BICEP2 data.

## Full Text

## Preamble

### Reconstruction of the Primordial Power Spectra with Planck and BICEP2

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Using the cubic spline interpolation method, we reconstruct the shape of the primordial scalar and tensor power spectra from the recently released Planck temperature and BICEP2 polarization cosmic microwave background data. We find that the vanishing scalar index running ($dns/d\ln k$) model is strongly disfavored at more than $3\sigma$ confidence level on the $k = 0.0002\,\mathrm{Mpc}^{-1}$ scale. Furthermore, the power-law parameterization gives a blue-tilt tensor spectrum, whether using only the first 5 bandpowers ($n_t = 1.20^{+0.56}_{-0.58}$ at 95% CL) or the full 9 bandpowers ($n_t = 1.24^{+0.51}_{-0.58}$ at 95% CL) of BICEP2 data sets. Unlike the large tensor-to-scalar ratio value ($r \sim 0.20$) under the scale-invariant tensor spectrum assumption, our interpolation approach gives $r_{0.002} < 0.060$ (95% CL) by using the first 5 bandpowers of BICEP2 data. After comparing results with and without BICEP2 data, we find that Planck temperature data with small tensor amplitude signals and BICEP2 polarization data with large tensor amplitude signals dominate the tensor spectrum reconstruction on large and small scales, respectively. Hence, the resulting blue tensor tilt actually reflects the tension between Planck and BICEP2 data.

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## Introduction

Recently, the BICEP2 experiment [?] reported an excess of Cosmic Microwave Background (CMB) B-mode polarization power spectrum over the base lensed-$\Lambda$CDM expectation in the range $30 < \ell < 150$, inconsistent with the null hypothesis at a significance of $> 5\sigma$. Since the single-field slow-roll inflationary model predicts a peak around multipole $\ell \sim 80$ in the B-mode auto-correlation (BB) spectrum seeded by primordial gravitational wave/tensor perturbation modes, the BICEP2 results are believed to be the first indirect detection of primordial gravitational waves. Under the assumption of power-law scalar and scale-invariant tensor spectra, the observed B-mode power spectrum is well-described by a lensed-$\Lambda$CDM+tensor theoretical model with tensor-to-scalar ratio $r = 0.20^{+0.07}_{-0.05}$, with $r = 0$ disfavored at $7.0\sigma$ confidence level.

However, the scientific results from BICEP2 data are in tension with those from other CMB experiments, such as Planck [?]. The first discrepancy concerns the amplitude of the scale-invariant tensor spectrum, described by the tensor-to-scalar ratio $r \equiv A_t/A_s$. Unlike scalar perturbations, due to the absence of an acoustic oscillation mechanism, tensor contributions to the temperature CMB spectrum are rapidly washed out inside the horizon at the electron-proton recombination epoch ($\ell \geq 200$) [?]. Hence, temperature anisotropies on large scales are a mixture of scalar and tensor contributions. Furthermore, if one assumes the simple power-law form of the primordial scalar power spectrum –i.e., no scalar index running $dns/d\ln k = 0$–the precisely measured higher multipoles by Planck put stringent constraints on the scalar amplitude $A_s$ and index $n_s$. Therefore, to explain the observed power deficit in the low-$\ell$ regime by Planck, one must suppress the tensor spectrum amplitude. Consequently, from only the temperature anisotropy measured by Planck, one obtains $r < 0.11$ at

95% confidence, which is in "very significant" tension (around 0.1% unlikely) with BICEP2 results [?]. As stressed by the BICEP2 team, however, this tension could be reconciled by adding the running of the scalar index, which provides the degree of freedom to suppress scalar temperature anisotropy in the low-$\ell$ regime. Then, combining Planck with WMAP low-$\ell$ polarization [?] and ACT [?]/SPT [?] high-$\ell$ data, one could obtain $r < 0.26$ at 95% confidence. Besides this, several other possible solutions to this tension have been proposed, such as step feature spectra [?], fast-slow roll models [?], anti-correlated scalar isocurvature initial conditions [?, ?], sterile neutrino species [?], and sudden changes in the speed of the inflaton or Lorentz violation [?]. See also [?] for other possibilities.

The second tension is between the observed blue-tilt tensor spectrum ($n_t > 0$) [?] by BICEP2 and the red-tilt one ($n_t = -r/8$) predicted by the standard inflationary paradigm. Generally, a blue tensor spectrum requires violation of the Null Energy Condition (NEC), which is equivalent to $\rho + P < 0$ (or $\dot{H} > 0$) in a flat universe. Several NEC-violating inflationary models exist in the literature, such as super-inflation [?], phantom inflation [?], G-inflation [?], etc. Some alternative paradigms of inflation, such as "string gas cosmology" [?], bouncing universe [?], or other possibilities [?, ?] (see references therein), might help resolve this tension.

In this paper, we adopt a purely phenomenological approach to reconstruct the shape of primordial scalar and tensor spectra from Planck temperature and BICEP2 polarization data.

## II. Parameterization of Primordial Spectra and Datasets

Since the amplitude of CMB anisotropy is tiny ($\delta T/\bar{T} \sim \mathcal{O}(10^{-5})$) and CMB observational windows cover several orders of magnitude in spatial scale, it is reasonable to parameterize the logarithms of primordial spectra—which seed the CMB anisotropy—in terms of the logarithm of fluctuation wavenumber, $\ln k$. The cubic spline interpolation method can be summarized as follows.

To reconstruct a smooth spectrum with continuous first and second derivatives, we adopt the cubic spline interpolation method, which has been used to analyze WMAP or Planck temperature and polarization data [?]. Besides cubic spline interpolation reconstruction, other model-independent algorithms exist, such as Bayesian evidence-selected linear interpolation [?, ?]. First, we uniformly sample $N_{\rm bin}$ points in the logarithmic scale of wavenumber. Second, inside the sampled bins $\ln k_i < \ln k < \ln k_{i+1}$, we use cubic spline interpolation to determine the logarithmic values of the primordial power spectrum. Third, we adopt boundary conditions where the second derivative is set to zero. For $k < k_1$ or $k > k_{N_{\rm bin}}$, we fix the slope of the primordial power spectrum at the boundaries and linearly extrapolate to the outside regimes. Mathematically, the corresponding formula can be written as:

$$\ln P(k) = \begin{cases} \frac{d\ln P(k)}{d\ln k}\Big|_{k_1} \ln\left(\frac{k}{k_1}\right) + \ln P(k_1), & k < k_1; \\ \ln P(k_i), & k \in \{k_i\}; \\ \text{cubic spline}, & k_i < k < k_{i+1}; \\ \frac{d\ln P(k)}{d\ln k}\Big|_{k_{N_{\rm bin}}} \ln\left(\frac{k}{k_{N_{\rm bin}}}\right) + \ln P(k_{N_{\rm bin}}), & k > k_{N_{\rm bin}}. \end{cases}$$

This reconstruction method has three advantages: first, it is easy to detect deviations from a scale-invariant or power-law spectrum because both are just straight lines in the $\ln k$-$\ln P$ plane. Second, negative values of the spectrum can be avoided by using $\ln P(k)$ instead of $P(k)$ for splines with steep slopes. Finally, the shape of the power spectrum reduces to the scale-invariant or power-law spectrum as special cases when $N_{\rm bin} = 1, 2$, respectively.

Since our purpose is to reconstruct both scalar and tensor spectra, we adopt different sampling logarithms based on different observational windows. For the primordial scalar curvature spectrum, constraints are mainly driven by CMB temperature modes. With Planck sensitivities, we uniformly sample 3 bins ranging in $\ln k \in (-8.517, -1.609)$, corresponding to $k \in (0.0002, 0.2)\,\mathrm{Mpc}^{-1}$. For the tensor spectrum, we adopt two uniform logarithmic sampling strategies: one corresponding to scales $k \in (0.002, 0.03)\,\mathrm{Mpc}^{-1}$, and the other to $k \in (0.002, 0.02)\,\mathrm{Mpc}^{-1}$. This is because the BICEP2 B-mode polarization data, which provide a very sensitive probe of the primordial tensor spectrum, show excess B-mode power across the full range of polarization multipoles ($20 \leq \ell \leq 340$). Moreover, compared with the first 5 bandpowers (in the range $20 \leq \ell \leq 200$), the power in the second 4 bandpowers shows extraordinary excess over the base lensed-ΛCDM expectation. This extraordinary excess might arise from exotic physics beyond the standard inflationary paradigm or from unresolved foreground contamination. Given this consideration, in this work we make two different choices of BICEP2 data: using the full 9 bandpowers, or using only the selected first 5 bandpowers. Consequently, we adjust our sampling logarithms as mentioned above.

In the remainder of this section, we briefly review the datasets used. First, we utilize the Planck TT power spectra: for low-$\ell$ modes ($2 \leq \ell < 50$) via all 9 frequency channels ranging from 30–353 GHz, and for high-$\ell$ modes ($50 \leq \ell \leq 2500$) through 100, 143, and 217 GHz frequency channels [?]. Second, to break the well-known parameter degeneracy between reionization optical depth and CMB temperature anisotropy amplitude, we also include WMAP9 low-$\ell$ temperature/polarization spectra ($2 \leq \ell \leq 32$) [?]. In addition, we use BICEP2 polarization (EE, EB, BB) spectra from 9 (or 5) bandpowers of multipoles in ($20 \leq \ell \leq 340$ or 200) at 150 GHz channels [?]. For the data analysis numerical package, we compute CMB angular power spectra using the public Einstein-Boltzmann solver CAMB [?] and explore the cosmological parameter space with a Markov Chain Monte Carlo sampler, namely CosmoMC [?].

[TABLE I]

**Table I:** List of primordial spectrum parameters used in the Monte Carlo sampling. The left parameter ranges are for chains from Planck+WP+BICEP2 data compilation, and the right ones are for those without BICEP2 data.

## III. Results and Discussions

In this section we begin with scalar spectrum reconstruction and then turn to the tensor spectrum case. The prior ranges of primordial spectrum parameters studied are listed in Table I. We emphasize that differences in the prior of tensor spectrum amplitude at our cubic spline sampling knots when BICEP2 data are included arise from the tension between Planck and BICEP2 data (as we will show later). When our MCMC sampler explores the wide parameter space spanned by $(\ln B_1, \ln B_2, \ln B_3)$, at some points the resulting spectra become inconsistent with Planck TE cross-correlation data. To avoid this problem we adjust the prior ranges, but the widths remain large enough and the tensor amplitudes $(\ln B_1, \ln B_2, \ln B_3)$ become well-constrained within these ranges, as shown in Fig. 9. We therefore conclude that our prior choices do not significantly affect the results.

### A. Scalar Spectrum Reconstruction

Since scalar spectrum reconstruction is primarily driven by CMB temperature data, we first study the case without BICEP2 polarization data, i.e., using only Planck temperature and WMAP9 low-$\ell$ polarization (WP) datasets. As mentioned in the previous section, we uniformly sample 3 points in the logarithmic scale of wavenumber, located at $k_1 = 0.0002$, $k_2 = 0.0063$, and $k_3 = 0.2\,\mathrm{Mpc}^{-1}$ with logarithmic amplitudes $\ln A_1$, $\ln A_2$, and $\ln A_3$, respectively. We then sample the parameter space spanned by vanilla ΛCDM parameters but replacing the scalar amplitude $\ln A_s$ and its tilt $n_s$ with $\ln A_1$, $\ln A_2$, and $\ln A_3$. Hereafter we call this parameter compilation $\Lambda\mathrm{CDM}{-}\ln A_s - n_s + \ln A_1 + \ln A_2 + \ln A_3$.

[Figure 1: see original paper]

The marginalized mean scalar spectrum reconstructed from Planck+WP data is represented by the black solid curve in Fig. 1, with corresponding $1 \sim 3\sigma$ error bars at the sampling points denoted by blue, red, and green segments, respectively. For comparison, we also show the primordial scalar spectrum from the Planck marginalized mean vanilla ΛCDM and ΛCDM+$dns/d\ln k$ (scalar index running) models with blue dashed and red dotted-dashed curves. From Fig. 1 we see that, first, our cubic spline interpolation result mimics the ΛCDM+$dns/d\ln k$ case; second, on the $k = 0.0002\,\mathrm{Mpc}^{-1}$ scale, the simplest vanilla model is disfavored at nearly $2\sigma$ level.

Adding BICEP2 polarization data, we show results in Fig. 2 for models including the tensor-to-scalar ratio $r$, i.e., $\Lambda\mathrm{CDM}{-}\ln A_s{-}n_s{+}r{+}\ln A_1{+}\ln A_2{+}\ln A_3$. We find that, first, due to the anti-correlation between scalar and tensor amplitudes (see the bottom left sub-panel of Fig. 8 in Appendix A), the large tensor-to-scalar ratio discovered by BICEP2 data leads to suppression of scalar amplitude on large scales. This is also explicitly demonstrated in the top sub-panel of Fig. 8 in Appendix A. As a result of this scalar power deficit on large scales, the vanilla ΛCDM model (blue curve) is strongly disfavored at $> 3\sigma$ confidence level on the $k = 0.0002\,\mathrm{Mpc}^{-1}$ scale, as shown in Fig. 2. A similar result was obtained by the authors of [**?**], who found a distinct preference for suppression of power in the scalar spectrum at large scales ($k \leq 10^{-3}\,\mathrm{Mpc}^{-1}$) via a linear spline reconstruction method using Planck and BICEP2 data. Second, assuming a scale-invariant tensor spectrum, our scalar spectrum cubic spline interpolation parameterization still yields a large tensor-to-scalar ratio $r = 0.21^{+0.10}_{-0.09}$ at 95% C.L., as shown in Table II in Appendix A.

### B. Tensor Spectrum Reconstruction

In the previous subsection we assumed a scale-invariant tensor spectrum. Here we relax this assumption and use the same cubic spline interpolation method to reconstruct the shape of the tensor spectrum. Unlike the CMB temperature spectrum, which is mainly sourced by primordial scalar perturbations, the B-mode polarization anisotropy seeded by tensor perturbations is only detected by BICEP2 on large scales with polarization spherical harmonic multipoles ranging $20 \leq \ell \leq 340$ (9 bandpowers). As with the scalar spectrum, we uniformly sample the tensor spectrum with 3 points in the logarithmic scale of wavenumber, but only on scales covered by BICEP2 observations. Consequently, we sample them at $k_1 = 0.002$, $k_2 = 0.0077$, and $k_3 = 0.03\,\mathrm{Mpc}^{-1}$, respectively. The cosmological parameters we estimate are the 6 vanilla ΛCDM model parameters plus the 3 extra tensor amplitudes $\ln B_1$, $\ln B_2$, and $\ln B_3$.

[Figure 3: see original paper]

In Fig. 3 we plot the primordial tensor spectra from the best-fit model of $\Lambda\mathrm{CDM}+\ln B_1+\ln B_2+\ln B_3$ (black solid curve) and $\Lambda\mathrm{CDM}+r+n_t$ (red dotted-dashed curve), along with error bars at the sampling points of the cubic spline interpolation method. First, both the standard power-law and our cubic spline parameterizations favor a blue-tilt tensor spectrum. The former (see Table III in Appendix B) reports $r_{0.002} < 0.061$ (95% CL, 9 bandpowers), $n_t = 1.24^{+0.51}_{-0.58}$ (95% CL, 9 bandpowers), while our cubic spline interpolation method gives $r_{0.002} < 0.064$ (95% CL, 9 bandpowers). Second, we notice that in our cubic spline interpolation method the slope of the tensor spectrum becomes larger in the low-$k$ regime, but remains consistent with the power-law parameterization at the $2\sigma$ confidence level.

As argued in the previous section, there exists extraordinary power excess in the higher wavenumber regimes of BICEP2 data. Given this consideration, we subsequently adopt only the selected first 5 band-power data of BICEP2 for our reconstruction. Based on the multipole ranges covered by these power bands ($20 \leq \ell \leq 200$), we sample points at $k_1 = 0.002$, $k_2 = 0.0063$, and $k_3 = 0.02\,\mathrm{Mpc}^{-1}$, respectively.

[Figure 4: see original paper]

The reconstructed tensor spectra and error bars are shown in Fig. 4. First, from the power-law parameterization we see that blue tensor spectra remain favored but with a smaller tilt: $r_{0.002} < 0.067$ (95% CL, 5 bandpowers), $n_t = 1.20^{+0.56}_{-0.64}$ (95% CL, 5 bandpowers). Second, with only the first 5 bandpower data, unlike the simplest power-law parameterization, a non-trivial shape of tensor spectrum is obtained. Specifically, in the range $k \in (0.002, 0.006)$, BICEP2 data favor a large tensor blue-tilt, while for $k > 0.0063\,\mathrm{Mpc}^{-1}$ the spectrum becomes nearly flat. This is due to our exclusion of the last 4 bandpower data. The resulting tensor-to-scalar ratio from our cubic spline interpolation method is $r_{0.002} < 0.060$ (95% CL, 5 bandpowers).

[Figure 5: see original paper]

Because the above blue tensor tilt is significantly inconsistent with standard inflationary predictions, we must determine its origin at the data analysis level. Note that we have thus far always used the Planck+WP+BICEP2 data compilation, so one natural guess is that this blue tensor tilt might reflect tension among the Planck, WMAP polarization, and BICEP2 datasets. To test this conjecture, we remove datasets one by one. Since we vary both primordial spectrum and standard ΛCDM parameters such as baryon density ($\Omega_b h^2$) and cold dark matter density ($\Omega_c h^2$), we must keep the robust Planck temperature data to obtain well-constrained ΛCDM parameters. Therefore, we first remove BICEP2 (i.e., using Planck+WP), and further discard WMAP polarization data (i.e., using only Planck temperature data). However, because CMB temperature data are insensitive to the reionization optical depth ($\tau$), discarding WMAP polarization data requires including a Gaussian prior on $\tau$ to break the well-known degeneracy between $\tau$ and scalar amplitude $A_s$. Here we adopt the Gaussian prior $\tau = 0.089 \pm 0.013$.

Additionally, because tensor spectrum contributions to CMB temperature anisotropies are only significant in the multipole range $2 \leq \ell \leq 100$, when using Planck+WP or Planck+$\tau$ prior datasets, we sample the $k$ knots of tensor spectra in the range $(0.0002, 0.01)$. The resulting primordial tensor spectrum shape, corresponding marginalized 1D/2D posterior distributions, and parameter constraints are shown in Fig. 5, Fig. 10, and Table V. First, we see that reconstructed tensor spectra from Planck+WP and Planck+$\tau$ prior are very similar in both central values and marginalized error bars. We conclude that WMAP low-$\ell$ polarization data are not crucial for tensor reconstruction results. Second, as shown in Fig. 6, error bars from Planck temperature data alone are quite large compared with those from the Planck+WP+BICEP2 compilation (compare error bars at sampling knot $k = 0.01\,\mathrm{Mpc}^{-1}$ in the dashed black curve with those at the second knot in the black solid curve in Fig. 6). This means current Planck temperature data are not robust enough to determine the shape of the primordial tensor spectrum; the resulting tensor spectrum from Planck temperature data alone could be red, blue, or scale-invariant. Third, comparing the second knot in Planck+WP (dashed black curve) and the leftmost knot in Planck+WP+BICEP2 results (solid black curve), Fig. 6 shows that both central values and marginalized error bars are very close. This reflects that the reconstructed tensor spectrum in the low-$k$ regime is actually driven by Planck temperature data. Furthermore, considering that BICEP2 data dominate the high-$k$ part, we conclude that our reconstructed blue tensor tilt arises from tension between Planck and BICEP2 datasets: small tensor amplitude signals from Planck temperature data dominate large-scale reconstruction, while large tensor amplitude signals from BICEP2 B-mode polarization data dominate small-scale reconstruction, leading to the resulting blue tensor tilt.

[Figure 6: see original paper]

Finally, to provide an intuitive impression of our reconstruction result, in Fig. 7 we plot the BB auto-correlation power spectrum of our marginalized mean models (listed in Table IV of Appendix B) along with the scale-invariant tensor spectrum with $r = 0.2$ model against the BICEP2 bandpower datasets. We see that to fit the BICEP2 data in the last 4 bandpowers, compared with the 5-bandpower reconstruction result (green curve) and the scale-invariant case (blue), the 9-bandpower curve (red) grows significantly in the high-$\ell$ regime. This requires more careful cross-check with future experiments, such as Planck polarization data and the Keck Array. On the other hand, once this discovery is confirmed, it will lead to a paradigm revolution in our understanding of the early universe.

[Figure 7: see original paper]

## IV. Conclusions

Adopting a purely phenomenological approach, in this paper we have reconstructed the shape of primordial scalar and tensor spectra using the cubic spline interpolation method with Planck temperature and BICEP2 B-mode polarization datasets. We find that, due to anti-correlation between scalar and tensor amplitudes on large scales, the large tensor-to-scalar ratio discovered by BICEP2 leads to suppression of scalar amplitude in this regime. Specifically, the vanishing scalar index running model is strongly disfavored by the Planck+WP+BICEP2 data compilation at more than $3\sigma$ confidence level on the $k = 0.0002\,\mathrm{Mpc}^{-1}$ scale. Furthermore, for tensor spectrum reconstruction, a blue-tilt spectrum is obtained whether using only the first 5 bandpowers ($n_t = 1.20^{+0.56}_{-0.64}$ at 95% CL) or the full 9 bandpowers ($n_t = 1.24^{+0.51}_{-0.58}$ at 95% CL) of BICEP2 datasets. Because of the large tensor tilt, compared with the large tensor-to-scalar ratio value ($r \sim 0.20$) under the scale-invariant assumption, our cubic spline interpolation method gives $r_{0.002} < 0.060$ (95% CL) and $r_{0.002} < 0.064$ (95% CL) using the datasets Planck+WP+BICEP2 (5 bandpowers) and (9 bandpowers), respectively. Finally, we also studied results without BICEP2 and found that our resulting blue tensor tilt actually reflects the tension in tensor amplitude between Planck (small amplitude but dominates reconstruction on large scales) and BICEP2 (large amplitude but dominates reconstruction on small scales) datasets.

Our results show that the conclusion of a blue-tilt tensor spectrum is very significant and independent of whether power-law or cubic spline parameterizations are used. More importantly, this blue-tilt spectrum is inconsistent with the prediction of standard single-field inflationary paradigm $n_t = -r/8$. On one hand, this requires more careful cross-check with future experiments such as Planck polarization data and the Keck Array. On the other hand, once this discovery is confirmed, it will lead to a paradigm revolution in our understanding of the early universe.

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## Appendix A: Marginalized Statistics in Scalar Spectrum Reconstruction

Here we list various marginalized statistical results for cubic spline interpolation and power-law parameterizations of the scalar spectrum, including 1D and 2D marginalized posterior distributions, marginalized mean values, and 68% (or 95%) confidence levels.

[Figure 8: see original paper]

**Table II:** Mean values and 68% (or 95%) confidence limits for primary/derived parameters in the cubic spline and power-law parameterization of the scalar spectrum.

## Appendix B: Marginalized Statistics in Tensor Spectrum Reconstruction

Here we list various marginalized statistical results for cubic spline interpolation and power-law parameterizations of the tensor spectrum, including 1D and 2D marginalized posterior distributions, marginalized mean values, and 68% (or 95%) confidence levels.

[Figure 9: see original paper]

**Table III:** Mean values and 68% (or 95%) confidence limits for primary/derived parameters in the power-law parameterization of the tensor spectrum.

**Table IV:** Mean values and 68% (or 95%) confidence limits for primary/derived parameters in the tensor spectrum cubic spline reconstruction.

[Figure 10: see original paper]

**Table V:** Mean values and 68% (or 95%) confidence limits for primary/derived parameters in the tensor spectrum cubic spline reconstruction.

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