The Gravitational-Wave Physics Postprint

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Abstract

The direct detection of gravitational waves by the Laser Interferometer Gravitational-Wave Observatory heralds the era of gravitational-wave astronomy and gravitational-wave cosmology. It is expected that an increasing number of gravitational-wave events will be detected by currently existing and planned gravitational-wave detectors. Gravitational waves open a new window to explore the Universe, and various mysteries will be unveiled through gravitational-wave detection, combined with other cosmological probes. Gravitational-wave physics is not only related to gravitation theory, but also is closely tied to fundamental physics, cosmology and astrophysics. In this review article, three kinds of sources of gravitational waves and relevant physics will be discussed, namely gravitational waves produced during the inflation and preheating phases of the Universe, gravitational waves produced during the first-order phase transition as the Universe cools down, and gravitational waves from the three phases: inspiral, merger and ringdown of a compact binary system, respectively. We will also discuss the gravitational waves as a standard siren to explore the evolution of the Universe.

Full Text

The Gravitational-Wave Physics

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Abstract

The direct detection of gravitational waves by the Laser Interferometer Gravitational-Wave Observatory heralds the dawn of gravitational-wave astronomy and gravitational-wave cosmology. It is expected that increasingly more gravitational-wave events will be detected by existing and planned gravitational-wave detectors. Gravitational waves open a new window for exploring the Universe, and various cosmic mysteries may be unveiled through gravitational-wave detection combined with other cosmological probes. Gravitational-wave physics is not only related to gravitational theory but is also intimately connected to fundamental physics, cosmology, and astrophysics. In this review article, we discuss three types of gravitational-wave sources and the relevant physics: gravitational waves produced during the inflation and preheating phases of the Universe, gravitational waves produced during first-order phase transitions as the Universe cools, and gravitational waves from the three phases—inspiral, merger, and ringdown—of compact binary systems. We also discuss gravitational waves as standard sirens for probing the evolution of the Universe.

Keywords: gravitational waves, inflation, reheating, first-order phase transition, binary black holes, standard sirens

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Introduction

On 11 February 2016, the LIGO Scientific Collaboration and the Virgo Collaboration [**?**] announced that on 14 September 2015 at 09:50:45 UTC, the two detectors of the Laser Interferometer Gravitational-Wave Observatory (LIGO) simultaneously observed a transient gravitational-wave signal. This event, named GW150914, exhibited a signal frequency that increased from 35 to 250 Hz, with a peak gravitational-wave strain of 1.0×10^{-21} at 150 Hz. The signal is consistent with the predictions of general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The source is located at a luminosity distance of 410^{+160}_{-180} Mpc, corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. The initial black hole masses are $36^{+5}_{-4}M_\odot$ and $29^{+4}_{-4}M_\odot$, and the resulting black hole mass is $62^{+4}_{-4}M_\odot$, with $3.0^{+0.5}_{-0.5}M_\odot$ radiated in the form of gravitational waves. This represents the first direct detection of gravitational waves and the first observation of a binary black hole merger.

On 15 June 2016, the same team announced the second gravitational-wave event, GW151226 [**?**]. This time, the observed signal lasted approximately 1 s, with frequency increasing from 35 to 450 Hz over 55 cycles, and the peak gravitational strain reaching $3.4^{+0.7}_{-0.9} \times 10^{-22}$ at 450 Hz. The source is again the merger of two black holes with masses $14.2^{+8.3}_{-3.7} M_{\odot}$ and $7.5^{+2.3}_{-2.3} M_{\odot}$, respectively, with the final black hole having mass $20.8^{+6.1}_{-1.7} M_{\odot}$. This event occurred at a luminosity distance of 440^{+180}_{-190} Mpc, corresponding to a redshift of $0.09^{+0.03}_{-0.04}$.

Gravitational waves were predicted by Albert Einstein in 1916 [**?**, **?**], one year after he finally formulated his theory of gravitation—general relativity. However, the physical reality of gravitational wave solutions to the Einstein field equations was not established until the Chapel Hill conference in 1957 [**?**]. In [**?**, **?**], it was shown that gravitational waves carry energy and, when passing through spacetime in the form of a sandwich wave, affect test particles. More than a century has passed since Einstein' s proposal of general relativity. Although it has passed various precise tests, some alternative theories survive, such as scalar-tensor gravity, $f(R)$ gravity, and modified gravity with higher curvature terms. On the other hand, based on general relativity, we now have a standard model of cosmology—the ΛCDM model—which is quite consistent with various astronomical observations made so far. We now understand well that gravitational waves exist not only in general relativity but also in other relativistic covariant gravity theories. Due to limited space, this review is confined to gravitational waves in general relativity.

Gravitational-wave sources can be roughly classified into two categories: cosmological origin and relativistic astrophysical origin. In the cosmological case, gravitational waves can be produced in the early stages of the Universe, for example, during the inflation and reheating epochs. Such waves are called primordial gravitational waves and will leave a unique imprint on the cosmic microwave background (CMB)—the so-called B-mode polarization. On the other hand, during the evolution of the Universe, various phase transitions are expected to have occurred as the temperature decreased, such as symmetry breaking in grand unified theories, electroweak phase transitions, and quantum chromodynamics phase transitions. Gravitational waves with different features are also expected to be produced during these transitions. Additionally, interactions of topological defects such as cosmic strings and domain walls generated during phase transitions will create gravitational waves. Therefore, detecting gravitational waves from these cosmological origins can reveal physics associated with the evolution of the Universe. On the astrophysics side, gravitational waves can be produced in various processes, such as the rotation of non-symmetric neutron stars, supernova explosions, and the inspiral, merger, and ringdown of compact binaries including white dwarfs, neutron stars, and/or black holes. In particular, compact binary systems are the main sources for ground-based gravitational-wave experiments such as LIGO, Virgo, KAGRA, and the Einstein Telescope, as well as space-based experiments such as the Laser Interferometer Space Antenna (LISA), Deci-Hertz Interferometer Gravitational wave Observatory (DECIGO), Big Bang Observer (BBO), Taiji, and Tianqin.

Thus, gravitational-wave physics is closely related to fundamental physics, cosmology, and astrophysics. The direct detection of gravitational waves [**?**, **?**] heralds the era of gravitational-wave astronomy and gravitational-wave cosmology. Gravitational-wave detection opens a new window to explore the Universe. By combining electromagnetic radiation, neutrinos, cosmic rays, and gravitational waves, we can expect to reveal various mysteries concerning the early evolution of the Universe, the properties of dark matter, and the nature of dark energy.

In this brief review, we summarize some important aspects relevant to gravitational-wave physics. The outline is as follows. Section 2 introduces gravitational waves produced in the primordial Universe and their detection through the CMB. We emphasize the properties of gravitational waves created during inflation and preheating processes and the current status of primordial gravitational-wave detection. Section 3 discusses gravitational waves produced during cosmic phase transitions, including bubble nucleation, expansion, and percolation. During strong first-order phase transitions, bubble collisions, turbulent magnetohydrodynamics (MHD), and sound waves are all sources of gravitational waves. Section 4 is devoted to gravitational waves from the dynamics of compact binary systems, introducing three main methods for solving binary systems: the post-Newtonian (PN) approximation, numerical relativity, and black hole perturbation theory, corresponding to the inspiral, merger, and ringdown phases of a binary black hole system, respectively. We also discuss the possibility of using gravitational waves from binary systems as a cosmological probe, which is expected to provide strong constraints on cosmological parameters when combined with other probes.

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2 Gravitational Waves From the Primordial Universe

To solve problems in big-bang cosmology such as the horizon and flatness problems, the inflationary scenario was introduced \cite{8-10}, in which a period of accelerated expansion occurred at early times. Inflation not only predicts primordial scalar perturbations, which provide a natural mechanism for generating the anisotropies of CMB radiation and the initial tiny seeds of the large-scale structure observed today, but also generates a stochastic background of primordial gravitational waves. Although such a stochastic background has not yet been observed, its detection would open a new window for understanding the physics of the early Universe and its origin and evolution. In this section, we first review the properties of primordial gravitational waves produced during inflation and preheating (see Fig. 1 [Figure 1: see original paper]), and then discuss observational implications.

Gravitational waves are described by a transverse-traceless gauge-invariant tensor perturbation h_{ij} in a Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right], \tag{2.1}$$

where τ is conformal time, a is the scale factor, and h_{ij} satisfies $\partial_i h_{ij} = 0$ and $\delta^{ij} h_{ij} = 0$. To first order in h_{ij}, the perturbed Einstein equation reads

$$h''_{ij} + 2\mathcal{H} h'_{ij} - \nabla^2 h_{ij} = \frac{2}{M_{\rm pl}^2} \Pi_{ij}^{\rm TT}, \tag{2.2}$$

where the prime denotes derivative with respect to τ, $\mathcal{H} \equiv a'/a$ is the Hubble parameter in τ, $M_{\rm pl} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass, and the source term $\Pi_{ij}^{\rm TT}$ is the transverse-traceless projection of the anisotropic stress tensor T_{ij}. Since h_{ij} is symmetric, transverse, and trace-free, tensor modes have two physical degrees of freedom, which are expanded in Fourier space as

$$h_{ij}(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \left[h_k^+(\tau) e_{ij}^+(\hat{\mathbf{k}}) + h_k^\times(\tau) e_{ij}^\times(\hat{\mathbf{k}}) \right], \tag{2.3}$$

where $e_{ij}^{+,\times}$ are the polarization tensors with two polarization states $(+, \times)$ of gravitational waves. We define the power spectrum of gravitational waves as

$$\mathcal{P}_T(\tau, k) = \frac{2\pi^2}{k^3} \left(|h_k^+|^2 + |h_k^\times|^2 \right), \tag{2.4}$$

and the energy spectrum as $\Omega_{\rm gw}(\tau, k) \equiv d\rho_{\rm gw}/d\ln k/\rho_c$, where

$$\rho_{\rm gw}(\tau, k) = \frac{1}{4a^2 M_{\rm pl}^2} \langle h'_{ij} h'^{ij} \rangle \tag{2.5}$$

is the energy density and $\rho_c = 3H^2 M_{\rm pl}^2$ is the critical density of the Universe.

2.1 During Inflation

In the standard single-field slow-roll inflationary scenario, at first order in perturbation theory Eq. (2.2) reduces to a free wave equation. This can be solved analytically in the slow-roll approximation. The Bunch-Davies vacuum condition is imposed in the asymptotic past because the modes lie well inside the Hubble radius. Of course, if a general vacuum condition is imposed, additional features appear in the power spectrum. During inflation, quantum fluctuations are amplified and stretched, then nearly frozen on super-Hubble scales. Single-field slow-roll inflation predicts a slightly red-tilted spectrum:

$$\mathcal{P}_T(k) = \left(\frac{H}{\pi M_{\rm pl}} \right)^2, \tag{2.6}$$

and a consistency relation $n_T = -r/8$ between the tensor spectral index n_T and the tensor-to-scalar ratio r. Here, n_T and r are evaluated when a given perturbation mode leaves the Hubble horizon, i.e., $k = aH$. We define the tensor-to-scalar ratio $r \equiv A_T/A_S$ at a pivot scale k_*, representing the amplitude of tensor perturbations A_T relative to that of scalar perturbations A_S.

The red-tilted spectrum means that the amplitude of tensor perturbations becomes small on small scales because large-scale modes are stretched across the Hubble horizon earlier than small-scale modes as the energy density slowly decreases during inflation. From the wave equation we see that the evolution of h_{ij} depends explicitly on $a(\tau)$ and implicitly on the inflationary potential through the FRW equations. Hence, the power spectrum of gravitational waves encodes useful information about the evolution of the scale factor. Moreover, the tensor-to-scalar ratio is related to the energy scale of inflation by $V = 3\pi^2 M_{\rm pl}^4 A_s r/2 = (1.88 \times 10^{16}\,\mathrm{GeV})^4 r/0.10$, where we have adopted the estimated value of the amplitude of scalar perturbations from Planck 2015 data [?]. Different models predict different values of the tensor-to-scalar ratio. For example, the simplest chaotic inflation with a quartic potential [?] predicts a large value $r \approx 0.26$, while R^2 inflation [?] predicts a small value $r \approx 0.0033$ to lowest order in slow-roll parameters assuming $N_* = 60$ e-folds. With the help of the scalar spectral index, the estimated value of r robustly discriminates slow-roll inflationary models. In summary, measuring the power spectrum of gravitational waves helps us to: - test the vacuum initial condition, - detect the evolution of the scale factor, - determine the energy scale of inflation, - discriminate inflationary models.

If primordial gravitational waves are detected, the next important question concerns the shape of their power spectrum and whether additional features exist. In the slow-roll inflationary scenario, the shape of the tensor power spectrum is characterized by the tensor spectral index n_T since the running of the spectral index is negligible to lowest order in slow-roll parameters. More general shapes beyond slow-roll may be reconstructed using a binning method with cubic spline interpolation in logarithmic wavenumber space \cite{13-15}. Checking the consistency relation between n_T and r provides a powerful test of the single-field slow-roll inflationary scenario. Violation of the consistency relation could arise from two aspects.

First, the second and third terms on the left-hand side of Eq. (2.2) are modified in general inflationary models, such as k-inflation [?], Gauss-Bonnet inflation \cite{17-19}, and generalized G-inflation [?]. In k-inflation, the consistency relation becomes $n_T = -r/8c_S$, where c_S is the sound speed of scalar perturbations. In Gauss-Bonnet inflation, the consistency relation is broken because $n_T = -r/8 - \delta_1$, where δ_1 is determined by the Gauss-Bonnet coupling term. In generalized G-inflation, the tensor spectral index depends not only on the evolution of the scale factor but also on higher-order derivative terms, which admits a blue-tilted power spectrum of gravitational waves. Second, a non-negligible source term exists on the right-hand side of Eq. (2.2) during inflation. In this case, the wave equation is solved using the Green' s function method. Possible sources for generating gravitational waves include first-order scalar perturbations [?], perturbations of extra fields such as the curvaton [?] and spectator fields [?, ?], and particle production during inflation [?].

2.2 During Preheating

In the inflationary scenario, at the end of inflation the inflaton field begins to oscillate around the minimum of its potential. These coherent oscillations produce elementary particles and eventually reheat the Universe, a process called reheating [?]. Preheating provides a more rapid and efficient mechanism for extracting energy from the inflaton field through parametric resonance [?]. This process is so rapid that the produced particles are not in thermal equilibrium. Preheating leads to large, time-dependent inhomogeneities of the stress tensor that source a stochastic background of gravitational waves [?]. Unlike gravitational waves produced during inflation, they are generated and remain inside the Hubble horizon until today, so their wavelengths are smaller than the Hubble radius at production. Consequently, the peak frequency of this stochastic background is typically of order 10^3 Hz or higher, making detection particularly challenging. For example, for ϕ^4 and ϕ^2 chaotic inflationary models, lattice simulations [?] show that preheating can produce gravitational waves with frequencies around $10^6 \sim 10^8$ Hz and peak power $\Omega_{\rm gw}h^2 \approx 10^{-9} \sim 10^{-11}$ today [?]. For hybrid inflation, gravitational waves cover a larger frequency range, with the peak wavelength depending essentially on the coupling constant [?]. See [?] for a recent review on gravitational waves from the inflationary era and preheating.

2.3 Observational Implications

Observational constraints on gravitational waves produced during inflation mainly come from the B-mode polarization of CMB anisotropies. Such gravitational waves generate a quadrupolar anisotropy with momentum $m = 2$ in the photon intensity field at recombination, while scalar perturbations generate only a quadrupolar anisotropy with $m = 0$. Importantly, only the quadrupolar anisotropy with $m = 2$ causes B-mode polarization. Therefore, measuring B-mode polarization allows us to probe gravitational waves from inflation. Planck 2015 data give a 95% confidence level upper bound $r_{0.002} < 0.10$ at pivot scale $k_* = 0.002\,\mathrm{Mpc}^{-1}$ [?], improved to $r_{0.002} < 0.08$ by adding the BKP cross-correlation likelihood [?]. With the scalar spectral index n_S, slow-roll inflationary models are discriminated in the $r - n_S$ plane, as shown in Fig. 2 [Figure 2: see original paper]. For example, the inflationary model with a quartic potential [?] is strongly disfavored by recent CMB data. Future ground-based and space-based CMB experiments will provide high-precision measurements of B-mode polarization. However, constraints from CMB anisotropies are limited to a narrow scale range of $10^{-4} \sim 10^{-1}\,\mathrm{Mpc}^{-1}$, corresponding to $\Delta N \approx 8$ e-folds, making it impossible to detect the global shape of the tensor spectrum using only CMB data. Laser interferometer experiments provide the possibility of measuring the stochastic background at small scales.

During inflation, quantum fluctuations of h_{ij} are amplified and stretched across the Hubble horizon, then nearly frozen on super-Hubble scales. After re-entering the Hubble horizon during radiation or matter domination, tensor perturbations begin to oscillate with amplitude damped by a factor a^{-1}. Unfortunately, for the nearly scale-invariant tensor spectrum predicted by single-field slow-roll inflation, the energy spectrum in the frequency range $10^{-10} \sim 10^{3}$ Hz today becomes too weak for direct detection by pulsar timing array experiments or laser interferometers [?]. If a blue-tilted tensor spectrum is predicted by inflationary models beyond slow-roll or caused by source terms, direct detection might become possible in the future [?].

For gravitational waves produced during preheating, since their wavelengths are smaller than the Hubble radius at production, the corresponding peak frequencies are typically of order 10^{3} Hz or higher, making detection by ground-based laser interferometer experiments particularly challenging.

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3 Gravitational Waves From Phase Transitions

It is believed that our observable Universe has experienced several phase transitions (PTs), during which an unknown high-degree symmetry is subsequently relaxed down to the broken electroweak symmetry described by the Standard Model of particle physics. If the transition from a high-temperature symmetric phase to a low-temperature symmetry-broken phase is first-order, true vacuum bubbles nucleate within the false vacuum, leading to expanding, colliding, and merging bubbles that generate a stochastic background of gravitational waves (see [?] for a recent review and [?] for discussions of gravitational waves from cosmic strings and domain walls, which are not covered here). The primary motivations for studying gravitational waves from phase transitions are twofold: First, the electroweak phase transition of the Standard Model is a crossover according to current Higgs mass measurements. Therefore, any detection of gravitational waves from phase transitions would necessarily provide a unique probe beyond the Standard Model that cannot be directly probed by particle colliders in the foreseeable future. Second, stochastic gravitational-wave backgrounds also include contributions from the inflationary and reheating eras and other cosmological defects from phase transitions like cosmic strings and domain walls, so studying them helps extract astrophysical gravitational-wave signals from the stochastic background. However, detection topics are not discussed in this review, which would require a separate paper on detecting stochastic backgrounds and distinguishing phase transition signals from reheating or other cosmological sources.

3.1 Bubble Nucleation

In seminal papers [?, ?], the first semiclassical description of vacuum decay in flat spacetime was developed for a single scalar field without derivative interactions. Vacuum decay occurs through barrier penetration from the unstable false vacuum to the stable true vacuum, with the field configuration captured by the bounce equation. The bounce equation describes true vacuum bubbles nucleated during barrier penetration in the surrounding false vacuum with probability per unit time per unit volume $\Gamma = A\exp(-B/\hbar)[1 + O(\hbar)]$. The coefficient B was derived in [?] as the on-shell Euclidean action of the bounce solution, while A was derived in [?] to account for quantum corrections. In the special case where the potential barrier is larger than the energy density difference between false and true vacua, a thin-wall approximation was proposed in [?] to evaluate the bounce action in closed form, including contributions from both the bubble interior and wall. This approximation provides a physical picture where released energy from the spreading true vacuum accelerates the bubble wall until a zero-energy critical bubble forms. The size of such a critical bubble is determined by the stationary point of the Euclidean action, giving rough cancellation between interior and wall energies.

The seminal works [?, ?] were later extended to vacuum decay in curved spacetime [?] and thermal decay in flat spacetime [?]. Although gravity does not change the general picture, it lowers the vacuum decay probability by nucleating larger bubbles. In the extreme case of nearly degenerate vacua, gravity can even stabilize the false vacuum by preventing the formation of zero-energy bubbles that would be possible in its absence by simply expanding the interior to balance surface tension. At finite temperature, instead of an $O(4)$-symmetric bubble in Euclidean spacetime, an $O(3)$-symmetric bubble solution periodic in the time direction with period T^{-1} is expected, with critical bubble size determined by the maximum point of total energy. The tunneling rate was estimated in [?, ?] as

$$\Gamma(T) \simeq T^4 \left(\frac{S_3[\phi_B(r), T]}{2\pi}\right)^{3/2} \exp\left(-\frac{S_3[\phi_B(r), T]}{T}\right), \tag{3.1}$$

where the Euclidean action

$$S_3[\phi(r), T] = 4\pi \int_0^\infty dr\, r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi, T)\right] \tag{3.2}$$

is evaluated at the bounce profile $\phi_B(r)$ of the equation of motion,

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{\partial V}{\partial \phi}. \tag{3.3}$$

The effective potential usually consists of tree-level potential, zero-temperature corrections, and finite-temperature corrections including daisy resummation. Recently, a new thermal resummation procedure was proposed in [?], enabling matching of theories to an effective field theory at finite temperature. Future work can proceed along this direction.

The naive strategy for solving the bounce equation is the shooting algorithm, illustrated in Fig. 3 [Figure 3: see original paper]. When multiple fields are involved, four approaches have been introduced in the literature: the early trial method [?], the undamping/damping algorithm [?], the widely adopted path deformation algorithm [?], and the recently proposed semi-analytic perturbative approach [?].

The generalization of [?] to non-zero cosmological constant allowing both false and true vacua was carried out in [?]. Vacuum decay is enhanced by gravity when the average of the two vacua is positive and suppressed when both vacua are negative. Decay for a positive false vacuum but negative average is enhanced or suppressed depending on whether gravitational effects become important. Currently, there is no satisfactory description of thermal decay in curved spacetime; however, we will clarify in future work that gravity corrections scale as $\kappa/(TR^3)$ and are unimportant because the characteristic bubble size R is much larger than the Planck length for phase transition scales below the Planck scale.

3.2 Bubble Expansion

After bubble nucleation within the false vacuum, the bubble wall expands rapidly until approaching the speed of light [?] or colliding with other walls. However, the realistic background is a thermal plasma full of relativistic particles. In this review, we focus on gravitational waves from phase transitions in the radiation-dominated era; for inflationary and matter-dominated eras, see \cite{49-51} and [?], respectively, and [?] for non-standard cosmology with additional components redshifting faster than radiation.

The central problem of bubble expansion in plasma is to determine the interconnections among: the bubble wall velocity v_w, friction η on the wall from the plasma, the phase transition strength α measuring released vacuum energy density relative to radiation energy density, and efficiency factors κ_ϕ and κ_v measuring the capability of transferring liberated vacuum energy into bubble wall expansion and bulk fluid motion, respectively. These quantities serve as the interface between particle physics models and bubble collision simulations.

After nucleation in thermal plasma, bubble growth stops when friction balances the net pressure across the wall. If external pressure exceeds internal pressure while external fluid velocity is smaller than internal velocity, the bubble wall behaves as a deflagration; the opposite defines a detonation. Both deflagrations and detonations are further classified as weak, Jouguet, and strong types according to whether the internal fluid velocity is smaller than, equal to, or larger than the speed of sound. In [?], it was proposed that with the Jouguet condition, the bubble wall velocity v_w is given by a simple formula expressed in terms of α alone, as is the efficiency factor κ_v. The Jouguet detonation formula was extensively used in early studies of gravitational waves from phase transitions, despite the fact [?] that Jouguet detonations can be unrealistic in cosmological phase transitions. Since [?], several parallel explorations have been undertaken:

- **Beyond Chapman-Jouguet condition**: Generalizing the model-independent parametrization of friction from [?], [?] explored the full range of bubble wall velocities for both deflagrations and detonations, finding analytic approximations for non-relativistic and ultra-relativistic cases. The state-of-the-art work [?] provides a unified picture with user-friendly fitting formulas for bubble expansion dynamics. A different but more accurate approach was adopted in [?] with microscopic considerations of particle content in specific models, revisited and improved in [?].

- **Criteria for runaway bubble walls**: Apart from stationary solutions with terminal velocity, runaway solutions exist when friction is too small to prevent the wall from approaching light speed. A simple criterion was found in [?]: bubble walls run away if the effective potential in the true vacuum remains deeper than in the false vacuum even after replacing the thermal potential by its second-order Taylor expansion term in the false vacuum. This was reformulated in [?] by comparing α to a critical value α_∞. Combined hydrodynamic and microscopic considerations were later extended to the runaway regime in [?, ?].
- **Reconciliation with baryogenesis**: Large gravitational-wave signals from phase transitions require fast-moving bubble walls, while baryogenesis scenarios need slow walls for effective diffusion. Reference [?] opened the possibility of reconciling baryogenesis with gravitational waves in the presence of fermions with large Yukawa coupling and heavier stabilizing bosons. Later, [?] pointed out that the relevant velocity for baryogenesis is the relative velocity between the wall and plasma, which can be much smaller than the wall velocity when the phase transition strength increases. Thus, large gravitational waves can be generated without jeopardizing baryogenesis.

3.3 Bubble Percolation

After bubble nucleation and expansion, percolation begins with colliding bubbles until the phase transition completes. When the initial nucleated bubble size is negligible, the transition duration can be estimated by the mean bubble radius at collision, characterized by a single parameter β. Both β and α are evaluated at the nucleation temperature T_*, defined as the temperature at which the number of bubbles generated per unit time per Hubble volume is of order unity. The old picture of bubble percolation included two sources: colliding bubble walls and turbulent bulk fluid motions with associated magnetic fields. The new picture adds a third source: sound waves from bulk motion resulting from bubble collisions, which persist long after percolation.

- **Bubble collisions**: Witten first realized in [?] that QCD phase transitions might leave detectable gravitational waves from violent bubble collisions, with peak frequency characterized by bubble size at collision and peak amplitude estimated by relative bubble size to Hubble horizon. This was generalized to electroweak phase transitions by Hogan [?]. Preliminary simulations in \cite{66–69} captured general features of gravitational waves from phase transitions. Remarkably, [?, ?] found that the spectrum from simulating two vacuum bubble collisions in Minkowski space depends only on the grossest features—α and β—similar to results for hundreds of bubbles [?]. As shown in Fig. 4 [Figure 4: see original paper], the envelope approximation \cite{66-68} proposes that gravitational waves are mainly generated from uncollided envelopes of colliding bubble walls, with contributions from overlap regions neglected. Extension to thermal bubble collisions [?] used Jouguet detonation and appreciated turbulent fluid motion. State-of-the-art results for the spectrum from colliding walls were settled in [?], achieving consensus with earlier studies [?, ?]. Recently, under thin-wall and envelope approximations in flat space, [?] claimed the spectrum can be estimated analytically without simulations.

- **Turbulent MHD**: The possibility of generating gravitational waves from turbulent fluid motion was mentioned in [?] as a remnant of bubble collisions, first estimated in [?] as a Kolmogorov spectrum under quadrupole approximation. Fully ionized plasma can also generate turbulent magnetic fields, which are themselves gravitational-wave sources. Early MHD analyses [?, ?] exhibited three problems [?]: large-scale (addressed in [?, ?]), time-evolution (persisted in [?] but addressed in [?]), and dispersion-relation (corrected in [?, ?]). The state-of-the-art result for the spectrum from non-helical MHD turbulence was established analytically in [?], ignoring circularly polarized waves \cite{79-85} from helical turbulence due to macroscopic parity violation. Numerical simulations of relativistic MHD turbulence are needed for confirmation.
- **Sound waves**: The possibility of generating gravitational waves from sound waves was pointed out in [?] but forgotten until the recent revelation in [?], which abandoned envelope approximation for colliding walls and showed gravitational waves should be dominated by overlapping sound waves in bulk fluid. These breakthrough findings were quantitatively understood in updated simulations [?, ?] and theoretically modeled in [?], after investigating inverse acoustic cascade [?] suggesting potential strong enhancement of sound wave density at small wavenumber.

3.4 Gravitational Wave Spectra

Gravitational waves from strong first-order phase transitions are guaranteed through three processes: bubble nucleation requiring a potential barrier for tunneling, bubble expansion requiring fast-moving walls for strong signals, and bubble percolation requiring efficient collisions to dissipate released vacuum energy into bulk fluid motion. Fitting formulas for gravitational-wave spectra from numerical simulations are well summarized in [?] and can be applied directly to particle physics models, where α and β evaluated at T_* along with wall velocity v_w are obtained from microscopic models, while efficiency factors κ_ϕ and κ_v are approximated using fitting formulas from [?].

We now discuss which particle physics models can exhibit first-order phase transitions capable of generating gravitational waves. The Standard Model with a phenomenological Higgs mechanism undergoes a crossover from high- to low-temperature phases [?] if the Higgs boson mass exceeds the W boson mass. To obtain gravitational waves from first-order transitions, one must go beyond the Standard Model (BSM). We provide an incomplete list of models with explicit gravitational-wave discussions, to be revisited when constructing reliable templates for extracting signals from stochastic backgrounds:

- **Higher-dimensional operators**: The simplest example is adding a cubic term, expected in supersymmetric extensions for strong first-order transitions. Using polynomial fitting formulas [?] for bounce action from general quartic potentials with cubic terms, semi-analytic calculations of electroweak phase transition signals were performed in [?]. Another important example is dimension-six operators [?, ?], first analyzed in [?] and revisited in [?] with full one-loop effective potential at finite temperature. Both examples considered detonations and runaway walls [?]. Fig. 5 [Figure 5: see original paper] shows the spectrum from dimension-six operators, discussed in detail in future work [?].

- **Additional scalar sectors**: The simplest and most studied example is the gauge singlet scalar extension \cite{59,101–107}, naturally fitting the cubic term [?] and testable at future colliders through precise triple Higgs coupling measurement [?]. Another important example is charged scalars under SM gauge groups, simplest realized in the two-Higgs-doublet model (2HDM). Gravitational waves from 2HDM were preliminarily analyzed in [?], further studied in [?] for CP-conserving cases and recently revisited in [?] for CP-violating cases.

- **Supersymmetric extensions**: Capabilities of detecting gravitational waves from phase transitions in Minimal Supersymmetric Standard Model (MSSM) and Next-to-Minimal Supersymmetric Standard Model (NMSSM) were first estimated in [?] and explored in [?]. Both MSSM [?] and NMSSM [?] transitions were thought insufficiently strong, but parameter spaces with strong first-order transitions have been identified for modified MSSM [?] and general NMSSM [?].

- **Hidden dark sectors**: Cosmological implications of possible gravitational-wave production from hidden dark sectors were first explored in [?], later discussed for light GeV scalars [?], vector thermal dark matter [?], UV-conformal dark sectors [?], $SU(N)$ dark sectors with n_f flavors [?], dark $U(1)$ gauge complex scalar singlets [?], hidden sectors with runaway bubble walls [?], and transitions involving successive hidden gauge symmetry breaking [?], dark matter asymmetry [?], and two-step transitions [?]. Gravitational waves from first-order transitions could uniquely probe these hidden sectors.

- **Other BSM extensions**: Gravitational waves from first-order transitions have also been analyzed in extra dimensions \cite{124–127}, Peccei-Quinn transitions [?], non-linear electroweak transitions [?], and QCD transitions \cite{130–133}.

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4 Gravitational Waves From Binary Systems

Gravitational-wave sources can be divided into deterministic and stochastic categories. For example, primordial gravitational waves from quantum fluctuations and phase transitions in the early Universe are stochastic. Due to their intrinsic random nature, we cannot predict waveforms from stochastic sources. Deterministic sources include predictable and unpredictable types. Supernovae are believed to produce gravitational waves, but their dynamics is too complex for waveform prediction. Another example is LISA' s foreground noise from white dwarf binaries in our galaxy \cite{134–136}; the combination of numerous signals makes the waveform unpredictable. This section focuses on predictable sources, for which we can construct theoretical waveform models before detection [?]. When experimental data are available, matched filtering techniques can improve detection sensitivity and enable astronomical detection of sources [?, ?].

Binary systems are among the most important sources for gravitational-wave detection projects including PTA, space-based laser interferometers (eLISA, Taiji, Tianqin), and ground-based interferometers (Advanced LIGO, Advanced Virgo, KAGRA, etc.). The gravitational-wave frequency of a binary near merger can be characterized by the innermost stable circular orbit (ISCO) frequency:

$$f_{\rm ISCO} = \frac{6^{3/2}}{2\pi M}, \tag{4.1}$$

where M is the total mass and we use geometric units $c = G = 1$. For $M = M_\odot$, $f_{\rm ISCO} \approx 2000$ Hz. Binary systems are predictable sources [?]. A typical gravitational waveform and frequency evolution are shown in Fig. 6 [Figure 6: see original paper], with different time scales for inspiral and merger/ringdown stages. Binary components can be white dwarfs, neutron stars, black holes, or even quark stars. If gravitational theories beyond general relativity are valid, other objects such as axion stars [?] and gravastars [?] might also be components.

Based on current stellar evolution understanding, a star' s final fate as a white dwarf, neutron star, or black hole is determined by its mass. Binaries can form through capture or many-body interactions. Due to gravitational slingshot effects in three-body interactions, binary systems can be formed efficiently and are expected to be common in our Universe.

Direct gravitational-wave detection opens gravitational-wave astronomy. Through this new window, gravitational waves will bring new observations of various objects and insights into fundamental physics. Black holes remain mysterious from quantum gravity perspectives, particularly regarding the information loss puzzle. Could the horizon be replaced by a firewall or other structure? Can experiments prove black hole horizons exist? Unfortunately, electromagnetic observations cannot provide proof of event horizons [?], but gravitational-wave observations might give direct evidence [?]. Currently, these aims are most achievable with predictable sources, especially binary systems. To realize them, we must construct accurate waveform models. Three methods are available: post-Newtonian approximation, numerical relativity, and black hole perturbation theory, used for inspiral, merger, and ringdown phases, respectively.

4.1 Post-Newtonian Approximation

Based on the quadrupole formula [?], we can estimate gravitational waves from a binary system:

$$h_+ = -\frac{2\Omega^2 R^2}{r}(1+\cos^2\theta)\cos[2\Omega(t-r)], \tag{4.2}$$

$$h_\times = -\frac{4\Omega^2 R^2}{r}\cos\theta\sin[2\Omega(t-r)], \tag{4.3}$$

where (r, θ, ϕ) is the observer position relative to the binary, and M, R, and Ω are the total mass, separation, and orbital frequency. The modes h_+ and $h_\times$ correspond to two gravitational-wave polarizations, related to the metric perturbation in transverse-traceless gauge [?]. Two points should be noted: First, the plane-wave approximation is reasonable because the field point is far from the source. Second, this estimation is only the leading order of post-Newtonian approximation, valid for weak, slowly-moving sources. For most realistic sources, these approximations break down, and higher-order corrections are needed for detection requirements [?, ?].

The post-Newtonian framework for processing binary systems consists of dynamics and waveform parts. To construct models, we solve binary dynamics and substitute solutions into waveform theory. Dynamical equations form a highly nonlinear system of ordinary differential equations requiring numerical solution. Time-domain models include TaylorT2, TaylorT4, etc. [?]. Through stationary phase approximation, post-Newtonian equations can be transformed to frequency domain, yielding approximate analytical expressions like the TaylorF2 model. Corrected for spin dynamics and orbital precession, TaylorF2 is modified into single- and double-precession models [?, ?]. TaylorT2 is replaced by the X model for elliptical orbits [?], with TaylorF2 extended to post-circular [?] and enhanced post-circular models [?, ?].

4.2 Numerical Relativity

Applying numerical methods to solve Einstein' s equations constitutes numerical relativity. Currently, it can only model the plunge and merger phases of binary systems. Since numerical relativity solves Einstein' s equations without approximation (up to numerical error), it is a universal tool for various sources. Fig. 7 [Figure 7: see original paper] shows a binary black hole evolution simulated numerically. However, diffeomorphism invariance of general relativity creates special numerical difficulties. Numerical relativity began in the 1960s [?], but numerical instability caused codes to crash after few steps. In the 1990s, LIGO construction demanded source modeling, leading to the Binary Black Hole Grand Challenge Project, which did not solve the stability problem. Unexpectedly, Pretorius first solved the stability problem in 2005 [?], followed by independent solutions by the Baker and Campanelli groups in 2006 [?, ?]. Now over ten groups worldwide have solved the stability problem, including Princeton, Caltech, Jena, Max Planck, and the Academy of Mathematics and Systems Science, CAS [?].

It remains an open question which tips and treatments are necessary and/or sufficient for stable computation. From partial differential equation theory, hyperbolicity analysis can only be applied to linearized Einstein equations [?]. In practice, Pretorius' s 2005 success depended heavily on the Einstein equation formalism and adaptive mesh refinement. The BSSN and generalized harmonic coordinate equations are widely used. Adaptive mesh refinement effectively handles multiscale problems. Parallel adaptive mesh refinement codes for Einstein equations include BAM (Bruegmann), AMSS-NCKU (Cao) [?], PAMR (Pretorius), and Carpet (Schnetter).

Beyond stability, major issues are accuracy and efficiency. Investigations show Z4c and CCZ4 formalisms are better than BSSN \cite{163-165}. As an example, the Z4c formalism proposed in 3D in [?] is:

$$\partial_t \chi = \frac{2}{3}\chi[\alpha(\hat{K} + 2\Theta) - D_i\beta^i], \tag{4.5}$$

$$\partial_t \tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij} + 2\tilde{\gamma}_{k(i}\partial_{j)}\beta^k - \tilde{\gamma}_{ij}\partial_k\beta^k + \beta^k\partial_k\tilde{\gamma}_{ij}, \tag{4.6}$$

$$\partial_t \hat{K} = -D_iD^i\alpha + \alpha\left[\tilde{A}_{ij}\tilde{A}^{ij} + \frac{(\hat{K}+2\Theta)^2}{3} + \kappa_1(1-\kappa_2)\Theta\right] + 4\pi\alpha[S+\rho_{\mathrm{ADM}}] + \beta^i\partial_i\hat{K}, \tag{4.7}$$

$$\partial_t \tilde{A}_{ij} = \chi[-D_iD_j\alpha + \alpha(R_{ij} - 8\pi S_{ij})]^{\mathrm{tf}} + \alpha[(\hat{K}+2\Theta)\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k{}_j] + 2\tilde{A}_{k(i}\partial_{j)}\beta^k - \tilde{A}_{ij}\partial_k\beta^k + \beta^k\partial_k\tilde{A}_{ij}, \tag{4.8}$$

$$\partial_t \Theta = \alpha\left[R - \tilde{A}_{ij}\tilde{A}^{ij} + \frac{(\hat{K}+2\Theta)^2}{3} - 16\pi\rho_{\mathrm{ADM}} - 2\kappa_1(2+\kappa_2)\Theta\right] + \beta^i\partial_i\Theta, \tag{4.9}$$

$$\partial_t \tilde{\Gamma}^i = \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k - 2\tilde{A}^{ij}\partial_j\alpha + 2\alpha\left[\tilde{\Gamma}^i_{jk}\tilde{A}^{jk} - \frac{3}{2}\tilde{A}^{ij}\partial_j\ln\chi - \tilde{\gamma}^{ij}\partial_j(2\hat{K}+\Theta) - 8\pi\tilde{\gamma}^{ij}S_j\right] + \beta^j\partial_j\tilde{\Gamma}^i - \tilde{\Gamma}^j\partial_j \tag{4.10}$$

The unknown functions to be solved numerically are listed on the left-hand side; notation details are in [?].

Spectral, finite difference, and finite element methods are three categories for partial differential equations. Most groups use finite difference, while Caltech' s SpEC code uses spectral methods. Finite element applications remain few [?]. Spectral methods have good efficiency due to exponential convergence but limited parallel scalability due to global data exchange. Finite difference methods with domain decomposition achieve good parallel efficiency but are limited by single-data-layer multiscale refinement. Finite element methods can combine spectral convergence with high parallel scalability, but constructing weak forms of Einstein' s equations for large-scale computation remains challenging [?, ?].

Current major challenges are simulating binaries with mass ratios above 100:1 and achieving well-converged simulations containing neutron stars [?].

4.3 Black Hole Perturbation

After merger, the system rings down to a Kerr black hole (or a stable neutron star for white dwarf/neutron star binaries). Here we focus on Kerr black hole final states, where ringdown is described by black hole perturbation theory. Regge, Wheeler, and Zerilli pioneered perturbation around Schwarzschild black holes [?, ?], while Teukolsky investigated Kerr perturbations [?, ?]. The former yields the Regge-Wheeler-Zerilli equation, the latter the Teukolsky equation. For Schwarzschild black holes, both methods are equivalent [?]. In the early 1970s, scholars began using these equations to study gravitational waves from test particles falling into black holes \cite{172–174}. The Teukolsky equation' s applicability to spinning black holes makes it more general.

The Teukolsky equation can be solved via post-Newtonian approximation (applied to the Teukolsky equation itself, not Einstein' s equations) or numerical methods. The PN method was developed by Mano, Suzuki, Takasugi, and others using hypergeometric and Coulomb wave functions for homogeneous solutions [?, ?]. Fujita and collaborators applied this to calculate waveforms and energy fluxes for extreme mass-ratio binaries, achieving 22PN accuracy for Schwarzschild and 11PN for Kerr black holes [?, ?], though still insufficient for detection requirements [?].

Numerical methods divide into frequency- and time-domain approaches. Frequency-domain methods separate variables and Fourier-expand to obtain ordinary differential equations [?]. The radial equation can be transformed to the Sasaki-Nakamura equation before numerical solution [?]. To provide test particle orbits, Hughes and coworkers applied adiabatic approximation to geodesic equations [?], now important for extreme mass-ratio binaries. Fujita and Tagoshi used hypergeometric and Coulomb expansions for faster, more accurate numerical solutions [?]. Time-domain methods solve 2+1 partial differential equations, primarily using finite difference methods (but see [?] for finite element). Due to extreme mass ratios, signals may last years, making computational efficiency a major challenge despite the Teukolsky equation being simpler than full numerical relativity.

4.4 Cosmological Probes

In 1986, Schutz showed that gravitational waves from binary systems encode absolute distance information, enabling inference of the Hubble constant [?]. Inspiraling and merging compact binaries can be considered standard candles, or "standard sirens" —the term reflecting that gravitational-wave detectors are omni-directional and coherently detect wave phase, making them more like microphones than telescopes. In an expanding Universe, the chirp waveform generalizes cosmologically by multiplying all masses by $1+z$ and replacing physical distance D with luminosity distance d_L [?, ?]. The waveform during inspiral is theoretically described by:

$$h = \frac{1}{d_L(z)} \left(\frac{G\mathcal{M}_c(z)}{c^3} \right)^{5/3} \left(\frac{\pi f}{c} \right)^{2/3} \cos\iota \, \sin[\Phi(f)], \tag{4.11}$$

valid at lowest (Newtonian) order for the "cross" polarization (the "plus" polarization has different dependence on binary orbital plane inclination ι). Here $\mathcal{M}_c(z)$ is the redshifted chirp mass, f the observed frequency, and $\Phi(f)$ the phase. For standard sirens, luminosity distance $d_L(z)$ is the crucial parameter. Parameter estimation from observed signals directly yields d_L with uncertainty from detector noise, enabling "self-calibrating" distance measurement without cosmic distance ladders. However, redshift z cannot be inferred from black hole binary signals because all observed parameters are redshifted by the same factor $1+z$. Independent redshift determination is needed for cosmography.

Before considering redshift acquisition, we must assess distance measurement accuracy. Performance is limited by: (1) intrinsic amplitude uncertainty equal to the inverse signal-to-noise ratio (SNR) [?, ?], related to detector sensitivity; (2) weak gravitational lensing distorting d_L measurements by a few percent \cite{189-192}; and (3) limited direction and orientation sensitivity creating large correlations between distance, sky position, and source orientation. A detector network is needed to measure position and orientation and break degeneracies. Simultaneous electromagnetic counterpart detection can accurately determine sky position and improve distance measurements.

The most traditional redshift method uses accompanying electromagnetic signals (EM counterparts). Binary neutron star (BNS) or black hole-neutron star (BHNS) mergers are hypothesized to produce short gamma-ray bursts (SGRBs) [?]. EM counterparts like SGRBs provide redshift if the host galaxy is identified. SGRBs are likely strongly beamed [?], constraining binary inclination and breaking distance-inclination degeneracy. Gravitational waves with SGRBs or other counterparts as standard sirens have been studied extensively \cite{187,188,190,191,195-210}. For example, Nissanke et al. [?] found that a network of advanced LIGO detectors can constrain the Hubble constant to 5% accuracy using MCMC methods. Reference [?] demonstrated that 1000 events from the Einstein Telescope could constrain h_0 and Ω_m to $\Delta h_0 \sim 5 \times 10^{-3}$ and $\Delta\Omega_m \sim 0.02$ using Fisher matrix analysis, confirmed by Cai et al. [?] using MCMC. Furthermore, Cai et al. used Gaussian Process methods to constrain the dark energy equation of state to $\Delta w(z) \sim 0.03$ at low redshift, improving on [?]. For space-based LISA, expansion of the Universe and interacting dark energy have been studied in [?, ?].

Other redshift inference methods include galaxy catalogs [?], neutron star mass distributions [?, ?], and tidal deformations [?, ?]. Measuring redshift is a major challenge [?], but new probes using black hole binaries as tracers of large-scale structure without redshift information have been proposed [?]. Black hole spin can also help estimate parameters. Gravitational-wave standard sirens provide an independent, complementary alternative to current experiments, and combining them with other observations will constrain cosmological parameters more precisely.

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5 Conclusions

The direct detection of gravitational waves by LIGO initiates a new era of gravitational-wave astronomy and cosmology. Gravitational-wave physics is closely related not only to gravitational physics but also to particle physics, cosmology, and astrophysics. Gravitational waves provide a powerful new tool for revealing nature' s secrets, and many relevant papers have appeared since the detection announcement.

In this review, we briefly introduced three gravitational-wave source types and relevant physics: waves from inflation and preheating in the early Universe, from cosmic phase transitions, and from compact binary dynamics. We also discussed gravitational waves as standard sirens for cosmic evolution. Due to space limitations, we could not cover all aspects of gravitational-wave physics but focused on main issues. We apologize for any incomplete reference list.

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