Parametrization of Quintessence and Its Potential: Postprint

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Date: 2017-09-27T00:00:00+00:00

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Full Text

Preamble

Parametrization of Quintessence and Its Potential

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Abstract

We develop a theoretical method for constructing the quintessence potential directly from the effective equation of state function $w(z)$, which describes the properties of dark energy. We apply our method to four parametrizations of the equation of state parameter and discuss the general features of the resulting potentials. In particular, we show that the constructed quintessence potentials all take the form of a runaway type.

PACS number(s): 98.80.Cq, 98.65.Dx

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Recent observations of type Ia supernovae suggest that the expansion of the universe is accelerating and that two-thirds of the total energy density exists in a dark energy component with negative pressure [?, ?]. In addition, measurements of the cosmic microwave background [?] and the galaxy power spectrum [?] also indicate the existence of dark energy. The simplest candidate for dark energy is a cosmological constant Λ, which has pressure $P_\Lambda = -\rho_\Lambda$. Specifically, a reliable model should explain why the present amount of dark energy is so small compared with the fundamental scale (the fine-tuning problem) and why it is comparable with the critical density today (the coincidence problem).

The cosmological constant suffers from both these problems. One possible approach to constructing a viable model for dark energy is to associate it with a slowly evolving and spatially homogeneous scalar field ϕ, called "quintessence" [?, ?, ?, ?, ?]. Such a model, for a broad class of potentials, can yield an energy density that converges to its present value for a wide set of initial conditions in the past and possesses tracker behavior.

The dark energy is characterized by its equation of state parameter w, which is generally a function of redshift z in quintessence models. The quintessence potential $V(\phi)$ and the equation of state $w_\phi(z)$ may be reconstructed from supernova observations [?, ?, ?]. In this letter, we develop a theoretical method for constructing the quintessence potential $V(\phi)$ directly from the dark energy equation of state function $w_\phi(z)$. We apply this method to four typical parametrizations that fit the data well [?, ?, ?, ?, ?, ?] and discuss the general features of the resulting potentials. The typical behavior of the constructed potentials is found to be of a runaway type.

We consider a spatially flat FRW universe dominated by non-relativistic matter and a spatially homogeneous scalar field ϕ. The Friedmann equation can be written as

$$H^2 = \frac{\rho_m + \rho_\phi}{3M_{\rm pl}^2},$$

where $M_{\rm pl} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass and ρ_m is the matter density. The energy density ρ_ϕ and pressure P_ϕ of the evolving scalar field ϕ are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi),$$

respectively, where $V(\phi)$ is the scalar field potential. The corresponding equation of state parameter is

$$w_\phi = \frac{P_\phi}{\rho_\phi}.$$

Using Eqs. (2) and (3), we obtain

$$\dot{\phi}^2 = (1 + w_\phi)\rho_\phi, \quad V(\phi) = \frac{1}{2}(1 - w_\phi)\rho_\phi.$$

The evolution of the quintessence field is governed by the equation of motion

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0,$$

which yields

$$\rho_\phi(z) = \rho_{\phi 0} \exp\left[3\int_0^z \frac{1+w_\phi(z')}{1+z'} dz'\right] \equiv \rho_{\phi 0} E(z),$$

where z is the redshift given by $1+z = a_0/a$ and the subscript 0 denotes the value of a quantity at redshift $z = 0$ (present). In terms of $w_\phi(z)$, the scalar field potential can be written as a function of redshift:

$$V[\phi(z)] = \frac{1}{2}[1 - w_\phi(z)]\rho_{\phi 0} E(z).$$

With the help of $\rho_m = \rho_{m0}(1+z)^3$ and Eq. (8), the Friedmann equation (1) becomes

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\phi 0} E(z)},$$

where $\Omega_{m0} \equiv \rho_{m0}/(3M_{\rm pl}^2 H_0^2)$ and $\Omega_{\phi 0} \equiv \rho_{\phi 0}/(3M_{\rm pl}^2 H_0^2)$. Using Eq. (5), we have

$$\dot{\phi} = \pm\sqrt{(1+w_\phi)}\rho_\phi^{1/2},$$

and from $dt = -dz/[(1+z)H(z)]$, we obtain

$$\frac{d\phi}{dz} = \mp \frac{\sqrt{3}M_{\rm pl}(1+w_\phi)^{1/2}(1+z)}{1 + r_0(1+z)^3 E^{-1}(z)}\sqrt{E(z)},$$

where $r_0 \equiv \Omega_{m0}/\Omega_{\phi 0}$ is the present energy density ratio of matter to quintessence. The sign is arbitrary, as it can be changed by the field redefinition $\phi \to -\phi$. So we choose the upper sign in the following discussions. Substituting the relevant expressions gives the construction equations.

We define the dimensionless quantities

$$\tilde{V} \equiv \frac{V}{\rho_{\phi 0}}, \quad \tilde{\phi} \equiv \frac{\phi}{M_{\rm pl}}.$$

The construction equations can then be written as

$$\tilde{V}[\phi(z)] = \frac{1}{2}[1 - w_\phi(z)]E(z),$$

$$\frac{d\tilde{\phi}}{dz} = \frac{\sqrt{3}(1+w_\phi)^{1/2}(1+z)}{1 + r_0(1+z)^3 E^{-1}(z)}\sqrt{E(z)},$$

which relate the quintessence potential $V(\phi)$ to the equation of state function $w_\phi(z)$. Given an effective equation of state function $w_\phi(z)$, these construction equations allow us to determine the quintessence potential $V(\phi)$.

Our method is novel in that it directly relates the quintessence potential to the equation of state function, thereby enabling straightforward construction of the potential without assuming its form. For instance, in the reconstruction method discussed in Ref. [?], the reconstruction equations relate the potential and equation of state to measurements of the luminosity distance. The potential may thus be reconstructed from supernova data via the luminosity distance, typically requiring an assumed functional form for $V(\phi)$. The dark energy properties are well described by the effective equation of state parameter $w_\phi(z)$, which generally depends on redshift.

Let us now consider the following four cases [?, ?, ?, ?, ?, ?]: a constant equation of state parameter and three two-parameter parametrizations.

Case I: $w_\phi = w_0$ (Ref. [?])

$$\tilde{V}(z) = \frac{1}{2}(1 - w_0)(1+z)^{3(1+w_0)},$$

$$\frac{d\tilde{\phi}}{dz} = \frac{\sqrt{3}(1+w_0)^{1/2}(1+z)}{1 + r_0(1+z)^{-3w_0}}\sqrt{(1+z)^{3(1+w_0)}}.$$

Case II: $w_\phi = w_0 + w_1 z$ (Ref. [?])

$$\tilde{V}(z) = \frac{1}{2}(1 - w_0 - w_1 z)(1+z)^{3(1+w_0-w_1)}e^{3w_1 z},$$

$$\frac{d\tilde{\phi}}{dz} = \frac{\sqrt{3}(1+w_0+w_1 z)^{1/2}(1+z)}{1 + r_0(1+z)^{-3(w_0-w_1)}e^{-3w_1 z}}\sqrt{(1+z)^{3(1+w_0-w_1)}e^{3w_1 z}}.$$

Case III: $w_\phi = w_0 + w_1 \frac{z}{1+z}$ (Ref. [?, ?, ?])

$$\tilde{V}(z) = \frac{1}{2}\left(1 - w_0 - w_1\frac{z}{1+z}\right)(1+z)^{3(1+w_0+w_1)}\exp\left(-\frac{3w_1}{1+z}\right),$$

$$\frac{d\tilde{\phi}}{dz} = \frac{\sqrt{3}\left(1+w_0+w_1\frac{z}{1+z}\right)^{1/2}(1+z)}{1 + r_0(1+z)^{-3(w_0+w_1)}\exp\left(\frac{3w_1}{1+z}\right)}\sqrt{(1+z)^{3(1+w_0+w_1)}\exp\left(-\frac{3w_1}{1+z}\right)}.$$

Case IV: $w_\phi = w_0 + w_1 \ln(1+z)$ (Ref. [?])

$$\tilde{V}(z) = \frac{1}{2}[1 - w_0 - w_1\ln(1+z)](1+z)^{3(1+w_0)+\frac{3}{2}w_1\ln(1+z)},$$

$$\frac{d\tilde{\phi}}{dz} = \frac{\sqrt{3}[1+w_0+w_1\ln(1+z)]^{1/2}(1+z)}{1 + r_0(1+z)^{-3w_0-\frac{3}{2}w_1\ln(1+z)}}\sqrt{(1+z)^{3(1+w_0)+\frac{3}{2}w_1\ln(1+z)}}.$$

We have numerically evaluated these equations. Figure 1 [Figure 1: see original paper] shows the evolution of the quintessence energy density $\rho_\phi(z)$, where we choose $w_0 = -0.8$, $w_1 = 0.1$ and $r_0 = 3/7$. At low redshift, all models obey the same evolution law, but deviations become clearly visible at redshift $z > 1$. Figure 2 [Figure 2: see original paper] shows the constructed quintessence potential $V(\phi)$, which takes the form of a runaway potential.

In evaluating these equations, we have chosen the initial value of the quintessence field $\tilde{\phi}_0 = 0.8$ at redshift $z = 0$ (present). This value is selected for definiteness; shifting it simply translates the scalar field horizontally in Figure 2, with no influence on the evolution of the universe or the shape of the quintessence potential. In general, ϕ decreases monotonically as z increases from 0, while the potential increases. This means that the potential decreases as the universe expands.

As shown in Figure 2, the four cases exhibit the same asymptotic behavior in the region $0.4 < \tilde{\phi} < 0.8$, corresponding to low redshift $0 < z < 1$. We can provide an approximate analytic form for the potential, which is exponential:

$$\tilde{V}(\tilde{\phi}) \approx \frac{1}{2}(1 - w_0) \exp\left[-3(1 + w_0)(1 + r_0)\tilde{\phi}\right].$$

The potentials differ at high redshift. In this region, we find that the field ϕ becomes small and the quintessence potential takes a power-law form for the first case:

$$\tilde{V}(\tilde{\phi}) \approx \frac{1}{2}(1 - w_0) \left(\frac{w_0\sqrt{3r_0}}{2\sqrt{1 + w_0}}\tilde{\phi}\right)^{2(1+w_0)/w_0},$$

while the potentials are more complicated for the remaining three cases.

It is possible to derive an exact analytic form for the potential in Case I. Integrating the equation for $d\tilde{\phi}/dz$ yields

$$\tilde{\phi}(z) = \frac{\sqrt{1 + w_0}}{\sqrt{3}w_0} \ln \frac{\sqrt{1 + r_0(1 + z)^{-3w_0}} - 1}{\sqrt{1 + r_0(1 + z)^{-3w_0}} + 1}.$$

Solving this for $1 + z$ and substituting into the expression for $\tilde{V}(z)$ gives

$$\tilde{V}(\tilde{\phi}) = \frac{1}{2}(1 - w_0) \left[r_0 \sinh^2\left(\frac{\sqrt{3}w_0}{2\sqrt{1 + w_0}}\tilde{\phi}\right)\right]^{-(1+w_0)/w_0}.$$

This is consistent with the asymptotic forms given above. This potential was also obtained in Ref. [**?**]. We thus confirm that the potential is indeed of the runaway type for $-1 < w_0 < 0$, with an asymptotic value of zero. This is a very interesting result, as such behavior arises generically in supersymmetric theories. This type of potential is also expected for tachyons in unstable D-brane systems in superstring theories [**?**].

In conclusion, we have developed a method for constructing the quintessence potential directly from the effective equation of state function $w_\phi(z)$, which describes the properties of dark energy. We have applied this method to four parametrizations of the equation of state parameter and shown that the constructed quintessence potential takes the form of a runaway type. The future Supernova/Acceleration Probe with high-redshift observations [?], in combination with Planck CMB observations [?], will be able to determine the parameters in the dark energy parametrization with high precision. Through precision mapping of the recent expansion history, we hope to learn more about the nature of dark energy.

Acknowledgements

This project was supported in part by the National Basic Research Program of China under Grant No. 2003CB716300 and by NNSFC under Grant No. 90403032. The work of N.O. was supported in part by the Grant-in-Aid for Scientific Research Fund of the JSPS No.

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Note: Figure translations are in progress. See original paper for figures.