Higgs in ation in Gauss-Bonnet braneworld postprint

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Date: 2017-09-27T00:00:00+00:00

Abstract

The measured masses of the Higgs boson and top quark indicate that the e#11;ective potential of the standard model either develops an unstable electroweak vacuum or stands stable all the way up to the Planck scale. In the latter case in which the top quark mass is about 2#27; below its present central value, the Higgs boson can be the inaton with the help of a large nonminimal coupling to curvature in four dimensions. We propose a scenario in which the Higgs boson can be the in aton in a#12;ve-dimensional Gauss-Bonnet braneworld model to solve both the unitarity and stability problems which usually plague Higgs ination. We #12;nd that in order for Higgs ination to happen successfully in the Gauss-Bonnet regime, the extra dimension scale must appear roughly in the range between the TeV scale and the instability scale of standard model. At the tree level, our model can give rise to a naturally small nonminimal coupling #24; #24; O(1) for the Higgs quartic coupling #21; #24; O(0:1) if the extra dimension scale lies at the TeV scale. At the loop level, the in ationary predictions at the treelevel are preserved. Our model can be confronted with future experiments and observations fromboth particle physics and cosmology.

Full Text

Preamble

Higgs Inflation in Gauss-Bonnet Braneworld

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(Dated: September 15, 2015)

The measured masses of the Higgs boson and top quark indicate that the effective potential of the Standard Model either develops an unstable electroweak vacuum or remains stable all the way up to the Planck scale. In the latter case, where the top quark mass is about 2 below its present central value, the Higgs boson can serve as the inflaton with the help of a large nonminimal coupling to curvature in four dimensions. We propose a scenario in which the Higgs boson can be the inflaton in a five-dimensional Gauss-Bonnet braneworld model to solve both the unitarity and stability problems that typically plague Higgs inflation. We find that for Higgs inflation to occur successfully in the Gauss-Bonnet regime, the extra dimension scale must lie roughly in the range between the TeV scale and the instability scale of the Standard Model. At tree level, our model can give rise to a naturally small nonminimal coupling O(1) for the Higgs quartic coupling O(0.1) if the extra dimension scale lies at the TeV scale. At the loop level, the inflationary predictions at tree level are preserved. Our model can be confronted with future experiments and observations from both particle physics and cosmology.

Introduction

The recently released Planck 2015 data [?] provide growing evidence that our observable universe experienced an inflationary era, stretching primordial quantum fluctuations to cosmic scales, leaving distinct imprints on the Cosmic Microwave Background Radiation and seeding the formation of cosmic structures. The currently favored inflationary scenarios [?] are single-field slow-roll inflationary models, where a scalar field plays the role of the inflaton. Despite the phenomenological success of inflation, there is growing theoretical interest in connecting inflation with low-energy particle physics, among which Higgs inflation is the most attractive model due to its minimality.

Higgs inflation [?] utilizes a nonminimal coupling of the Standard Model (SM) Higgs boson to four-dimensional Einstein gravity. At high energy scales, the Higgs boson decouples from the SM and slowly rolls down an exponential plateau-like potential in the Einstein frame. The Planck normalization requires a large nonminimal coupling 5×10 for tree-level analysis, given the Higgs quartic coupling 1.8×10 estimated from the Higgs mass m_h 125 GeV and vacuum expectation value (VEV) v 246 GeV. At intermediate energy scales where preheating [?]/reheating [?] occurs, the Higgs boson oscillates along a quadratic potential and decays into SM particles. At low energy scales, the potential transitions to the usual SM quartic potential. The cosmological predictions of Higgs inflation can fit the Planck 2015 data well and exhibit insensitivity to its reheating processes [?]. However, two major problems plague Higgs inflation: the unitarity problem and the stability problem.

The stability problem [?, ?] states that for successful Higgs inflation, the top quark mass must be about 2 below its present central value for the measured Higgs mass. This reveals a potential tension between constraints from particle physics and those from cosmology. To stabilize the SM electroweak vacuum in Higgs inflation, one either introduces new particle thresholds such as scalar fields \cite{9-12}, fermion fields \cite{13-15}, and vector fields [?], or invokes new physics such as asymptotically safe Higgs inflation \cite{17-20}. It is worth noting that Higgs inflation can also be realized [?] in the case of a metastable electroweak vacuum if one takes into account the unknown finite parts of counterterms and finite temperature corrections to the effective potential.

The unitarity problem \cite{22-30} states that the tree-level analysis is already invalid even before Higgs inflation can take place at the scale M_P/ due to unitarity violation at the scale M_P/ by naive power-counting. Restoring unitarity above M_P/ introduces either new particles or new interactions, both of which might spoil the flatness of the inflationary potential in an uncontrollable manner. There are three ways to address the unitarity problem: First, introducing new interactions such as new Higgs inflation [?], unitary Higgs inflation [?], the Higgs model [?], and its variant [?]. However, there is no guarantee [?] whether the quantum corrections of these new interactions are under control. Second, recognizing the background-dependent cutoff [?, ?, ?] above which strong dynamics should enter to restore unitarity. However, there is also no guarantee [?] whether the strong dynamics would call for new physics. Third, fine-tuning the Higgs mass and top quark mass to achieve an extremely small Higgs quartic coupling around the Planck scale as in critical Higgs inflation \cite{34-38}. However, an unnaturally small requires the top quark mass to be about 2 below its present central value, and can only be made O(1) if one allows a large r 0.1 in direct conflict with Planck 2015 TT, TE, EE + lowP constraints [?]. We report in this paper an alternative: extra dimensions.

The idea of extra dimensions stemmed from the attempt by Kaluza and Klein to unify gravitational and electromagnetic interactions. Although the idea failed, the formalism survived. Later it was found that string theory can only be defined consistently in higher dimensions while the compactification scale is too high to be tested experimentally. However, large extra dimension scenarios renewed interest in extra dimensions, particularly the Arkani-Hamed, Dimopoulos and Dvali (ADD) model [?, ?] and Randall and Sundrum (RS) models [?, ?], opening new doors to tackle profound mysteries in particle physics and cosmology. In five dimensions, it is natural to include the Gauss-Bonnet term for four reasons: First [?], it presents a unique combination of a second-order symmetric and divergence-free tensor that can lead to second-order field equations in bulk metric components. Second [?], it arises in heterotic string theory as next-to-leading order corrections with the Gauss-Bonnet coupling identified with the Regge slope. Third [?], it leads to ghost-free nontrivial gravitational self-interactions for dimensions higher than four. Fourth \cite{46-52}, the zero mode of the graviton is localized on the brane at low energy with only two independent degrees of freedom corresponding to the usual four-dimensional graviton. As a result, there have been extensive studies on the Gauss-Bonnet braneworld scenario.

In this paper, we realize Higgs inflation in five-dimensional Gauss-Bonnet braneworld cosmology. We find that for Higgs inflation to occur in the Gauss-Bonnet regime, the combined parameter $/ ^2$ could increase by many orders of magnitude with decreasing energy scale of the extra dimension, and the extra dimension scale must lie roughly in the range between the TeV scale and the SM instability scale. For the extra dimension scale near the TeV scale, the nonminimal coupling can be made O(1) for the Higgs quartic coupling O(0.1) with tensor-to-scalar ratio r 10 ^{12} safely inside the Planck 2015 TT, TE, EE + lowP bound r 0.1. The prediction of scalar spectral index 0.960 n_s 0.968 and its running −0.0008 _s −0.0005 remains almost the same as in the four-dimensional case for all possible extra dimension scales. Furthermore, the inflationary predictions are preserved beyond tree-level analysis.

This paper is organized as follows. In Section II, we review the general formalism of the five-dimensional Gauss-Bonnet braneworld scenario. In Section III, we propose Higgs inflation in the five-dimensional Gauss-Bonnet braneworld model. In Section IV, the tree-level results are summarized. In Section V, we go beyond tree-level analysis. The final section is devoted to conclusions.

II. Gauss-Bonnet Braneworld Cosmology

We briefly review in this section the general formalism of the five-dimensional Gauss-Bonnet braneworld scenario.

The total action of the Gauss-Bonnet braneworld model reads (we neglect possible boundary terms)

$$S = \int d^5x\sqrt{-g_5}\left[-2\Lambda_5 + R_5 - 4\alpha R^{(5)}_{ab}R^{ab}_{(5)} + \alpha R^{(5)}_{abcd}R^{abcd}_{(5)}\right] + \int d^4x\sqrt{-g_4}\left[-m_4^2\sigma + \mathcal{L}_{\rm matter}\right],$$

which contains a five-dimensional anti-de Sitter (AdS) bulk with a negative cosmological constant Λ and a four-dimensional Friedmann-Robertson-Walker (FRW) brane with a positive tension m ^{2} . The confined matter field with Lagrangian density can be approximated as a perfect fluid by assumption. The Gauss-Bonnet term is weighted by , which should be positive from the perspective of stringy generalization of general relativity for Einstein-Gauss-Bonnet gravity. We will see that the Planck scale M (^{2} = 8 G) = 2.435 × 10^1 GeV on the four-dimensional FRW brane can be derived from the more fundamental Planck scale M (^{2} = 8 G) in the five-dimensional AdS bulk, where ^{2} = 1/M ^{3} and ^{2} = 1/M ^{2}.

The field equation and junction equation of the action (1) admit an FRW brane solution

$$ds_4^2 = -dt^2 + a(t)^2\gamma_{ij}dx^idx^j,$$

which can be induced from the AdS bulk metric,

$$ds_5^2 = -f(a)d\tau^2 + \frac{da^2}{f(a)} + a^2\gamma_{ij}dx^i dx^j,$$

by requiring

$$-\frac{f(a)}{\dot{\tau}(t)^2} + \frac{\dot{a}(t)^2}{f(a)} = -1,$$

with respect to the embedding coordinates (t) and a(t). Therefore, the scale factor a(t) on the brane can be interpreted as the motion of the brane a( ) in the bulk. Here describes a maximally symmetric 3-hypersurface with spatial curvature constant k = 0, ±1 and f(a) can be solved for pure AdS spacetime [**?**, **?**] as

$$f(a) = k_3 + a^2\mu^2,$$

where has two branches for > 0,

$$\mu^2 = \frac{1}{4\alpha}\left(1 \pm \sqrt{1 + \frac{4}{3}\alpha\Lambda_5}\right),$$

and the negative branch of (6), Λ $= -6$ $^2(1 - 2$ $^2)$, has the RS limits Λ $= -6$ ^{2} by taking $\to 0$. is usually associated with the bulk curvature scale |R | ^{2}. Introducing a dimensionless Gauss-Bonnet coupling 4 ^{2}, the subdominant Gauss-Bonnet term |R ^{2}| |R | requires 4. The negative bulk cosmological constant Λ < 0 requires < 2 from (7), and the negative branch $1 - 4$ $^2 < 0$ requires < 1 from (6).

The modified FRW equation now reads [**?**, **?**]

$$H^2 = \frac{\mu^2}{2\beta}\left[\left(1 + \frac{\rho + m_4^2\sigma}{2\mu m_4^2}\right)^{2/3} - 1\right]^2 - \frac{k_3}{a^2}.$$

To match standard cosmology on the brane with a vanishing cosmological constant in the limits H^2/ ^{2} 1, one requires

$$m_4^2\sigma = 2\mu m_4^2(3 - \beta), \quad \mu\kappa_5^2 = (1 + \beta)\kappa_4^2.$$

In the high Hubble scale H^2/ ^{2} 1, the modified FRW equation (10) describes the GB regime

$$H^2 \approx \left(\frac{1+\beta}{12\beta}\right)^2 \frac{(\rho + m_4^2\sigma)^2}{m_4^4},$$

while in the low Hubble scale H²/ ² 1, the modified FRW equation (10) describes the GR regime

$$H^2 \approx \frac{\rho + m_4^2\sigma}{3m_4^2} - \frac{k_3}{a^2}.$$

The RS regime emerges when the RS energy scale is smaller than the GB energy scale m_ < m_ , which occurs when 0.151. We will set 0.151 from now on to simplify the evolution of the brane universe with the GB regime followed immediately by the GR regime. The full evolution of the brane universe is presented in Fig. 1 [Figure 1: see original paper] with several typical choices of .

III. Higgs Inflation in the Gauss-Bonnet Braneworld

We first review Higgs inflation in four-dimensional Einstein gravity. The action in the Jordan frame is

$$S_4 = \int d^4x\sqrt{-g}\left[\frac{M_4^2 + \xi h^2}{2}R_4 - \frac{1}{2}(\partial h)^2 - V(h)\right],$$

where Ω² = M ² + h² and V(h) = (/4)(h² − v²)². The four-dimensional Planck mass is recovered via M ² = M² + v² when the Higgs field is at its VEV v = 246 GeV. As long as the nonminimal coupling M ²/v² 10³², one can safely approximate M² = M ² − v² M ².

Therefore, after making the conformal transformation g̃ = Ω²g and field redefinition

$$\frac{d\chi}{dh} = \sqrt{\frac{1+\xi(1+6\xi)h^2/M_4^2}{(1+\xi h^2/M_4^2)^2}},$$

where we adopt the convention ² = 1, one obtains the action in the Einstein frame

$$\tilde{S}_4 = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_4^2}{2}\tilde{R} - \frac{1}{2}(\tilde{\partial}\chi)^2 - U(\chi)\right],$$

with

$$U(\chi) = \frac{V(h(\chi))}{\Omega^4(h(\chi))} \approx \frac{\lambda M_4^4}{4\xi^2}\left(1 + e^{-\chi/(\sqrt{6}M_4)}\right)^{-2}.$$

Here we have used the large field solution h M exp(/(√6 M)) of the field redefinition equation (20) in the large field limit h M /√ .

We then uplift the Ricci scalar curvature in (21) as if it is reduced from the five-dimensional Gauss-Bonnet gravity when the extra dimension emerges at high energy scale. The action of our model then reads

$$S = \int d^5x\sqrt{-g_5}\left[-2\Lambda_5 + R_5 - 4\alpha R^{(5)}_{ab}R^{ab}_{(5)} + \alpha R^{(5)}_{abcd}R^{abcd}_{(5)}\right] + \int d^4x\sqrt{-g_4}\left[-m_4^2\sigma - \frac{1}{2}(\partial\chi)^2 - U(\chi)\right],$$

by choosing the matter field Lagrangian on the brane in (1) as the canonically normalized Higgs field, = −½( )² − U( ).

It is worth noting that, unlike previous works [?, ?] where a bulk/brane scalar field is nonminimally coupled to bulk/brane curvature in the Jordan frame, the canonically normalized Higgs field in the Einstein frame is minimally coupled to the five-dimensional Gauss-Bonnet gravity in our model. We argue that the action (1) with (23) is actually a natural choice from an effective field theory perspective. At low energy scales, the extra degrees of freedom due to the presence of the extra dimension should be integrated out and the physics should be well described by the SM with a nonminimal coupling term. When the energy scale increases, the physical Higgs boson starts to decouple from the SM, and it is the canonically normalized Higgs field that plays the role of inflaton. Therefore, the effect of the Gauss-Bonnet braneworld needs to be accounted for only when one goes to higher energy in the Einstein frame if extra dimensions really exist. Thus we directly uplift the curvature term in (21) as if it is reduced from the five-dimensional Gauss-Bonnet gravity at leading order.

The inflationary predictions [?] can be carried out directly just as those done in [?]. First, solving the Hubble parameter from the modified FRW equation (10) by replacing with U( ), and calculating the slow-roll parameters [?]

$$\epsilon(\chi) = \frac{U'(\chi)H'(\chi)}{3H(\chi)^3}, \quad \eta(\chi) = \frac{U''(\chi)}{3H(\chi)^2},$$

to find the endpoint of inflation by solving max[( ), | ( )|] = 1. Second, solving _N and / ² from the combined equations [?]:

$$N = \int_{\chi_N}^{\chi_{\rm end}} \frac{3H(\chi)^2}{U'(\chi)}d\chi, \quad \frac{U'(\chi_N)^2}{H(\chi_N)^6} = \frac{12\pi^2 A_s}{M_4^6},$$

for a given e-folding number N and Planck normalization ln(10¹ A_s) = 3.094. Third, with _N and / ² solved above, we can easily obtain the scalar spectral index n_s = 1 − 6 (_N) + 2 (_N), its running

$$\alpha_s = \frac{M_4^2 U'(\chi_N)}{3H^2(\chi_N)} \left(6\epsilon'(\chi_N) - 2\eta'(\chi_N)\right),$$

and the tensor-to-scalar ratio r = A_t/A_s, where the amplitude of gravitational waves is given by [?]

$$A_t = \frac{2H(\chi_N)^2}{\pi^2 M_4^2} F(x)^2,$$

with suppression factor

$$F(x)^2 = \left(\sqrt{1+x^2} - \frac{1-\beta}{1+\beta} x^2 \sinh^{-1}\frac{1}{x}\right)^{-1}, \quad x \equiv \frac{H}{\mu}.$$

The pivot scale is chosen as k = 0.05 Mpc ¹ and the e-folding number is taken in the range N = 50–60.

IV. Tree-Level Results

In Fig. 2 [Figure 2: see original paper], we present various characteristic features of Higgs inflation with respect to the energy scale of the extra dimension in the Gauss-Bonnet braneworld.

The first result is that the extra dimension scale must be below the SM instability scale to have Higgs inflation in the GB regime. This can be seen from the first panel in Fig. 2: the GB regime with H/ 1 is found for log (/M) −6, which is roughly the energy scale 10^{12} GeV where the Higgs quartic coupling would become negative. This result can also be derived formally as follows: in the GB regime one can use the modified FRW equation (15) in the Planck normalization (26) and find that

$$\frac{\lambda}{\xi^2} \approx \frac{12\pi^2 A_s}{M_4^4} \left(\frac{1+\beta}{12\beta}\right)^2 \left(1 + e^{-\chi_N/(\sqrt{6}M_4)}\right)^{-2}.$$

Since _N 5M from the second panel of Fig. 2, one immediately obtains 3.4 × 10^{12} GeV.

The second result is that the energy scale of the extra dimension lies below not only the SM instability scale but also the inflationary Hubble scale in the GB regime. This can be seen from the third panel of Fig. 2 where the purple line is obtained by = H. The GB regime lies on the left-hand side of the purple line, where the extra dimension scale is less than the inflationary Hubble scale. Therefore, one might worry whether Kaluza-Klein (KK) modes excited during inflation would spoil the flatness of the inflationary potential. Fortunately, the spectrum of KK modes consists only of the massless four-dimensional graviton and a continuum of states with masses larger than the inflationary Hubble scale [**?**], which are too heavy to be excited during inflation in any real processes. Furthermore, scattering processes involving these heavy KK modes running in loops are highly suppressed by their masses in the propagators. Therefore, the flatness of the inflationary potential is preserved. However, the GR regime lies on the right-hand side of the purple line, where the extra dimension scale is above the inflationary scale. Consequently, the extra dimension is invisible to the Higgs boson during inflation, and Higgs inflation in the five-dimensional GR regime is effectively the same as four-dimensional Higgs inflation, which is why we are not interested in Higgs inflation in the GR regime.

The third result is that the Hubble scale remains almost unchanged during inflation in the GB regime. This can be seen from the third panel of Fig. 2 where H_start H_end for a given in the GB regime. This can be understood as follows: On the one hand, we can see from the second panel of Fig. 2 that the field values during inflation in the GB regime are deeper into the exponential plateau-like potential than in the GR regime; therefore, the potential change is rather small during inflation. On the other hand, the modified FRW equation H^2 $^{2/3}$ in the GB regime suppresses the contributions to the Hubble parameter from potential changes. In general, the inflationary Hubble scale is H_inf 10^{13} GeV.

The fourth result is that the extra dimension scale must be above the TeV scale. This can be seen from the last panel of Fig. 2 where we require that the inflationary Hubble scale is below the five-dimensional Planck scale H_inf M , namely, 1 TeV. This is consistent with the requirement that the energy density on the brane should be limited by the induced four-dimensional Planck scale _inf M , which also leads to 1 TeV. For the extra dimension scale 1 TeV, the five-dimensional Planck scale M 1.7×10^{13} GeV is very close to the inflationary Hubble scale. With the upper bound (32), the five-dimensional Planck scale can also be bounded from above, namely, M 2.6×10^{1} GeV.

In Fig. 3 [Figure 3: see original paper] we present the combined parameter $/ ^2$ for Higgs inflation with respect to the extra dimension scale in the Gauss-Bonnet braneworld. In the GR regime with modified FRW equation (17), the combined parameter $/ ^2$ 10^{1} remains almost the same as in the four-dimensional case. In the GB regime, $/ ^2$ surprisingly increases by many orders of magnitude with decreasing extra dimension scale. However, with modified FRW equation (15), the product $(/ ^2)$ 10^{1} remains constant with decreasing extra dimension scale in the GB regime. In the case with the extra dimension scale near the TeV scale, which is of experimental interest, $/ ^2$ can achieve order O(0.1), which can lead to a naturally small O(1) for O(0.1). This naturally solves the unitarity problem without fine-tuning and violating the Planck 2015

TT, TE, EE + lowP bound r 0.1, as shown below.

Next we explain why we are only interested in Higgs inflation in the GB regime instead of the RS regime. In the GB regime, the requirement that the inflationary Hubble scale is below the five-dimensional Planck scale,

$$H^3 \approx \left(\frac{1+\beta}{12\beta}\right)^3 \frac{(\rho + m_4^2\sigma)^3}{m_4^{12}} \lesssim M_5^3 = \frac{M_4^3}{1+\beta}\mu,$$

namely,

$$\frac{\lambda}{\xi^2}\frac{\mu}{M_4} \lesssim \frac{16\beta}{(1+\beta)^2} \approx 1.82,$$

is satisfied as long as the extra dimension scale is just larger than the TeV scale. However, if we allow to take other values near 0, then Higgs inflation can also take place in the RS regime with modified FRW equation (16), which can give

$$\frac{\lambda}{\xi^2} \approx \frac{12\pi^2 A_s}{M_4^4}\left(\frac{12(3-\beta)}{1+\beta}\right)^2 \left(1 + e^{-\chi_N/(\sqrt{6}M_4)}\right)^{-2}.$$

To naturally solve the unitarity problem, one needs / 2 of order O(0.1), which requires both the extra dimension scale and inflationary Hubble scale to be extremely near the four-dimensional Planck scale. Therefore, Higgs inflation in the RS regime is less interesting than in the GB regime from an experimental point of view.

In Fig. 4 [Figure 4: see original paper], we present the inflationary predictions of n_s, r, and _s for Higgs inflation in the Gauss-Bonnet braneworld. Both 0.960 n_s 0.968 and −0.0008 _s −0.0005 are stable against changes in the extra dimension scale. Only in the GR regime does r 10 3 as in the four-dimensional case. In the GB regime, r drops significantly with decreasing extra dimension scale. Unlike critical Higgs inflation \cite{34-38} in four dimensions where r 0.1 for O(1), our r can be as small as 10 12, safely inside the Planck 2015 TT, TE, EE + lowP bound r 0.1 for extra dimension scales around the TeV scale. Recall that the field excursion during inflation is roughly 5M for all extra dimension scales; this explicitly evades the usual Lyth bound argument that a super-Planckian field excursion during inflation corresponds to an observable tensor-to-scalar ratio. The Lyth bound is modified in the presence of extra dimensions in our model:

$$\Delta\chi \approx \int_0^N \sqrt{\frac{r}{8F^2}}dN,$$

where the suppression factor (30) is F² 1 for the GR regime and F² (1 +) 1(H/) 1 for the GB regime, namely,

$$\Delta\chi \approx \int_0^N \sqrt{\frac{r}{8}} \left(\frac{H}{\mu}\right)^{1/2} (1+\beta)^{1/2} dN.$$

With decreasing extra dimension scale, the tensor-to-scalar ratio can be dragged down to unobservable levels even if the field excursion during inflation is super-Planckian.

V. Going Beyond Tree-Level Analysis

In the previous section we saw that for Higgs inflation to occur in the GB regime, the extra dimension scale is below the inflationary Hubble scale. Thanks to the fact that the masses of extra KK modes are larger than the inflationary Hubble scale, the flatness of the inflationary potential is preserved since these heavy KK modes certainly cannot be excited at external legs and any contributions from these heavy KK modes running in loops are suppressed by their masses in the propagators. When going beyond tree-level analysis of the renormalization-group (RG)-improved effective potential, we can actually follow the methods \cite{7,8,63-65} developed for four-dimensional Higgs inflation. The net effect of adding the extra dimension to Higgs inflation is the change of normalization condition (26) for scalar spectrum amplitude due to the modified FRW equation (10) at the background level. We present below the procedures to obtain the predictions of , , n_s, _s, and r at the inflationary scale with respect to the top quark mass for a given Higgs mass at electroweak scale and extra dimension scale.

First, the initial conditions at¯ = m_t for the MS SM couplings are taken from [?], which are repeated here for convenience:

$$g'(m_t) = 0.3587, \quad g(m_t) = 0.6483, \quad g_s(m_t) = 1.1666 + 0.00314\left(\frac{\alpha_s(m_Z)-0.1184}{0.0007}\right) - 0.00046\left(\frac{m_t-173.35}{0.5}\right)$$

$$y_t(m_t) = 0.93697 + 0.00550\left(\frac{m_t-173.35}{0.5}\right) - 0.00042\left(\frac{\alpha_s(m_Z)-0.1184}{0.0007}\right) \pm 0.00050_{\text{th}},$$

$$\lambda(m_t) = 0.12710 + 0.00206\left(\frac{m_t-173.35}{0.5}\right) - 0.00004\left(\frac{m_h-125.66}{0.3}\right) \pm 0.00030_{\text{th}}.$$

Second, the three-loop RG equations for SM couplings, three-loop RG equation for the Higgs anomalous dimension   = d ln h/d ln¯, and two-loop RG equation for the nonminimal coupling are used in our analysis from the Appendix of Ref. [?]. We omit here the complete expressions for these RG equations. However, it is worth noting that the s-factor [?, ?]

$$s(h) = \frac{1 + \xi h^2/M_4^2}{1 + (1 + 6\xi)\xi h^2/M_4^2}$$

will be important in our case of small , unlike four-dimensional Higgs inflation with large which renders a chiral electroweak theory at high energy scale. We solve these RG equations with the above initial conditions and input parameters within corresponding uncertainties [?]:

$$m_h = (125.66 \pm 0.34)\ \text{GeV}, \quad m_t = (173.36 \pm 0.65 \pm 0.3)\ \text{GeV}, \quad \alpha_s(m_Z) = 0.1184 \pm 0.0007,$$

to obtain the running couplings and anomalous dimension as functions of renormalization scale ¯ = m_t e^t.

Third, we do not include two-loop radiative corrections in our effective potential since the tree-level potential U ( ) = M /(4 ²)(1 + e /(√ M)) ² and one-loop Coleman-Weinberg potential

$$U_1(\chi) = \sum_i \frac{n_i M_i^4(\chi)}{64\pi^2} \left[\ln \frac{M_i^2(\chi)}{\bar{\mu}^2} - c_i\right],$$

where

$$M_W^2 = \frac{g^2}{4}\Omega^2, \quad M_Z^2 = \frac{g^2 + g'^2}{4}\Omega^2, \quad M_t^2 = \frac{y_t^2}{2}\Omega^2, \quad M_h^2 = \frac{\lambda}{2}\Omega^2,$$

are sufficient for our purpose. Note that we are working in prescription I, where quantum corrections are computed in the Einstein frame. After the replacements [?, ?]

$$h \to Z(\bar{\mu})h, \quad \Omega(h) \to \Omega(h)\left(1 + \frac{3\xi h^2}{M_4^2 + \xi h^2}\right)^{1/2},$$

where Z(¯) = exp((¯ ‘) d ln¯’), we obtain the RG-improved effective potential U_eff((h)) = U ((h)) + U ((h)), where (h) is the solution of field redefinition (20).

Finally, the initial value = (m_t) can be fixed by matching the Planck normalization (26) with U( ) replaced by U_eff( ). To be more specific, for given input parameters of m_h, m_t, _s(m_Z), , and extra dimension scale , we compute the initial conditions and then solve the RG equations to obtain the running couplings and anomalous dimension. We then compute the effective potential and scalar spectrum amplitude. We repeat this procedure by choosing different until the Planck normalization is fulfilled. Once is determined, we follow the above procedures once again to obtain _inf = (h_N), _inf = (h_N) at e-folding number N = 60. We can also obtain the corresponding values for n_s and r with U( ) replaced by U_eff( ).

In Fig. 5 [Figure 5: see original paper], with input Higgs mass m_h = 125.66 GeV, we present the numerical results of the nonminimal coupling _inf, Higgs quartic coupling _inf, scalar spectral index n_s, and tensor-to-scalar ratio r during inflation with respect to the top quark mass. The extra dimension scale is 10 TeV for the blue region and 50 TeV for the red region respectively, where the 1 uncertainties are mainly from the strong coupling _s along with other theoretical uncertainties from threshold corrections (48) and (49). During inflation, the nonminimal coupling _inf O(0.1) and the Higgs quartic coupling _inf O(0.01). The inflationary predictions of n_s and r remain the same as the tree-level results.

We also find the following upper bound for the top quark mass as a function of the Higgs mass, strong coupling, and other theoretical uncertainties when the extra dimension scale is fixed at = 50 TeV:

$$m_t < 171.179 + 0.4816 \left(\frac{m_h - 125.66}{0.3} \right) + 0.283 \left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007} \right) \pm 0.162.$$

In Fig. 6 [Figure 6: see original paper], we present the allowed region in the m_h-m_t plane for our model, where the 1 uncertainties from the strong coupling _s along with other theoretical uncertainties from threshold corrections have been properly accounted for. We also provide the upper bound set by four-dimensional Higgs inflation and the experimental constraint on m_t and m_h for comparison. We find that with increasing energy scale of the extra dimension, the upper bound on the top quark mass in our model approaches those in four-dimensional Higgs inflation. Therefore, the stability problem persists as in four-dimensional Higgs inflation, although the unitarity problem is indeed solved.

Fortunately, the first nonzero KK mode appears above the inflationary scale even though the extra dimension scale can be as low as the TeV scale; therefore, adding extra dimensions only changes the background dynamics of the universe without jeopardizing low-energy particle physics. To solve the stability problem, one could follow the same spirit of stabilizing the SM effective potential, and further generalizations of our model should be considered. For example, it was found in \cite{13-15} that a TeV-scale type III seesaw mechanism can simultaneously account for neutrino oscillations and stabilize the SM effective potential without introducing any additional scalar fields. Therefore, Higgs inflation in the Gauss-Bonnet braneworld can be self-consistent up to the inflationary Hubble scale, free of both the unitarity problem and stability problem. Although shift symmetry may still be needed to preserve the flatness of the potential above the inflationary scale—a common problem shared by many other inflation models—Higgs inflation in the Gauss-Bonnet braneworld might be more conveniently embedded into underlying UV theories.

VI. Conclusions

In this paper, we realize Higgs inflation in the five-dimensional Gauss-Bonnet braneworld scenario. We find that for Higgs inflation to take place in the GB regime, the extra dimension scale must be in the range between the TeV scale and the instability scale of the SM. Furthermore, the intriguing improvement of many orders of magnitude for / ^{2} with decreasing extra dimension scale comes as a nice surprise. For the extra dimension scale around the experimentally interesting TeV scale, the nonminimal coupling can be made of order O(1) for the Higgs quartic coupling O(0.1). The predicted scalar spectral index 0.960 n_s 0.968 and its running −0.0008 $_s$ −0.0005 are well within the Planck 2015 constraints on inflation. Unlike critical Higgs inflation, the tensor-to-scalar ratio r 10 ^{12} is safely inside the Planck 2015 TT, TE, EE + lowP bound r 0.1. We also investigate the inflationary predictions beyond tree-level analysis and find that the predictions remain almost the same as the tree-level results. However, to avoid the stability problem, one must follow the same spirit of stabilizing the SM effective potential by using, for instance, the TeV-scale type III seesaw mechanism.

Acknowledgments

We would like to thank Mikhail Shaposhnikov for helpful discussions during the International Conference on Gravitation and Cosmology and the Fourth Galileo-Xu Guangqi Meeting at the Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, China.

R.G.C. is supported by the Strategic Priority Research Program of the Chinese Academy of Sciences, Grant No. XDB09000000. Z.K.G. is supported by the National Natural Science Foundation of China, Grants No. 11175225 and No. 11335012.

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