
AI translation · View original & related papers at
chinaxiv.org/items/chinaxiv-201709.00145

Unified Approach to the Universe Model and the Local Gravitation Problem Postprint

Authors: Guang-Wen Ma, Zong-Kuan Guo

Date: 2017-09-27T00:00:00+00:00

Abstract

A united approach of the large-scale structure of a closed universe and the local spherically symmetric gravitational field is given by supposing an appropriate boundary condition. The general feature of the model obtained are the following. The universe is approximately homogeneous and isotropic on the average on large scale and is expanding at present, as described by the standard model; while locally, the small exterior region of a star started long ago to contract, as expected by the gravitational collapse theory.

Full Text

Preamble

United Approach to the Universe Model and the Local Gravitation Problem

Guang-Wen Maa and Zong-Kuan Guoa,b

aDepartment of Physics, Zhengzhou University, Zhengzhou, Henan 450052, PR China

bInstitute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100080, China

E-mail: guangwenma@sohu.com, guozk@itp.ac.cn

Abstract

A unified approach to the large-scale structure of a closed universe and the local spherically symmetric gravitational field is developed by imposing an appropriate boundary condition. The general features of the resulting model are as follows: The universe is approximately homogeneous and isotropic on average at large scales and is currently expanding, as described by the standard model;

while locally, the small exterior region of a star began contracting long ago, as expected from gravitational collapse theory.

1 Introduction

In traditional approaches, cosmological models and local gravitation problems are treated separately. In various universe models, only the average large-scale structure is considered, with matter distribution imagined as homogenized or smoothed—effectively treating a galaxy as a perfect fluid element. Conversely, when investigating local gravitational problems such as spherically symmetric gravitational fields, the region is typically regarded as asymptotically flat space-time, as if no other matter exists in the universe. While this treatment has its merits—the model is simple, and ignoring minor details does not impair our understanding of large-scale structure—there are fundamental limitations. Birkhoff’s theorem [1] appears to justify leaving the remaining matter of the universe out of consideration for local problems. However, no matter how reasonable this may seem, the asymptotically flat behavior of local gravitational field solutions is incompatible with a closed universe model, while observational evidence suggests the real universe is most likely closed.

The cosmological redshift demonstrates that distances between galaxies are increasing, meaning the universe is expanding on intergalactic scales. On the other hand, the dominant form of cosmic matter is compact celestial bodies, indicating that in some small regions of this expanding universe, space began contracting long ago. To our knowledge, no existing model treats these two problems concurrently. In this paper, we attempt to provide a unified approach to cosmological models and local gravitational problems by investigating a spherically symmetric gravitational field in an expanding closed universe.

For practicality, we adopt a compromise proposal called the “homogeneous by areas” model of matter distribution, wherein the universe is divided into several spherically symmetric regions sharing the same center, with each region having constant matter density.

2 The Gravitational Field Equations and Their General Solutions

Consider a local dense-matter region that appears in a closed Friedmann universe due to some mechanism. For simplicity, we assume its density is constant. Naturally, a spherically symmetric rare-matter or vacuum region may surround this dense area. This matter distribution model serves as our starting point. In this scenario, the three-dimensional spacelike hypersurface ($t = \text{constant}$) is a rotational hypersurface embedded in four-dimensional flat spacetime. In comoving coordinates, the line element can be written as:

$$ds^2 = dt^2 - U(\psi, t)d\psi^2 - V(\psi, t) \sin^2 \psi (d\theta^2 + \sin^2 \theta d\varphi^2)$$

where $0 \leq \psi \leq \pi$. The relationship between four-dimensional spherical coordinates R, ψ, θ, φ and Cartesian coordinates x, y, z, w is:

$$\begin{aligned}x &= R \sin \psi \sin \theta \cos \varphi \\y &= R \sin \psi \sin \theta \sin \varphi \\z &= R \sin \psi \cos \theta \\w &= R \cos \psi\end{aligned}$$

One often introduces $r = \sin \psi$ and rewrites the metric in the familiar form:

$$ds^2 = dt^2 - A(r, t)dr^2 - B(r, t)(d\theta^2 + \sin^2 \theta d\varphi^2)$$

However, it is crucial to emphasize that r is not a good coordinate due to the non-monotonicity of the transformation. The same value of r corresponds to two values of ψ , i.e., two positions in the universe. This means $A(r, t)$ and $B(r, t)$ should in principle have two sets of physical solutions corresponding to the two regions $0 \leq \psi \leq \pi/2$ and $\pi/2 \leq \psi \leq \pi$, respectively.

Consider the zero-pressure perfect-fluid model where $U^r = U^\theta = U^\varphi = 0$ and $U^t = 1$. Einstein's equations are [1]:

$$G_{\mu\nu} = 8\pi GT_{\mu\nu}$$

with $T_{\mu\nu} = \rho U_\mu U_\nu$. The field equations yield:

$$\frac{\dot{A}\dot{B}}{4AB} + \frac{\dot{B}^2}{4B^2} - \frac{\ddot{B}}{2B} = 4\pi G\rho A$$

$$\frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} - \frac{\ddot{A}}{2A} = 4\pi G\rho B$$

$$\frac{\ddot{B}}{2A} - \frac{\dot{A}\dot{B}}{4A^2} - \frac{\dot{B}^2}{4AB} = 0$$

From the last equation, we find:

$$B = S^2(r, t)r^2$$

where $K = K(r)$ is an arbitrary function of r . Using the first three equations yields:

$$\ddot{S} = -\frac{F(r)}{2S^2}$$

where $F = F(r)$ is an arbitrary function of r . The solution of this equation is cycloid-like:

$$S = \frac{F}{K}(1 - \cos \alpha)$$

with the time relation:

$$t - t_0 = \frac{F}{K^{3/2}}(\alpha - \sin \alpha)$$

where $C = C(r)$ is an arbitrary function. Substituting these into the field equations gives the density expression:

$$\rho = \frac{(F' - Kr^3)'}{4\pi GS'S^2r^3}$$

3 Homogeneous by Areas Model of Matter Distribution

When investigating spherically symmetric gravitational collapse of a celestial body in a closed universe, the “homogeneous by areas model” proves suitable. This model is expressed through the density distribution:

$$\rho(r, t) = \begin{cases} \rho_s(t) & 0 \leq \psi < \psi_1 \quad (\text{area I}) \\ 0 & \psi_1 < \psi < \psi_2 \quad (\text{area II}) \\ \rho_u(t) & \psi_2 < \psi \leq \pi \quad (\text{area III}) \end{cases}$$

where $\psi_1 = \arcsin r_1$ and $\psi_2 = \arcsin r_2$. The junction conditions at the boundaries $\psi = \psi_1$ and $\psi = \psi_2$ require that the metric $g_{\mu\nu}$ be continuous across these surfaces.

Substituting this density expression into the field equations and integrating yields:

$$K_I = \frac{4\pi G}{3} \rho_s F_I^2 (1 - \cos \alpha_I)^3 \quad (\text{area I})$$

$$K_{II} = \frac{\lambda}{F_{II} r^3} \quad (\text{area II})$$

$$K_{III} = \frac{4\pi G}{3} \rho_u F_{III}^2 (1 - \cos \alpha_{III})^3 \quad (\text{area III})$$

where λ is an integration constant for area II; for areas I and III, the integration constants have been set to zero to ensure K_I and K_{III} remain finite at $r = 0$ (i.e., $\psi = 0$ and $\psi = \pi$).

The junction conditions are:

$$\begin{aligned} A_I(r_1, t) &= A_{II}(r_1, t), & A_{II}(r_2, t) &= A_{III}(r_2, t) \\ B_I(r_1, t) &= B_{II}(r_1, t), & B_{II}(r_2, t) &= B_{III}(r_2, t) \end{aligned}$$

The parameter α in the solution determines whether the scale factor $S(r, t)$ is expanding or contracting. As an additional condition, we require:

$$\alpha_I(r_1, t) = \alpha_{II}(r_1, t), \quad \alpha_{II}(r_2, t) = \alpha_{III}(r_2, t)$$

These expressions provide determining conditions for $F(r)$, $K(r)$, and $C(r)$. From the continuity conditions we infer that $S(r_1, t)$ is continuous at the boundaries, which implies:

$$F_I(r_1) = F_{II}(r_1), \quad F_{II}(r_2) = F_{III}(r_2)$$

The time relation can be written as:

$$t_0 = \frac{F_i}{K_i^{3/2}}(\alpha_i - \sin \alpha_i)$$

where $i = I, II, III$ and $\alpha_{i0} = \alpha_i(r, t_0)$. The continuity conditions further imply:

$$K_I(r_1) = K_{II}(r_1), \quad K_{II}(r_2) = K_{III}(r_2)$$

Substituting these into the density expressions yields:

$$\rho_s F_I^3(r_1) r_1^3 (1 - \cos \alpha_I(r_1, t)) = \rho_u F_{III}^3(r_2) r_2^3 (1 - \cos \alpha_{III}(r_2, t))$$

From the first and last density relations we also obtain:

$$\rho_{s0} (1 - \cos \alpha_{I0})^3 = \rho_{u0} (1 - \cos \alpha_{III0})^3$$

In general, these conditions cannot completely determine F_i and K_i (or α_{i0}) because a certain freedom of coordinate transformation remains in comoving coordinates. In this sense, selecting the functional form of F_i and α_{i0} amounts to selecting a coordinate system.

We now construct explicit solutions based on the following considerations: (i) When $\psi_2 = \psi_1$ (i.e., $\rho_s = \rho_u$), the universe should be homogeneous and isotropic, so α_{I0} , α_{II0} , and α_{III0} should be approximately constant, as should F_{II} and F_{III} . (ii) When $\psi_1 < \psi < \psi_2$, both α_{I0} and α_{II0} should exceed π , indicating that the scale factor in this region has contracted, reflecting gravitational collapse of a celestial body.

The first condition suggests $\alpha_{I0} = \alpha_{II0} = \alpha_{III0} = \text{constant}$ and $F_I = F_{II} = F_{III} = \text{constant}$. We propose the following forms:

$$\alpha_{i0} = \beta + \delta\omega_i$$

$$F_i = a + b\omega_i$$

where β , δ , a , and b are constants, and ω_i are appropriate functions of the radial coordinate. The continuity conditions are satisfied by construction. To enforce the remaining junction conditions, we differentiate the time relation with respect to r and substitute into the field equations, obtaining continuity relations for the derivatives at the boundaries. Assigning any two of the four constants (e.g., β and a as initial conditions) allows determination of the remaining two, yielding a unique solution set.

4 Discussion

From the solution forms, when $r_2 = r_1$ we have $\alpha_{i0} = \beta$ and $F_i = a$, which exactly recovers the homogeneous and isotropic Friedmann universe model. In general, for $\psi < \psi_1$ and $\psi > \psi_2$, we find $\alpha_{III0} \approx \beta$ and $F_{III} \approx a$, showing that spacetime far from the star tends toward homogeneity and isotropy. In the region where $\psi_1 < \psi < \psi_2$, the scale factor contracts, reflecting stellar collapse.

Observational data indicate $\beta \approx \pi/2$. If $(r_2 - r_1)/(r_2 + r_1)\delta > \pi/2$, the celestial body is indeed collapsing.

In traditional treatments of spherically symmetric gravitational collapse, the exterior region is considered infinite and empty [2,3,4], with only the stellar surface as a boundary. This creates a puzzle: Suppose a star's surface with constant density is described by $x^2 + y^2 + z^2/b^2 = 1$ in some coordinate system. Under the transformation $x' = x$, $y' = y$, $z' = az/b$, the surface equation becomes $x'^2 + y'^2 + z'^2 = a^2$ in the new coordinates. What symmetry does the gravitational field possess—axisymmetric or spherically symmetric? Our new treatment eliminates this puzzle through the existence of a second boundary.

Birkhoff's theorem states that solutions to spherically symmetric vacuum gravitational field equations must be Schwarzschild. Therefore, the metric in area II should be transformable to Schwarzschild form. Such a transformation has been found:

$$\bar{r} = F_{II}r(1 - \cos \alpha_{II})$$

$$\bar{t} = 2GM \ln \left(\frac{\sqrt{F_{II}r} - GM \sin \alpha_{II}}{\sqrt{F_{II}r} + GM \sin \alpha_{II}} \right) + \sqrt{GM}(1 + \cos \alpha_{II})$$

where M is the mass parameter and G is the gravitational constant.

References

- [1] G. Birkhoff, *Relativity and Modern Physics*, Harvard University Press, Cambridge, 1923.
- [2] S. Weinberg, *Gravitation and Cosmology*, Wiley, New York, 1972.
- [3] J. R. Oppenheimer and H. Snyder, *Phys. Rev.* **56**, (1936) 455.
- [4] M. M. May and R. H. White, *Phys. Rev.* **141**, (1966) 1232.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv – Machine translation. Verify with original.