Reheating phase diagram for single- eld slow-roll in ationary models postprint

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Date: 2017-09-27T00:00:00+00:00

Abstract

We investigate the influence on the inflationary predictions from the reheating processes characterized by the e-folding number Nreh and the effective equation-of-state parameter wreh during the reheating phase. For the first time, reheating processes can be constrained in the Nreh-wreh plane from Planck 2015. We find that for Higgs inflation with a non-minimal coupling to gravity, the predictions are insensitive to the reheating phase for current CMB measurements. We also find that the spontaneously broken SUSY inflation and axion monodromy inflation with p=2/3 potential, which with instantaneous reheating lie outside or at the edge of the 95% confidence region in the ns-r plane from Planck 2015 TT,TE,EE+lowP, can well fit the data with the help of reheating processes. Future CMB experiments would put strong constraints on reheating processes.

Full Text

Preamble

Reheating Phase Diagram for Single-Field Slow-Roll Inflationary Models

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(Dated: September 10, 2015)

We investigate the influence of reheating processes on inflationary predictions, characterized by the e-folding number $N_{\rm reh}$ and the effective equation-of-state parameter $w_{\rm reh}$ during the reheating phase. For the first time, reheating processes can be constrained in the $N_{\rm reh}$-$w_{\rm reh}$ plane from Planck 2015. We find that for Higgs inflation with nonminimal coupling to gravity, the predictions are insensitive to the reheating phase for current CMB measurements. We also find that spontaneously broken SUSY inflation and axion monodromy inflation with

$\phi^{2/3}$ potential, which lie outside or at the edge of the 95% confidence region in the n_s-r plane from Planck 2015 TT,TE,EE+lowP when assuming instantaneous reheating, can fit the data well when reheating processes are properly taken into account. Future CMB experiments will place strong constraints on reheating processes.

Introduction

Single-field slow-roll inflationary models with a standard kinetic term provide an excellent fit to Planck data [?, ?]. In the inflationary scenario, the Universe expands quasi-exponentially as the scalar field rolls slowly along a very flat potential. After inflation ends, the inflaton field oscillates around the minimum of the potential. During this reheating period, the energy in the inflaton is transferred to a plasma of Standard Model particles. Reheating is an integral part of inflationary models; without it, the Universe after inflation would be empty and cold. However, the physics of reheating may be more complicated. As pointed out in Ref. [?], the equation-of-state parameter changes sharply during the reheating phase due to the out-of-equilibrium nonlinear dynamics of fields.

Predictions of slow-roll inflationary models are typically derived assuming a reasonable range for the e-folding number $N_{\rm inf}$ during inflation. With increasingly precise CMB measurements [?, ?], it becomes important to consider the impact of reheating processes on inflationary predictions [?, ?, ?, ?, ?, ?, ?, ?, ?]. Reheating processes provide additional constraints on inflationary models via the reheating temperature [?], and specific inflationary models have been investigated in Refs. [?, ?, ?, ?, ?].

Although the physics of reheating is highly uncertain and unconstrained due to its nonlinear backreaction and nonperturbative nature, the reheating phase can in principle be characterized by only two parameters: the e-folding number $N_{\rm reh}$ and the effective equation-of-state (EoS) parameter $w_{\rm reh}$. In terms of these two parameters, one can express observable quantities such as the scalar spectral index n_s and its running α_s, the tensor spectral index n_t and its running α_t, the tensor-to-scalar ratio r, the e-folding number $N_{\rm inf}$, and the reheating temperature $T_{\rm reh}$. Thus, reheating processes can be constrained in the $N_{\rm reh}$-$w_{\rm reh}$ plane by Planck constraints on n_s and r.

Following the approach proposed in Refs. [?, ?, ?], we study several single-field slow-roll inflationary models including Higgs inflation, power-law potentials, hilltop inflation, natural inflation, spontaneously broken (SB) SUSY inflation, and superconformal α-attractors. Although reheating/preheating mechanisms are well studied in Higgs inflation [?, ?, ?], the underlying mechanisms of unitarization and stabilization might substantially impact the reheating phase [?]. For Higgs inflation, we find that inflationary predictions are insensitive to reheating processes given current CMB measurement precision, a conclusion also reached in Ref. [?]. We find that SB SUSY inflation and power-law potential

$\phi^{2/3}$, which lie outside or at the edge of the 95% confidence region in the n_s-r plane from Planck 2015 TT,TE,EE+lowP, can fit the data well when reheating processes are taken into account. However, the constrained parameter space of reheating processes remains very large for most inflationary models due to current relatively weak constraints on inflation; future measurements of n_s and r will eventually narrow down this parameter space, thus revealing the physics of reheating.

The paper is organized as follows: In Sec. II we introduce the effective description of the reheating phase in terms of $N_{\rm reh}$ and $w_{\rm reh}$ for single-field slow-roll inflationary models. In Sec. III we study specific inflationary models. Section IV presents our conclusions.

II. Descriptions of the Reheating Phase

Following the method proposed in Refs. [?, ?, ?], we first derive a formula for the effective number of degrees of freedom $g_{\rm reh}$ at the end of the reheating phase. It is worth noting that one should not take this literally, since most observables are insensitive to the precise value of $g_{\rm reh}$ due to its logarithmic dependence. However, the derived formula for $g_{\rm reh}$ is essential for carrying out inflationary predictions in the $N_{\rm reh}$-$w_{\rm reh}$ plane, which can be used to constrain the parameter space of the reheating phase to satisfy current constraints on inflation.

The pivot scale is chosen as $k_* = 0.05\,\mathrm{Mpc}^{-1}$, expressed by

$$k_* = a_* H_* = a_0 H_*.$$

In what follows, the current scale factor $a_0 = 1$ and all quantities with subscript "$*$" are evaluated when the pivot scale crosses the horizon.

The first two factors of (1) can be computed by

$$\frac{a_{\rm reh}}{a_0} = e^{-(N_* + N_{\rm reh})}.$$

The third factor of (1) is computed as follows. Using the conservation equation of entropy $g_{\rm reh} a_{\rm reh}^3 T_{\rm reh}^3 = g_{S,0} T_0^3$ with $g_{S,0} = 2 + (7/8) \times 3 \times 2 \times (4/11) = 43/11$, and noting that $g_\gamma = 2$, $g_\nu = (7/8) \times 3 \times 2 = 21/4$, and $T_\nu = (4/11) T_\gamma$, we adopt $T_\gamma = 2.7255\,\mathrm{K}$. Thus, the third factor can be written as

$$\frac{T_{\rm reh}}{T_0} = \left(\frac{43}{11 g_{\rm reh}} \right)^{1/3} e^{N_{\rm reh}}.$$

Next, we apply this formalism to a general single-field slow-roll inflationary potential $V(\phi, p)$ with only one parameter p (for multiparameter potentials, one must fix some parameters). Inflation ends when the slow-roll condition is broken:

$$\epsilon(\phi_{\rm end}, p) = \epsilon_{\rm end} \Rightarrow \phi_{\rm end}(p).$$

Once we have the field value $\phi_{\rm end}(p)$ at the end of inflation, we obtain the potential energy density at that moment:

$$V(\phi_{\rm end}(p), p) \equiv V_{\rm end}(p).$$

To compute H_* via the slow-roll equation $3H_*^2 = V_*$, we require the field value ϕ_* when the pivot scale crosses the horizon. This can only be determined by inputting Planck observations on the scalar power spectrum amplitude via

$$A_s = \frac{H_*^2}{8\pi^2\epsilon_*}.$$

However, this requires full knowledge of the tensor-to-scalar ratio r_* and the consistency relation $r_* = 16\epsilon_*$, both of which have not been observed or confirmed yet. A more conservative approach to compute H_* is to use the slow-roll equation $3H_*^2 = V_*$.

Combining the relevant equations gives the final formula for the effective number of degrees of freedom at the end of the reheating phase:

$$g_{\rm reh}(p, N_{\rm reh}, w_{\rm reh}) = \left(\frac{T_\gamma}{T_0}\right)^{12} \left(\frac{43}{11}\right)^4 \left(\frac{3-\epsilon_{\rm end}}{\pi^2 V_{\rm end}(p)}\right)^3 V_*^6(p) \times \exp\left[9N_{\rm reh}w_{\rm reh} - 12N_*(p)\right].$$

Three comments on this result follow:

1. The general formalism presented above is insensitive to the precise values of $\epsilon_{\rm end}$ and $g_{\rm reh}$ due to logarithmic dependence. Therefore, it suffices to take fiducial values $\epsilon_{\rm end} = 1$, $g_{\rm reh} = 106.75$ for Higgs inflation and $g_{\rm reh} = 10^3$ for other single-field slow-roll inflationary models.
2. The case $w_{\rm reh} = 1/3$ should be seen as equivalent to $N_{\rm reh} = 0$, representing instantaneous reheating. As we will see in the next section, an instantaneous reheating process manifests as an asymptotic line in the $N_{\rm reh}$-$w_{\rm reh}$ plane.
3. The degeneracy between phases is the source of freedom in choosing N_* in the n_s-r plane. This can be seen from the fact that any shift ΔN_* from N_* can be compensated by shifts $\Delta N_{\rm reh}$ and $\Delta w_{\rm reh}$ from $N_{\rm reh}$ and $w_{\rm reh}$, provided that

$$9N_{\text{reh}}\Delta w_{\text{reh}} + 9\Delta N_{\text{reh}}\left(w_{\text{reh}} + \Delta w_{\text{reh}} - \frac{1}{3}\right) = 12\Delta N_*.$$

III. Reheating Phase Diagram

We begin with the usual n_s-r plane. By solving the equations in terms of reheating phase variables N_{reh} and w_{reh}, we can construct what we refer to as reheating phase diagrams in the N_{reh}-w_{reh} plane. These diagrams are insensitive to different input values of g_{reh}.

What priors should we choose for N_{reh} and w_{reh} in general? First, the inflation era ends when the EoS parameter equals $-1/3$, and the radiation era begins when the EoS parameter equals $1/3$. It might seem that w_{reh} should be in the range $[-1/3, 1/3]$. However, potential dominance (with EoS parameter -1) and kinetic dominance (with EoS parameter 1) are possible, assuming a massive inflaton.

Second, in the n_s-r plane, inflationary predictions are typically made by choosing N_{inf} in the range $[50, 60]$, or more generally $[40, 70]$, which is degenerate with N_{reh} and w_{reh}. As mentioned in the previous section, any shift ΔN_* from N_* can be compensated by shifts ΔN_{reh} and Δw_{reh}, provided that the degeneracy condition holds.

Assuming a maximum e-folding number during inflation $N_* = 70$ which can be shifted by $\Delta N_* = -30$, and a maximum EoS parameter during reheating $w_{\text{reh}} = 1$ which can be shifted by $\Delta w_{\text{reh}} = -2$, one can easily work out the minimum e-folding number during reheating $N_{\text{reh}} = 0$, which can be shifted by

$$\Delta N_{\text{reh}} = \frac{12\Delta N_* - 9N_{\text{reh}}\Delta w_{\text{reh}}}{9\left(w_{\text{reh}} + \Delta w_{\text{reh}} - \frac{1}{3}\right)} = 30.$$

Therefore, without prior knowledge of the reheating phase, one can generally choose N_{reh} in the prior $[0, 30]$ and w_{reh} in the prior $[-1, 1]$, which covers N_* in the range $[40, 70]$. A constant EoS parameter w_{reh} should be viewed as an effective parameter time-averaging the EoS parameter during the entire reheating process.

A. Reheating Phase Diagram for Higgs Inflation

In Higgs inflation, the Higgs field with a large nonminimal coupling to Einstein gravity in the Jordan frame gives rise to an exponential plateau-like potential in the large field region of the Einstein frame where the inflaton is defined. The action in the Jordan frame is

$$S_J = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}\Omega^2 R - \frac{1}{2}(\partial h)^2 - V(h)\right],$$

where $V(h) = (\lambda/4)(h^2 - v^2)^2$ with the electroweak vacuum expectation value $v = 246\,\text{GeV}$. The conformal factor $\Omega^2 = 1 + \xi h^2/M_P^2$ and the scalar field redefinition allow us to transform the action into the Einstein frame

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_P^2}{2}\tilde{R} - \frac{1}{2}(\tilde{\partial}\chi)^2 - U(\chi)\right],$$

where the kinetic terms for Einstein gravity and the new scalar field are both canonically normalized. The potential term

$$U(\chi(h)) = \frac{V(h)}{\Omega^4} = \frac{\lambda M_P^4}{4\xi^2}\left(1 - \frac{\xi v^2}{M_P^2 + \xi h^2}\right)^2$$

can be abbreviated as $U(\chi(\phi)) = \Lambda^4(1+\phi^2)^2 \equiv V(\phi)$ using the dimensionless scalar field $\phi = \xi h/M_P$ and the combined parameter $Z = \lambda/\xi^2$ for later convenience. Here we ignore v and set $M_P = 1$ from now on.

In the large field region $h \gg M_P/\sqrt{\xi}$, one can solve the scalar field redefinition to obtain $\chi \simeq \sqrt{6}M_P \ln\phi$; thus an exponential plateau-like potential in the large field region $U(\chi) = \Lambda^4(1 + e^{-2\chi/\sqrt{6}M_P})^{-2}$ is obtained as promised. The slow-roll dynamics with respect to the inflaton χ can be carried out directly by computing the slow-roll parameters

$$\epsilon(\phi) = \eta(\phi) = \frac{4\phi^2}{(\phi^2+1)^2}, \quad \zeta_2(\phi) = \frac{8\phi^2(\phi^2-1)}{(\phi^2+1)^3} + \frac{16\phi^4}{(\phi^2+1)^4},$$

the e-folding number during inflation

$$N(\phi_N) = \frac{\phi_N^2 - \phi_{\text{end}}^2}{4} + \ln\frac{1+\phi_{\text{end}}^2}{1+\phi_N^2},$$

and the scalar spectral index, its running, and the tensor-to-scalar ratio

$$n_s(\phi_N) = 1 - \frac{8\phi_N^2}{(\phi_N^2+1)^2} - \frac{12}{(\phi_N^2+1)^2} + \frac{8}{(\phi_N^2+1)^3},$$

$$r(\phi_N) = \frac{16\phi_N^2}{(\phi_N^2+1)^2}, \quad \alpha_s(\phi_N) = -\frac{32\phi_N^2}{(\phi_N^2+1)^3} - \frac{48}{(\phi_N^2+1)^3} + \frac{48}{(\phi_N^2+1)^4}.$$

The reheating phase diagrams for Higgs inflation showing n_s, r, α_s, Z, T_{reh}, and N_{inf} are presented simultaneously in Fig. 1 [Figure 1: see original paper] with input values $g_{\text{reh}} = 106.75$ and $\ln(10^{10}A_s) = 3.094 \pm 0.034$ from Planck 2015 normalization [?]. The dashed contour lines in the last panel reflect different input values of $\ln(10^{10}A_s) = 3.094 \pm 0.034$ from Planck 2015 normalization. However, the dashed contour lines in other panels are too close to the solid lines to distinguish any difference. Therefore, it suffices to take the mean value for $\ln(10^{10}A_s)$ when considering other inflationary models.

As seen in the first panel of Fig. 1, almost all possible reheating processes are allowed within the 1σ region $n_s = 0.9645 \pm 0.0049$ reported by Planck 2015 TT,TE,EE+lowP [?]. This insensitivity of cosmological predictions to the reheating phase is also evident in other panels. Therefore, the cosmological predictions of Higgs inflation (including the reheating temperature, as shown in Ref. [?] in the n_s-r plane) are insensitive to its reheating processes given current CMB measurement precision. However, reheating processes should be considered for future measurements of n_s with refined precision up to 1% and for direct detection of primordial gravitational waves.

B. Reheating Phase Diagrams for Other Models

Reheating phase diagrams can also be constructed for other single-field slow-roll inflationary models. Among those selected by the Planck Collaboration [?], we study power-law potentials, hilltop inflation, natural inflation, SB SUSY inflation, and superconformal α-attractors. Since the models closest to Higgs inflation (equivalent to R^2 inflation to lowest order in slow-roll approximation at tree level) in terms of Bayes evidence are brane inflation and exponential inflation, we expect these two models would yield essentially the same results as Higgs inflation. From now on, we adopt the fiducial value $g_{\rm reh} = 10^3$ and the mean value $\ln(10^{10}A_s) = 3.092$ from Planck 2015 TT,TE,EE+lowP for ΛCDM+r, for which $n_s = 0.9652^{+0.0093}_{-0.0091}$ and $r < 0.106$ with 95% limits at pivot scale $k_* = 0.05\,\mathrm{Mpc}^{-1}$ will be used to constrain the reheating phase diagrams. As an aside, reheating phase diagrams with respect to the e-folding number $N_{\rm inf}$ during inflation are also presented for all models.

1. Power-law potential Inflation models with power-law potential [?] motivated by axion monodromy [?, ?] take values such as $p = 4/3, 1, 2/3$ for $V(\phi) = \Lambda^4\phi^p$. The slow-roll approximations give

$$\epsilon(\phi) = \frac{p^2}{2\phi^2}, \quad \eta(\phi) = \frac{p(p-1)}{\phi^2}, \quad \phi_{\rm end} = \frac{p}{\sqrt{2}},$$

$$N = \frac{\phi_N^2 - \phi_{\rm end}^2}{2p},$$

which are necessary to construct their reheating phase diagrams in Fig. 2 [Figure 2: see original paper]. Requiring $0.9561 < n_s < 0.9745$ and $r < 0.106$ according to current constraints from Planck 2015 TT,TE,EE+lowP [?], one finds the parameter spaces of the reheating phase specified by the green regions in the $N_{\rm reh}$-$w_{\rm reh}$ plane. An interesting observation is that inflation models with larger green areas have larger Bayes factors, as shown in Table 6 of Ref. [?]. Notably, axion monodromy inflation with $\phi^{2/3}$ potential, which lies at the edge of the 95% confidence region in the n_s-r plane constrained by Planck 2015 TT,TE,EE+lowP, can actually satisfy current constraints when reheating processes [?] are taken into account. We do not present reheating phase diagrams for power-law potentials with $p \geq 2$, as their reheating phase diagrams have no green region at all (the regions allowed for $0.9561 < n_s < 0.9745$ and $r < 0.106$ have no intersection).

2. Hilltop inflation Hilltop inflation [?] with potential

$$V(\phi) = \Lambda^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p + \cdots\right]$$

has slow-roll parameters

$$\epsilon_1(\phi) = \frac{p^2}{2\mu^2}\left(\frac{\phi}{\mu}\right)^{2p-2}\left(1 - \left(\frac{\phi}{\mu}\right)^p\right)^2,$$

$$\epsilon_2(\phi) = \frac{p - 1 + (p/2)(\phi/\mu)^p}{\mu^2(1-(\phi/\mu)^p)^2}\left(\frac{\phi}{\mu}\right)^{p-2},$$

with $\phi_{\mathrm{end}} = \mu - 1/\mu$ and

$$N = \frac{\mu^2}{2p}\left[\left(\frac{\phi_N}{\mu}\right)^2 - \left(\frac{\phi_{\mathrm{end}}}{\mu}\right)^2\right] \quad \text{for } p \neq 2,$$

$$N = \frac{\mu^2}{4}\left[\left(\frac{\phi_N}{\mu}\right)^2 - \left(\frac{\phi_{\mathrm{end}}}{\mu}\right)^2 - 2\ln\frac{\phi_N/\mu}{\phi_{\mathrm{end}}/\mu}\right] \quad \text{for } p = 2.$$

Planck 2015 favors hilltop inflation with $\log_{10}\mu > 1.02(1.05)$ for $p = 2$, $w_{\mathrm{reh}} = 0$ (allowing w_{reh} to vary) and $\log_{10}\mu > 1.05(1.02)$ for $p = 4$, $w_{\mathrm{reh}} = 0$ (allowing w_{reh} to vary). However, the reheating phase diagrams for hilltop inflation presented in Fig. 3 [Figure 3: see original paper] slightly loosen the bound on $\log_{10}\mu$. Each colored region is specified by requiring $0.9561 < n_s < 0.9745$ and $r < 0.106$ for different values of parameter μ. Larger values of μ cover larger portions of parameter space in the N_{reh}-w_{reh} plane, while lower values of μ would require more exotic reheating processes beyond theoretically reasonable ones with $N_{\mathrm{reh}} \sim \mathcal{O}(1)$ and $w_{\mathrm{reh}} \in [-1/3, 1/3]$.

3. Natural inflation Natural inflation [?, ?] with periodic potential is expressed by

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right].$$

As far as n_s and r are concerned, natural inflation recovers quadratic chaotic inflation when the curvature scale of the potential $f \to \infty$. The slow-roll approximations give

$$\epsilon_1(\phi) = \frac{\sin^2(\phi/f)}{2f^2(1+\cos(\phi/f))^2}, \quad \epsilon_2(\phi) = \frac{\cos(\phi/f)}{f^2(1+\cos(\phi/f))},$$

with $\phi_{\rm end} = f \arccos\left[(1-2f^2)/(1+2f^2)\right]$ and

$$N = f^2 \ln\left[\frac{1-\cos(\phi_{\rm end}/f)}{1-\cos(\phi_N/f)}\right].$$

Planck 2015 favors natural inflation with $\log_{10} f > 0.84(0.83)$ for $w_{\rm reh} = 0$ (allowing $w_{\rm reh}$ to vary). As in hilltop inflation, the reheating phase diagram presented in Fig. 4 [Figure 4: see original paper] slightly loosens the bound on $\log_{10} f$, and smaller or larger values of f would require more exotic reheating processes. However, since there are currently no observational constraints on reheating phase variables, one cannot simply rule out these parameter spaces.

4. Spontaneously broken SUSY Spontaneously broken SUSY inflation [?] is described by the potential

$$V(\phi) = \Lambda^4 \left(1 + \alpha_h \log\frac{\phi}{\mu}\right),$$

considered here in the small field limit $\phi \ll \mu$ with super-Planckian VEV $\mu \gg 1$ and a flat prior $[-2.5, 1]$ for $\log_{10}\alpha_h$. The slow-roll approximations give

$$\epsilon(\phi) = \frac{\alpha_h^2}{2\phi^2(1+\alpha_h\log\phi)^2}, \quad \eta(\phi) = -\frac{\alpha_h}{\phi^2(1+\alpha_h\log\phi)},$$

with $\phi_{\rm end} = \sqrt{2/W(2\exp[2/\alpha_h])}$ and

$$N = \frac{1}{2\alpha_h}\left[\phi_N^2\log\phi_N^2 - \phi_{\rm end}^2\log\phi_{\rm end}^2 - (\phi_N^2 - \phi_{\rm end}^2)\right],$$

where $W(z)$ is the Lambert function defined by $z = W(z)\exp[W(z)]$. The reheating phase diagram for SB SUSY inflation is presented in Fig. 5 [Figure

5: see original paper]. As in natural inflation, smaller values of α_h are allowed if more exotic reheating processes are invoked. Although SB SUSY inflation lies outside the 95% confidence region in the n_s-r plane constrained by Planck 2015 TT,TE,EE+lowP, there exist parameter spaces in the $N_{\rm reh}$-$w_{\rm reh}$ plane that accommodate Planck constraints on inflation. Therefore, SB SUSY inflation cannot be simply ruled out when the reheating phase is taken into account.

5. α-attractors α-attractor E-models [**?**] with exponentially flat potential

$$V(\phi) = \Lambda^4 \left(1 - e^{-\sqrt{2/3\alpha}\,\phi}\right)^2$$

approach the predictions of quadratic inflation for $\alpha \to \infty$, the Starobinsky model ($n_s = 1 - 2/N$, $r = 12/N^2$) for $\alpha = 1$, and α-attractors ($n_s = 1 - 2/N$, $r = 0$) for $\alpha \to 0$. Planck 2015 favors α-attractor E-models with $\log_{10}\alpha^2 < 1.7(2.0)$ for $w_{\rm reh} = 0$ (allowing $w_{\rm reh}$ to vary). However, the reheating phase diagram presented in Fig. 6 [Figure 6: see original paper] slightly loosens the bound on α as expected.

α-attractor T-models [**?**] with potential

$$V(\phi) = \Lambda^4 \tanh^{2m}\left(\frac{\phi}{\sqrt{6\alpha}}\right)$$

approach the predictions of power-law potential ϕ^{2m} for $\alpha \to \infty$ and α-attractors ($n_s = 1 - 2/N$, $r = 0$) for $\alpha \to 0$. Planck 2015 favors α-attractor T-models with $\log_{10}\alpha^2 < 2.3(2.5)$ for $m = 1$, $w_{\rm reh} = 0$ (allowing $w_{\rm reh}$ to vary) and $0.2 < m < 1$ ($m < 1$) for $m \neq 1$, $w_{\rm reh} = 0$ (allowing $w_{\rm reh}$ to vary). The reheating phase diagram slightly loosens the bound on α as expected.

IV. Conclusions

In the n_s-r plane, uncertainties from the reheating phase are usually characterized by the freedom in choosing the e-folding number of inflation. In this paper, we characterize the reheating phase by only two effective parameters, $N_{\rm reh}$ and $w_{\rm reh}$. Thanks to the fact that observable quantities are insensitive to the effective number of degrees of freedom at the end of reheating, we can express all other inflationary observables in terms of these phase variables. Therefore, for the first time we are able to constrain the parameter space of the reheating phase in the $N_{\rm reh}$-$w_{\rm reh}$ plane using constraints on inflation from Planck 2015.

For Higgs inflation, the parameter space of the reheating phase covers almost the entire $N_{\rm reh}$-$w_{\rm reh}$ plane, indicating that inflationary predictions are insensitive to reheating processes given current CMB measurement precision. However, future refined measurements of the scalar spectral index and direct detection of primordial gravitational waves will constrain the reheating phase variables. For other inflationary models selected by the Planck Collaboration, the constrained parameter spaces of the reheating phase generally loosen the bounds on potential parameters if more exotic reheating processes are allowed. Inflationary models with larger parameter spaces in the reheating phase diagrams generally have larger Bayes factors. Since there are currently only theoretical considerations, not observational constraints, on possible reheating phases, one cannot simply rule out those parameter spaces with exotic reheating processes even if they lie outside the 95% confidence region in the n_s-r plane constrained by Planck 2015 TT,TE,EE+lowP. Only those inflationary models with no allowed parameter space in the $N_{\rm reh}$-$w_{\rm reh}$ plane can be definitively ruled out.

It should be noted that in warm inflation scenarios [**?**, **?**], a separate reheating phase is not necessary [**?**]. Unlike standard scenarios where the inflaton must be coupled to other degrees of freedom to transfer its vacuum energy to reheat the Universe until radiation finally takes over, the inflaton could slowly dissipate its kinetic energy into radiation, allowing the relative abundance of radiation to increase during inflation until it smoothly dominates. When these dissipative effects are considered, one can accommodate, for example, the $\lambda\phi^4$ model with Planck data for a nearly thermalized state in a supersymmetric realization of warm inflation with renormalizable interactions [**?**].

Acknowledgments

We thank Peng-Xu Jiang and Jian-Wei Hu for helpful discussions. R.G.C. is supported by the Strategic Priority Research Program of the Chinese Academy of Sciences, Grant No. XDB09000000. Z.K.G. is supported by the National Natural Science Foundation of China, Grants No. 11175225 and No. 11335012.

Note added: Reference [**?**] recently appeared on arXiv. Both works follow the same method to study a similar problem for Higgs inflation but from different angles. We characterize the reheating phase with $N_{\rm reh}$ and $w_{\rm reh}$ and express every observable in terms of these two phase variables. The impact of various reheating processes on inflationary predictions can be shown with respect to not only the reheating temperature but also other cosmological observables. We further study other inflationary models selected by the Planck Collaboration and find that reheating phase diagrams can constrain the parameter space of the reheating phase to satisfy current constraints on inflation.

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Note: Figure translations are in progress. See original paper for figures.