

# Lorentz invariance violation in the neutrino sector: a joint analysis from big bang nucleosynthesis and the cosmic microwave background Post-print

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## Abstract

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## Full Text

## Preamble

### Lorentz Invariance Violation in the Neutrino Sector: A Joint Analysis from Big Bang Nucleosynthesis and the Cosmic Microwave Background

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We investigate constraints on Lorentz invariance violation in the neutrino sector from a joint analysis of big bang nucleosynthesis and the cosmic microwave background. The effect of Lorentz invariance violation during the epoch of big bang nucleosynthesis changes the predicted helium-4 abundance, which influences the power spectrum of the cosmic microwave background at the recombination epoch. In combination with the latest measurement of the primordial helium-4 abundance, the Planck 2015 data of the cosmic microwave background anisotropies give a strong constraint on the deformation parameter since adding the primordial helium measurement breaks the degeneracy between the deformation parameter and the physical dark matter density.

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## Introduction

Neutrino oscillations have shown that there are small but nonzero mass squared differences between three neutrino mass eigenstates (see Ref. [1] and references therein), which imply the existence of physics beyond the standard model of particle physics. However, neutrino oscillation experiments cannot tell us the overall mass scale of neutrinos. Fortunately, cosmology provides a promising way to determine or constrain the total mass of neutrinos through the gravitational effects of massive neutrinos, which can significantly change the CMB power spectrum due to lensing [2–5] and the formation of large-scale structures [6, 7], thereby altering the cosmic microwave background (CMB) anisotropies and the large-scale structure distribution of matter (see [8] for a review). Recent Planck 2015 data combined with some low-redshift data give an upper limit on the total mass of neutrinos at 95% confidence level,  $\Sigma m < 0.19$  eV [5].

Another possible signal of new physics is Lorentz invariance violation in the neutrino sector. The observed neutrino oscillations may originate from a combination of effects involving neutrino masses and Lorentz invariance violation [9–14]. Lorentz invariance is a fundamental symmetry in the standard model of particle physics. Although present experiments have confirmed Lorentz invariance to good precision [15], it may be broken in the early Universe when energies approach the Planck scale. The standard model itself is believed to be a low-energy effective theory of an underlying unified theory. Lorentz invariance violation has been explored in quantum gravity [16], loop quantum gravity [17], non-commutative field theory [18], and doubly special relativity theory [19]. Various searches for Lorentz invariance violation have been performed with a wide range of systems [20, 21].

The cosmological consequences of neutrinos with the Coleman-Glashow type [10] dispersion relation have been studied in Ref. [22]. As expected, the Lorentz-violating term affects not only the evolution of the cosmological background but also the behavior of neutrino perturbations. The former changes the expansion rate prior to and during the epoch of photon-baryon decoupling, which alters the heights of the first and second peaks of the CMB temperature power spectrum,

while the latter alters the shape of the CMB power spectrum by changing neutrino propagation. Since these two effects can be distinguished from a change in the total mass of neutrinos or in the effective number of extra relativistic species, CMB data have been proposed as a probe of Lorentz invariance violation in the neutrino sector.

Moreover, since the Lorentz-violating term influences the abundances of light elements by altering the energy density of the Universe and weak reaction rates prior to and during the big bang nucleosynthesis (BBN) epoch, BBN provides a promising probe of Lorentz invariance violation in the neutrino sector in the early Universe [23]. In particular, the BBN-predicted abundance of helium-4 is very sensitive to the Lorentz-violating term. It is well known that the primordial helium-4 abundance plays a non-negligible role at the recombination epoch because it influences the number density of free electrons. Therefore, Lorentz invariance violation affects the CMB power spectrum by changing the BBN-predicted abundance of helium-4. A reasonable way to test Lorentz invariance violation in the neutrino sector with CMB data is to take into account the BBN-predicted helium-4 abundance as a prior rather than fixing the helium-4 abundance.

In this paper, we investigate constraints on Lorentz invariance violation in the neutrino sector from a joint analysis of BBN and CMB. In combination with the latest measurement of the primordial helium-4 abundance, the Planck 2015 data of the CMB anisotropies give a strong constraint on the deformation parameter. Adding the primordial helium measurement can effectively break the degeneracy between the deformation parameter and the physical dark matter density.

This paper is organized as follows. In Sect. 2, we parameterize Lorentz invariance violation in the neutrino sector. In Sect. 3, we derive the weak reaction rates in the Lorentz-violating extension of the standard model and calculate the BBN prediction of the helium-4 abundance. In Sect. 4, we derive the Boltzmann equation for neutrinos in the synchronous gauge and calculate the CMB power spectrum. In Sect. 5, we place constraints on the deformation parameter using the Planck 2015 data in combination with the latest measurement of the primordial helium-4 abundance. Sect. 6 is devoted to conclusions.

## 2. Deformed Dispersion Relation

At a phenomenological level, the deformed dispersion relation can be characterized by a power series of momentum. In this paper, we consider a simple instance of the dispersion relation for neutrinos constructed in [10], which can be parameterized as follows:

$$E^2 = m^2 + p^2 + \xi p^2,$$

where  $E$  is the neutrino energy,  $m$  the neutrino mass,  $p = \sqrt{p_i p^i}$  the magnitude of the 3-momentum, and  $\xi$  the deformation parameter characterizing the Lorentz

symmetry violation.

In the spatially flat Friedmann-Lemaître-Robertson-Walker metric with scale factor  $a$ , the number density  $n_\nu$ , energy density  $\rho_\nu$ , and pressure  $P_\nu$  for neutrinos with the above dispersion relation are given by:

$$n_\nu = \int \frac{d^3q}{(2\pi)^3} f_0(q),$$

$$\rho_\nu = \int \frac{d^3q}{(2\pi)^3} \epsilon f_0(q),$$

$$P_\nu = \int \frac{d^3q}{(2\pi)^3} \frac{(1 + \xi)q^2}{3\epsilon} f_0(q).$$

The phase space distribution for neutrinos is:

$$f_0(q) = g_s [1 + \exp(\epsilon/T_0)]^{-1},$$

where  $g_s = 2$  is the number of spin degrees of freedom,  $T_0$  the neutrino temperature today,  $q$  the magnitude of the comoving 3-momentum, and  $\epsilon = \sqrt{m^2 a^2 + (1 + \xi)q^2}$  the comoving energy.

### 3. BBN Prediction

The abundances of light elements produced during the BBN epoch depend on the competition between nuclear and weak reaction rates and the expansion rate of the Universe. In the standard cosmological scenario and in the framework of the electroweak standard model, the dynamics of this phase is controlled by only one free parameter: the baryon-to-photon number density.

We consider  $N$  species of nuclides whose abundances  $X_i$  are the number densities  $n_i$  normalized with respect to the baryon number density  $n_B$ . Their evolutions are governed by the following Boltzmann equations:

$$\dot{X}_i = \sum_{j,k,l} \left( \frac{\Gamma_{kl \rightarrow ij}}{N_k! N_l!} \prod_{k,l} X_N - \frac{\Gamma_{ij \rightarrow kl}}{N_i! N_j!} \prod_{i,j} X_N \right),$$

where  $\Gamma$  denotes the reaction rate and  $N_i$  is the number of nuclide  $i$  involved in the reaction.

In the case of Lorentz invariance violation in the neutrino sector, the changes in the reaction rates of the following weak reactions must be taken into account:

$$n + e^+ \leftrightarrow p + \bar{\nu}_e,$$

$$n + \nu_e \leftrightarrow p + e^-,$$

$$n \leftrightarrow p + \bar{\nu}_e + e^-,$$

which determine the neutron-to-proton ratio when baryons become uncoupled from leptons. The abundances of light elements depend on the neutron-to-proton ratio at the onset of BBN. As an example, let us compute the reaction rate of the process  $n + e^+ \leftrightarrow p + \bar{\nu}_e$  in the Lorentz-violating extension of the standard model [20, 24]. The reaction rate is [23]:

$$\Gamma = \frac{(C_V^2 + 3C_A^2)}{4}(1 + \xi)^{-3/2}\Gamma^{(0)},$$

where  $C_V$  and  $C_A$  are the vector and axial couplings of the nucleon, and  $\Gamma^{(0)}$  is the standard reaction rate per incident nucleon derived in Ref. [25]. The first prefactor on the right-hand side arises from the neutrino propagator and the  $e\nu W$  coupling in the Lorentz-violating extension of the standard model, while the second prefactor comes from the statistical distribution for neutrinos. For the other processes in the weak reaction network, their rates can be derived by properly changing the statistical factors and the delta function determined by energy conservation for each reaction. Therefore, the corrections to the conversion rate of neutrons into protons and its inverse rate are the same as in the equation above.

To calculate the abundances of light elements produced during the BBN epoch, we modified the publicly available PArthENoPE code [26] to appropriately incorporate the Lorentz-violating term in the neutrino sector. As shown in [23], the BBN-predicted abundance of helium-4 is sensitive to the deformation parameter  $\xi$  because it is mainly determined by the neutron-to-proton ratio, which is related to the weak reaction rate, neutrino number density, and expansion rate of the Universe. In our analysis, we focus on the helium-4 abundance that influences the CMB power spectrum.

Although we use an interpolation method to obtain the value of  $Y_p$  on a grid of points in  $(\Omega_b h^2, \xi)$  space (see Sect. 5), it is possible to describe the dependencies of the BBN-predicted helium-4 abundances upon these two parameters through simple linear fits that are accurate to a few percent over their ranges of applicability. Following [27], we obtain the following linear fit for  $Y_p$  versus  $\Omega_b h^2$  and  $\xi$ :

$$Y_p^{\text{FIT}} = 0.2334 + 0.52\Omega_b h^2 + 0.258\xi,$$

which is valid over our adopted ranges in  $\Omega_b h^2$  and  $\xi$ , as shown in Fig. 1 [Figure 1: see original paper]. This fit is a very good approximation over the

adopted parameter ranges, which is useful for facilitating studies of the viability of various options for non-standard physics and cosmology prior to undertaking detailed BBN calculations.

Since the neutrino species share the temperature with photons prior to neutrino decoupling, the prediction for the effective number of neutrino species is:

$$N_{\text{eff}} = 3.046(1 + \xi)^{-3/2}.$$

The total relativistic energy density  $\rho_r$  in neutrinos is given in terms of the photon density  $\rho_\gamma$  by:

$$\rho_r = N_{\text{eff}} \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \rho_\gamma.$$

#### 4. CMB Power Spectrum

The Lorentz-violating term affects not only the evolution of the cosmological background but also the behavior of neutrino perturbations. Following Ref. [28], we derive the Boltzmann equation for the distribution function of neutrinos with the deformed dispersion relation to linear order in perturbations. In the synchronous gauge, the perturbed energy density  $\delta\rho_\nu$ , pressure  $\delta P_\nu$ , energy flux  $\theta_\nu$ , and shear stress  $\sigma_\nu$  are respectively given by:

$$\begin{aligned} \delta\rho_\nu &= \int \frac{d^3q}{(2\pi)^3} \epsilon f_0(q) \Psi_0, \\ \delta P_\nu &= \int \frac{d^3q}{(2\pi)^3} \frac{(1+\xi)q^2}{3\epsilon} f_0(q) \Psi_0, \\ (\rho_\nu + P_\nu)\theta_\nu &= \int \frac{d^3q}{(2\pi)^3} \sqrt{1+\xi} q f_0(q) \Psi_1, \\ (\rho_\nu + P_\nu)\sigma_\nu &= \int \frac{d^3q}{(2\pi)^3} \frac{2(1+\xi)q^2}{3\epsilon} f_0(q) \Psi_2, \end{aligned}$$

where  $\Psi_l$  are the expansion coefficients of the perturbed neutrino distribution function expanded in a series of Legendre polynomials:

$$\Psi(\mathbf{k}, \hat{n}, q, \tau) = \sum_{\ell=0}^{\infty} (-i)^\ell (2\ell + 1) \Psi_\ell(\mathbf{k}, q, \tau) P_\ell(\hat{k} \cdot \hat{n}),$$

with  $\tau$  being conformal time. These perturbations  $\Psi_\ell$  satisfy the following Boltzmann equations:

$$\dot{\Psi}_\ell + (1 + \xi) \frac{qk}{\epsilon} \frac{1}{2\ell + 1} [(\ell + 1)\Psi_{\ell+1} - \ell\Psi_{\ell-1}] - \frac{1}{6} \dot{h} \frac{d \ln f_0}{d \ln q} \delta_{\ell 0} - \frac{1}{2} \dot{\eta} \frac{d \ln f_0}{d \ln q} \delta_{\ell 2} = 0,$$

in the synchronous gauge, where dots denote derivatives with respect to conformal time, and  $h$  and  $\eta$  are the two scalar modes of metric perturbations in Fourier space. To avoid reflections from high- $\ell$  equations, we truncate this Boltzmann hierarchy by setting  $\Psi_\ell = 0$  for  $\ell > \ell_{\max}$  and adopting the following scheme [28]:

$$\Psi_{\ell_{\max}+1} \approx \frac{2\ell_{\max} + 1}{(1 + \xi)qk\tau} \Psi_{\ell_{\max}} - \Psi_{\ell_{\max}-1}.$$

Making the transformation  $q \rightarrow \sqrt{1 + \xi}q$ , we note that the number density, energy density, and pressure of neutrinos are proportional to  $(1 + \xi)^{-3/2}$ , which means that increasing  $\xi$  decreases these quantities. Moreover, there is a factor  $1 + \xi$  in the second term of the Boltzmann equations, which arises from neutrino propagation.

For massless neutrinos, the Boltzmann equations are simplified by setting  $\epsilon = \sqrt{1 + \xi}q$ . The number of variables can be reduced by integrating out the  $q$  dependence in the neutrino distribution function:

$$F(\mathbf{k}, \hat{n}, \tau) = \frac{\int q^2 dq \epsilon f_0(q) \Psi}{\int q^2 dq \epsilon f_0(q)} = \sum_{\ell=0}^{\infty} (-i)^\ell (2\ell + 1) F_\ell(\mathbf{k}, \tau) P_\ell(\hat{k} \cdot \hat{n}).$$

Using the orthonormality and recursion relations of Legendre polynomials, we derive from the Boltzmann equations the evolution equations for the perturbed energy density  $\delta_\nu = \delta\rho_\nu/\rho_\nu$ , energy flux  $\theta_\nu$ , and shear stress  $\sigma_\nu$  that are independent of momentum  $q$ :

$$\dot{\delta}_\nu = -\frac{4}{3} \dot{\eta} - \frac{4}{3} \sqrt{1 + \xi} \theta_\nu - \frac{2}{3} \dot{h},$$

$$\dot{\theta}_\nu = \sqrt{1 + \xi} k^2 \left( \frac{1}{4} \delta_\nu - \sigma_\nu \right),$$

$$\dot{\sigma}_\nu = \frac{4}{15} \sqrt{1 + \xi} \theta_\nu - \frac{3}{10} \sqrt{1 + \xi} k F_3 + \frac{2}{15} \dot{\eta} + \frac{2}{5} \dot{h},$$

$$\dot{F}_\ell = \frac{\sqrt{1 + \xi} k}{2\ell + 1} [\ell F_{\ell-1} - (\ell + 1) F_{\ell+1}], \quad \ell \geq 3.$$

The truncation scheme for massless neutrinos is:

$$F_{\ell_{\max}+1} \approx \frac{2\ell_{\max} + 1}{\sqrt{1 + \xi}k\tau} F_{\ell_{\max}} - F_{\ell_{\max}-1}.$$

In the synchronous gauge, the adiabatic initial conditions for metric perturbations and massless neutrinos in the cold dark matter frame are:

$$h = Ck^2\tau^2 - C\omega k^2\tau^3,$$

$$\eta = 2C - \frac{5 + 4R_\nu}{3(15 + 4R_\nu)} Ck^2\tau^2,$$

$$\delta_\nu = -\frac{2}{3} Ck^2\tau^2,$$

$$\theta_\nu = -\frac{1}{18} \frac{15 + 4R_\nu}{(1 + \xi)} Ck^4\tau^3,$$

$$\sigma_\nu = \frac{4R_\nu}{3(15 + 4R_\nu)} \frac{1}{(1 + \xi)} Ck^2\tau^2,$$

where  $C$  is a dimensionless constant determined by the amplitude of fluctuations from inflation,  $R_\nu \equiv \rho_\nu/(\rho_\gamma + \rho_\nu)$ , and  $\omega \equiv \Omega_m H_0 / \sqrt{\Omega_\gamma + \Omega_\nu}$ , which corresponds to the matter contribution to the total energy density of the Universe.

In the radiation-dominated era, massive neutrinos are relativistic. The initial condition for  $\Psi_0$  of massive neutrinos is related to  $\delta_\nu$ . Then, using the Boltzmann equations and ignoring mass terms in the differential equations for high-order moments, we obtain the initial conditions for  $\Psi_\ell$  of massive neutrinos:

$$\Psi_0 = -\frac{1}{6} \frac{d \ln f_0}{d \ln q} Ck^2\tau^2,$$

$$\Psi_1 = -\frac{\sqrt{1 + \xi}qk}{36(1 + \xi)} \frac{d \ln f_0}{d \ln q} Ck^2\tau^3,$$

$$\Psi_2 = -\frac{1}{15} \frac{d \ln f_0}{d \ln q} Ck^2\tau^2,$$

$$\Psi_\ell = 0, \quad \ell \geq 3.$$

To compute the theoretical CMB power spectrum, we modified the Boltzmann CAMB code [29] to appropriately incorporate the Lorentz-violating term in the

neutrino sector. As pointed out in [22], the effects of the Lorentz-violating term on the CMB power spectrum are distinguishable from a change in the total mass of neutrinos or in the effective number of extra relativistic species [30–34]. Therefore, measurements of CMB anisotropies provide a cosmological probe of Lorentz invariance violation in the neutrino sector. Moreover, we calculate the matter power spectrum today for  $\xi = -0.1, 0, 0.1$ . From Fig. 2 [Figure 2: see original paper], we find that increasing  $\xi$  can enhance the matter power spectrum at small scales. It is well known that variations in the helium-4 abundance modify the density of free electrons between helium and hydrogen recombination and therefore influence the CMB power spectrum. However, the effects of the Lorentz-violating term on the BBN-predicted helium-4 abundance were not considered in [22]. In the next section, we shall place constraints on the deformation parameter from a joint analysis of BBN and CMB.

## 5. Joint Analysis from BBN and CMB

In our analysis, we use a modified version of the publicly available CosmoMC package to explore the parameter space using the Monte Carlo Markov chain technique [35]. Since the deformation parameter  $\xi$  is nearly uncorrelated with the total mass of neutrinos when CMB data are used to detect signatures of Lorentz invariance violation [22], we consider only massless neutrinos with vanishing chemical potentials. Our cosmological model is the spatially flat  $\Lambda$ CDM model plus three types of massless neutrinos with the deformed dispersion relation, described by the following seven parameters:

$$\{\Omega_b h^2, \Omega_c h^2, \theta_{MC}, \tau_{re}, n_s, A_s, \xi\},$$

where  $h$  is the dimensionless Hubble parameter defined by  $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $\Omega_b h^2$  and  $\Omega_c h^2$  are the physical baryon and dark matter densities relative to the critical density,  $\theta_{MC}$  approximates the ratio of the sound horizon to the angular diameter distance at photon decoupling,  $\tau_{re}$  is the reionization optical depth, and  $n_s$  and  $A_s$  are the spectral index and amplitude of primordial curvature perturbations at the pivot scale  $k_0 = 0.05 \text{ Mpc}^{-1}$ .

We first run the PArthENoPE BBN code with two free input parameters,  $\Omega_b h^2$  and  $\xi$ , to precalculate the primordial helium-4 abundance on a grid of points in  $(\Omega_b h^2, \xi)$  space. For each parameter pair, the helium-4 abundances are obtained from the grid via two-dimensional cubic spline interpolation, which is then used to calculate the CMB power spectrum. This method is called “BBN consistency.”

We use the recently released Planck 2015 likelihood code and data, including the Planck low- $\ell$  likelihood at multipoles  $2 \leq \ell \leq 29$  and Planck high- $\ell$  likelihood at multipoles  $\ell \geq 30$  based on pseudo- $C_\ell$  estimators [5]. The former uses foreground-cleaned LFI 70 GHz polarization maps together with the temperature map obtained from Planck 30 to 353 GHz channels by the Commander component separation algorithm over 94% of the sky. The latter uses 100, 143,

and 217 GHz half-mission cross-power spectra, avoiding the galactic plane as well as the brightest point sources and regions where CO emission is strongest. “Planck 2015” denotes the combination of the low- $\ell$  temperature-polarization likelihood and the high- $\ell$  temperature likelihood.

We use the “Joint Light-curve Analysis” (JLA) sample of type Ia supernovae, constructed from the SNLS and SDSS supernova data together with several samples of low-redshift supernovae [36]. Baryon acoustic oscillation (BAO) measurements are another important astrophysical dataset that is powerful for breaking parameter degeneracies from CMB measurements. We use BAO measurements of  $D_V/r_{\text{drag}}$  from 6dFGS at  $z_{\text{eff}} = 0.106$ , SDSS Main Galaxy Sample at  $z_{\text{eff}} = 0.15$ , BOSS LOWZ at  $z_{\text{eff}} = 0.32$ , and BOSS CMASS at  $z_{\text{eff}} = 0.57$  [5]. Here  $D_V$  is the effective distance measure for angular diameter distance,  $r_{\text{drag}}$  is the comoving sound horizon at the end of the baryon drag epoch, and  $z_{\text{eff}}$  is the effective redshift. In our analysis, we also use the helium-4 mass fraction measurement  $Y_p = 0.2449 \pm 0.0040$ , derived from helium and hydrogen emission lines from metal-poor extragalactic HII regions and from a regression to zero metallicity [37].

In Table I [TABLE:I], we list our results. With the combined data of Planck 2015+JLA+BAO, the deformation parameter is estimated to be  $\xi = 0.042 \pm 0.049$  if the helium-4 abundance is set from the BBN prediction. For comparison, if the helium-4 abundance is not varied independently of other parameters, with the same dataset we find  $\xi = 0.101 \pm 0.091$ , which indicates that the constraint on  $\xi$  is not improved compared to the result derived in [22] from the 7-year WMAP data [38] in combination with lower-redshift measurements of the expansion rate. The reason is that the estimated values of cosmological parameters are biased by fixing  $Y_p = 0.24$ . The gray and red regions in Fig. 3 [Figure 3: see original paper] show 68% and 95% contours in the  $\xi$ - $\Omega_c h^2$  plane when  $Y_p = 0.24$ , derived from Planck 2015+JLA+BAO and from WMAP7+ $H_0$ +BAO [22], respectively. We see that Planck 2015 data marginally favor a positive  $\xi$  compared to the 7-year WMAP data. The addition of the latest helium-4 abundance measurement leads to a strong constraint on the deformation parameter:  $\xi = -0.002 \pm 0.014$ . As shown in Figure 3, adding the  $Y_p$  data breaks the degeneracy between the deformation parameter and the physical dark matter density. This result shows that no signal of Lorentz invariance violation is detected by the joint analysis of CMB and BBN. Figure 4 [Figure 4: see original paper] shows the constraints in the  $\xi - \Omega_b h^2$ ,  $\xi - H_0$ ,  $\xi - n_s$ , and  $\xi - A_s$  planes from Planck 2015+JLA+BAO.

## 6. Conclusions and Discussions

We have studied cosmological constraints on Lorentz invariance violation in the neutrino sector through a joint analysis of CMB and BBN. Instead of fixing the helium-4 abundance, we have applied the BBN prediction of the helium-4 abundance determined by  $\xi$  and  $\Omega_b h^2$  to calculate the CMB power spectra. Using Planck 2015 data in combination with the JLA sample of type Ia supernovae and BAO features, we have placed constraints on the deformation parameter.

Adding the measurement of helium-4 abundance breaks the degeneracy between the deformation parameter and the physical dark matter density, thereby improving the constraint by a factor of 3.

As found in [22], the deformation parameter is nearly uncorrelated with the total mass of neutrinos  $\Sigma m_\nu$  when WMAP data are used to constrain  $\xi$ . Planck 2015 data are much more sensitive to the total mass of neutrinos than WMAP data. Therefore, we consider the effects of neutrino masses on our results. Figure 5 [Figure 5: see original paper] shows 68% and 95% contours in the  $\xi - \Sigma m_\nu$  plane for Planck 2015 data combined with lower-redshift measurements. We see that  $\xi$  is nearly uncorrelated with  $\Sigma m_\nu$ .

Compared to cosmological bounds on Lorentz-violating coefficients, observations of high-energy astrophysical neutrinos give stronger constraints. As listed in Table XIII of Ref. [13], the coefficient is constrained down to  $\mathcal{O}(10^{-9})$  from time-of-flight measurements, under the assumption that neutrino oscillations are negligible. Cohen and Glashow have argued that the observation of neutrinos with energies exceeding 100 TeV and a baseline of at least 500 km allows us to deduce that the Lorentz-violating parameter is less than  $\mathcal{O}(10^{-11})$  [39]. The present work provides a new way to probe signals of Lorentz invariance violation in the early Universe, which can in principle be used to constrain  $\xi$  in the sterile neutrino sector [40, 41]. Although present cosmological data are too weak to yield competitive constraints, future measurements of CMB and BBN offer prospects for placing stringent constraints.

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