Null test of the cosmic curvature using H(z) and supernovae data (postprint)

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Abstract

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Full Text

Preamble

Null Test of the Cosmic Curvature Using H(z) and Supernovae Data

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We introduce a model-independent approach to the null test of the cosmic curvature which is geometrically related to the Hubble parameter H(z) and luminosity distance d_L(z). Combining independent observations of H(z) and d_L(z), we use the model-independent smoothing technique of Gaussian processes to reconstruct these quantities and determine the cosmic curvature Ω_K^(0) in the null test relation. The null test is purely geometrical and does not assume any cosmological model. We show that the cosmic curvature Ω_K^(0) = 0 is consistent with current observational data sets, falling within the 1 limit. To demonstrate the effect on the precision of the null test, we produce a series of simulated data for models with different $\Omega_K^{(0)}$. Future observations of better quality can provide significant improvement to constrain or refute the flat universe with $\Omega_K^{(0)} = 0$.

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Introduction

Whether the space of our Universe is open, flat, or closed is one of the most fundamental problems in modern cosmology. The spatial topology of the Universe is also closely related to other important problems such as the evolution of the Universe and the properties of dark energy. The effect of allowing nonzero curvature on dark energy models has been studied extensively (see, for example, Refs. [1–4]), and the detection of a significant deviation from $\Omega_K^{(0)} = 0$ would have profound consequences for inflation models and fundamental physics. Consequently, this issue has attracted considerable attention [5–8].

The ΛCDM model is consistent with all current data, and a flat universe is preferred even in the recent Planck 2015 results [9]. However, almost all studies constraining the cosmic curvature assume specific models for dark energy, such as a particular equation of state $w(z)$. These are model-dependent and indirect methods. Notably, there exists some degeneracy between spatial curvature and the equation of state of dark energy in these studies. It would be preferable to detect the spatial curvature of the Universe through geometrical and model-independent methods.

In this paper, we utilize a model-independent geometrical relation among the cosmic curvature $\Omega_K^{(0)}$, Hubble parameter $H(z)$, and luminosity distance $d_L(z)$. By combining the Hubble rate and luminosity distance, we can directly determine and test whether the cosmic curvature deviates from zero [3, 10, 11]. To achieve this goal, we focus on two independent observations that directly provide $H(z)$ and $d_L(z)$, respectively. For $H(z)$ data, we use measurements derived from differential ages of galaxies ("cosmic chronometers") and from the radial baryon acoustic oscillation (BAO) scale in galaxy distributions. For $d_L(z)$, we employ the SNeIa Union 2.1 data sets.

We apply the model-independent Gaussian processes (GP) method for smoothing the observational data. The advantage of this approach is that it is entirely model-independent; hence, we need not assume any specific dark energy model or gravity theory. The method is purely geometrical and constrained directly by observational data. However, the precision of the null test is limited by the quality of observational data. Detecting a tiny cosmic curvature more precisely requires higher-quality observational data sets, which we also discuss in this paper.

This paper is organized as follows. In Sec. II, we introduce the theoretical method for the null test of cosmic curvature. In Sec. III, we first provide a brief introduction to Gaussian processes, then apply the GP method to the null test using two independent data sets: CC+BAO and Union 2.1, followed by tests with simulated data. We present discussions and conclusions in Sec. IV.

II. Theoretical Method

In a FLRW universe, the luminosity distance d_L can be expressed as

$$d_L(z) = \frac{c(1+z)}{H_0\sqrt{|\Omega_K^{(0)}|}}\mathrm{sinn}\left(\sqrt{|\Omega_K^{(0)}|}\int_0^z \frac{dz'}{E(z')}\right),$$

where $E(z) \equiv H(z)/H_0$, $\Omega_K^{(0)} \equiv -Kc^2/(a_0H_0)^2$, and $K = +1, -1, 0$ corresponds to a closed, open, and flat universe, respectively.

Differentiating Eq. (1) and writing $D(z) = (H_0/c)(1+z)^{-1}d_L(z)$ as the normalized comoving distance, we obtain

$$E^2(z)D'^2(z) - 1 = \frac{\Omega_K^{(0)}}{3}D^2(z).$$

We see that the cosmic curvature $\Omega_K^{(0)}$ can be directly determined from the Hubble parameter and luminosity distance using Eq. (2). Thus, the null test of $\Omega_K^{(0)}$ is straightforward. Note that $D(0) = 0$ introduces a singularity at $z = 0$. Therefore, for greater clarity, we transform Eq. (2) to

$$\frac{\Omega_K^{(0)}}{3}\frac{D^2(z)}{E(z)D'(z)+1} = E(z)D'(z) - 1.$$

Since the left-hand side (lhs) of Eq. (3) is nonzero when $z \neq 0$ if $\Omega_K^{(0)}$ is nonvanishing, the null test of cosmic curvature $\Omega_K^{(0)}$ is equivalent to testing whether the entire lhs of Eq. (3) vanishes. If we define

$$\mathcal{O}_K(z) \equiv \frac{\Omega_K^{(0)}}{3}\frac{D^2(z)}{E(z)D'(z)+1} = E(z)D'(z) - 1,$$

then a flat universe implies $\mathcal{O}_K(z) = E(z)D'(z) - 1 = 0$ at any redshift. A deviation from zero indicates a signal of nonvanishing cosmic curvature. Note that the theoretical value of $\mathcal{O}_K(z)$ is always zero at $z = 0$. Therefore, the null test simply checks whether there exists a signal of nonvanishing $\mathcal{O}_K(z)$ at nonzero redshifts.

Thus, we should use current observational data sets to reconstruct $E(z)$ and $D'(z)$ independently and combine these reconstructions to test whether the relation $E(z)D'(z) - 1 = 0$ holds at arbitrary redshifts. We emphasize that the null test is entirely geometrical and cosmological model-independent; the reconstructions of $E(z)$ and $D'(z)$ are derived directly from observational data sets.

III. Null Test Using H(z) and Supernovae Data

Given observational data sets, it is crucial to employ a model-independent method to reconstruct $E(z)$, $D(z)$, and its derivative $D'(z)$. Various methodologies exist for reconstructing functions from data (see [12] for a brief analysis). Since we require a nonparametric approach to smooth the data and reconstruct derivatives, Gaussian processes [13–16] are particularly suitable for our purpose.

A. Gaussian Processes

Gaussian processes allow one to reconstruct a function from data without assuming a specific parametrization. Here, we use Gaussian processes in Python (GaPP) [16], which has been employed in various studies for different purposes [16–24]. The distribution over functions provided by GP is well-suited for describing observational data. At each point z, the reconstructed function $f(z)$ follows a Gaussian distribution with a mean value and Gaussian error. The functions at different points z and $\tilde{z}$ are related by a covariance function $k(z, \tilde{z})$, which depends only on a set of hyperparameters ℓ and σ_f. Here, ℓ measures the coherence length of the correlation in the x direction, and σ_f denotes the overall amplitude of the correlation in the y direction. Both hyperparameters are optimized by GP using the observational data sets. Unlike actual parameters, GP does not specify the form of the reconstructed function; instead, it characterizes the typical variation of the function. Detailed analysis and description of the GP method can be found in [16, 19].

B. Hubble Rate Data and Union 2.1

Following [17, 18], we perform an analysis based on observational Hubble data compiled from several sources, independent of SNeIa. We combine measurements of $H(z)$ obtained through two methods. The first is cosmic chronometers, which primarily use passively evolving galaxies, providing 21 data points compiled by Moresco et al. [25, 26]. The second is radial baryon acoustic oscillations from galaxy clustering in redshift surveys, yielding seven data points of Hubble parameters from different experiments [27–30]. We summarize the total of 28 data points in Table I.

We normalize $H(z)$ using $H_0 = 70\,\text{km}/(\text{s Mpc})$, obtaining observational data points for $E(z)$. We then use the GP method to reconstruct $E(z)$. Note that H_0 is merely a normalization factor whose value does not influence our null test in Eq. (5).

To reconstruct $D(z)$, we use the SNeIa Union 2.1 data sets [31], which contain 580 SNeIa data points. We transform the distance modulus $m-M$ given in the data set to D using

$$m - M + 5\log_{10}\left(\frac{c}{H_0}\right) - 25 = 5\log_{10}[(1+z)D].$$

For consistency, we also use $H_0 = 70\,\mathrm{km/(s\ Mpc)}$ to normalize $d_L(z)$. After obtaining these 580 observational data points for $D(z)$, we apply the GP method to reconstruct $D(z)$ and its derivative $D'(z)$. Finally, we combine the reconstructions of $E(z)$ and $D'(z)$ and apply them to the null test of $\mathcal{O}_K(z)$ in Eq. (5). We again stress that the null test is model-independent, so we need not assume any cosmological model, and the two data sets for $H(z)$ and supernovae are independent of each other.

C. Null Test

Having obtained 28 data points for $E(z)$ and 580 points for $D(z)$, we now use the GP method to reconstruct them separately.

[Figure 1: see original paper] shows the Gaussian process reconstruction of $E(z)$ (left) from CC+BAO, $D(z)$ (middle), and $D'(z)$ (right) from Union 2.1. The shaded blue regions represent the 68% and 95% confidence levels (C.L.) of the reconstruction. The flat ΛCDM model (red line) with $\Omega_{m0} = 0.27$ is also shown for comparison. As expected, at higher redshifts, the errors become large due to poor data quality in that region.

Using the reconstructions of $E(z)$ and $D'(z)$, we apply Monte Carlo sampling to determine $\mathcal{O}_K(z)$ in Eq. (5) at each redshift point we wish to reconstruct. [Figure 2: see original paper] shows that the reconstructed $\mathcal{O}_K(z)$ is consistent with vanishing cosmic curvature, falling within the 1 limit. This indicates no significant signal of deviation of the cosmic curvature $\Omega_K^{(0)}$ from zero at the current observational data level [using $H(z)$ and supernovae]. Additionally, we note that the mean value of the cosmic curvature is negative in the high-redshift region, consistent with results from model-dependent constraints in the literature.

in [33] provides the errors of D, σ_D, and the corresponding numbers of SNeIa for each redshift bin.

D. Mock Data

As seen in [Figure 1: see original paper], all reconstructions of $E(z)$, $D(z)$, and $D'(z)$ are consistent with the flat ΛCDM model, which we use for comparison. To demonstrate how a larger quantity of data with varying accuracy and errors affects our null test when reconstructing $E(z)$ and $D(z)$, we first simulate a data set of 128 points for $E(z)$. Following the methodology in [32], we draw errors from a Gaussian distribution: $\sigma_E \sim \mathcal{N}(\bar{\sigma}, \epsilon)$ with $\bar{\sigma} = (\sigma_+ + \sigma_-)/2$ and $\epsilon = (\sigma_+ - \sigma_-)/4$, where σ_+ and σ_- are the two straight lines that bound the uncertainties $\sigma(z)$ of the observational $E(z)$ data from above and below, respectively. Then $E(z)_{\rm sim}$ is sampled from the Gaussian distribution $E(z)_{\rm sim} \sim \mathcal{N}(E(z)_{\rm fid}, \sigma_E)$, where $E(z)_{\rm fid}$ is the theoretical value from the fiducial model.

For simulated $D(z)$ data, we create mock data sets of future SNeIa according to the Dark Energy Survey (DES) [33]. DES is expected to obtain high-quality light curves for approximately 4000 SNeIas from $z = 0.05$ to $z = 1.2$. At each redshift point z, $D(z)_{\rm sim}$ is sampled from the normal distribution $D(z)_{\rm sim} \sim \mathcal{N}(D(z)_{\rm fid}, \sigma_D)$.

After obtaining the simulated data sets for $E(z)$ and $D(z)$, we use the GP method to reconstruct $E(z)$, $D(z)$, and $D'(z)$. We then combine the reconstructions of $E(z)$ and $D'(z)$, using Monte Carlo sampling to determine $\mathcal{O}_K(z)$ and check whether GP can recover its theoretical value and distinguish it from $\Omega_K^{(0)} = 0$.

We simulate data points for three different concordance models: a ΛCDM fiducial model with $\Omega_K^{(0)} = 0$ and $\Omega_m = 0.3$, and two models with nonvanishing cosmic curvature, $\Omega_K^{(0)} = \pm 0.16$ and $\Omega_m = 0.3$. We test whether the GP method can detect or recover all three models and distinguish them from each other. The results are shown in [Figure 3: see original paper]–[Figure 5: see original paper].

We see from [Figure 3: see original paper] that $E(z)$, $D(z)$, and $D'(z)$ are all reconstructed very well from the mock data sets assuming the concordance model $\Omega_K^{(0)} = 0$. The reconstructed $\mathcal{O}_K(z)$ is also consistent with the concordance model. As expected, at higher redshifts, errors become large due to poor data quality in that region.

Furthermore, [Figure 4: see original paper] and [Figure 5: see original paper] show that the reconstructions for models with $\Omega_K^{(0)} = \pm 0.16$ also recover the fiducial model very well, falling within the 1 limit and obviously deviating from the concordance model $\Omega_K^{(0)} = 0$ at 95% C.L. The large errors and less precise reconstructions at high redshifts ($z > 1.0$) are due to poor data quality in that region, as there are no simulated $D(z)$ data at $z > 1.2$. Nevertheless, we can claim that with the current quantity and errors of observational data for $E(z)$ and $D(z)$, the GP method can at least detect cosmic curvature of order $\Omega_K^{(0)} \geq 0.16$ at 2 and rule out $\Omega_K^{(0)} = 0$.

However, as noted, the theoretical value of $\mathcal{O}_K(z)$ is always zero at $z = 0$. Therefore, the theoretical values of $\mathcal{O}_K(z)$ for models with different $\Omega_K^{(0)}$ deviate only slightly from the model with $\Omega_K^{(0)} = 0$ at low redshifts. As z increases, the difference becomes larger. Consequently, we see from [Figure 4: see original paper] and [Figure 5: see original paper] that it is very difficult to rule out $\Omega_K^{(0)} = 0$ at low redshifts. It may be possible at intermediate redshifts ($0.6 < z < 1.0$) as these figures show, but this is not always effective when $\Omega_K^{(0)}$ is smaller. Therefore, to detect a model with $\Omega_K^{(0)} < 0.16$ or even smaller and rule out $\Omega_K^{(0)} = 0$ at 95% C.L., larger and higher-quality data sets are required.

[Figure 6: see original paper] shows the reconstructed $\mathcal{O}_K(z)$ using mock data sets for $E(z)$ and $D(z)$ but with errors reduced to a quarter of the original mock data errors. The reconstructions are more precise, and the null test is more accurate; it can detect a model with $\Omega_K^{(0)} = 0.05$ or even smaller, ruling out $\Omega_K^{(0)} = 0$ at 95% C.L.

IV. Discussions and Conclusions

In this paper, we have introduced a nonparametric approach to performing a null test of cosmic curvature. Using the Gaussian process method, we reconstructed the Hubble rate and distance-redshift relation [$E(z)$, $D(z)$, and $D'(z)$] independently. In the reconstruction, we need not assume any cosmological model. By combining the reconstructions of $E(z)$ and $D'(z)$, we can determine $\mathcal{O}_K(z)$, which is related to the cosmic curvature $\Omega_K^{(0)}$. We have shown that $\Omega_K^{(0)} = 0$ is consistent with current data sets (CC+BAO, Union 2.1), falling within the 1 limit, although the mean of the reconstructed curvature $\mathcal{O}_K$ is negative in the high-redshift region, consistent with results from model-dependent constraints in the literature. Additionally, note that the mean of the reconstructed curvature $\mathcal{O}_K$ in [Figure 3: see original paper] is positive in the high-redshift region for the fiducial flat ΛCDM model. In this sense, our result shown in [Figure 2: see original paper] indicates a slight possibility for a closed universe.

To demonstrate how larger data sets with different accuracies and errors affect our null test, we created mock data sets for $E(z)$ and $D(z)$ using the methodology proposed in Refs. [32] and [33]. We found that with the current quality of simulated data, the GP method can recover and distinguish models with cosmic curvature of order 10^{-1} from $\Omega_K^{(0)} = 0$. However, to detect even smaller $\Omega_K^{(0)}$, we must decrease the uncertainties of the data sets. Reducing the errors to a quarter of the mock data errors allows the method to rule out $\Omega_K^{(0)} = 0$ for cosmic curvature of order 10^{-2} at 95% C.L.

Based on a one-parameter extension to the six-parameter ΛCDM model, CMB data [9, 34, 35] provide strong constraints on $\Omega_K^{(0)}$. This constraint can be improved dramatically by adding BAO data that break the geometric degeneracy between $\Omega_K^{(0)}$ and H_0. However, model assumptions and priors on model parameters may bias estimates of cosmic curvature. Here, we propose a model-independent method for reconstructing the function $\mathcal{O}_K(z)$ with redshift, which characterizes deviations from a flat universe. As seen from Eq. (5), $\mathcal{O}_K(z)$ is completely and directly determined by the Hubble parameter and luminosity distance. Once we reconstruct these quantities, the reconstruction of cosmic curvature is straightforward. GP is suitable for our purpose because it can smooth data and reconstruct functions model-independently. Therefore, we can test cosmic curvature without assuming models or imposing priors. Although current low-redshift data place weaker constraints on cosmic curvature than

CMB data, this method provides a new cross-check of cosmic curvature using future data from large-scale structure measurements.

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