A Tracker Solution for a Holographic Dark Energy Model Postprint

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Abstract

We investigate a class of holographic dark energy models with the future event horizon as the IR cutoff and the equation of state −1. In this model, the constraint on the equation of state automatically specifies an interaction between matter and dark energy. With this interaction included, an accelerating expansion is obtained as well as the transition from deceleration to acceleration. It is found that there exists a stable tracker solution for the numerical parameter d > 1, while d less than one will not lead to a physical solution. This model provides another possible phenomenological framework to alleviate the cosmological coincidence problem in the context of holographic dark energy. Some properties of the evolution which are relevant to cosmological parameters are also discussed.

Full Text

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Abstract

We investigate a holographic dark energy model with the future event horizon as the IR cutoff. In this model, the constraint on the equation of state automatically specifies an interaction between matter and dark energy. With this interaction included, an accelerating expansion is obtained as well as the transition from deceleration to acceleration. It is found that there exists a stable tracker solution for the numerical parameter $d > 1$, while d smaller than one does not lead to a physical solution. This model provides another possible phenomenological framework to alleviate the cosmological coincidence problem in the context of holographic dark energy. Some properties of the evolution that are relevant to cosmological parameters are also discussed.

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1. Introduction

Numerous and complementary cosmological observations lend strong support to the present acceleration of our universe [?]. Some negative-pressure source is needed to meet these observational requirements. Since its energy density is unexpectedly small ($\rho_\Lambda \sim 10^{-47}\,\mathrm{GeV}^4$) in the framework of quantum field theory (QFT), understanding this amazing phenomenon represents a great challenge to fundamental physics [?]. Despite a variety of phenomenological dynamical fields (quintessence, phantom, k-essence, etc.) with suitably chosen potentials [?], the simplest cosmological constant remains attractive.

As a matter of fact, this is a long-standing topic and several proposals have been discussed in the past twenty years, such as the screening of the cosmological constant due to Hawking radiation or the existence of an infrared fixed point in effective theories of gravitation [?], the relaxation of the cosmological constant based on the coincidence limit of the graviton propagator growing in time [?], and the screening mechanism due to virtual gravitons [?]. Nevertheless, some of these mechanisms have the ability to be compatible with subsequent observations of type Ia supernovae which indicate present cosmic acceleration, although most of them were primarily focused on inflationary cosmology.

The theoretical appeal of Λ and recent remarkable observational evidence on the equation of state (EOS) w_{DE} around -1 [?, ?] motivates theorists to attempt reconciliation. Hereafter we mean a dynamical dark energy (DE) with a true cosmological constant $w_{DE} = -1$. In Ref. [?], a kind of back-reaction-induced cancellation mechanism is proposed to understand the seemingly varying cosmological constant. With the Hubble parameter H as the renormalization scale, a picture of the running cosmological constant has been developed in Ref. [?]. We note that some phenomenological approaches to study decaying vacuum cosmology [?, ?] have also emerged.

On the other hand, the holographic principle [?] has inspired great efforts to attack this fine-tuning problem of ρ_Λ. In a seminal paper by Cohen et al. [?], there is a suggestion that in QFT a short-distance cutoff is related to a long-distance cutoff due to the limit set by black hole formation, and this often-neglected IR limitation to QFT would correspond to an energy scale of $10^{-2.5}$ eV if the IR cutoff is our present Hubble scale $H^{-1} \sim 10^{28}$ cm.

According to Refs. [?]-[?], this choice of IR cutoff will alleviate the cosmological constant problem [?]. In line with this suggestion, Hsu [?] and Li [?] argued that this energy density could be viewed as the holographic DE density satisfying $\rho_{DE} = 3d^2 M_P^2 L^{-2}$, where M_P is the reduced Planck mass. Li also demonstrated that only by identifying L with the radius of the future event horizon R_e can we obtain the EOS $w_{DE} < -1/3$ and an accelerating universe.

Generally speaking, the universe was in the matter-dominated era until recently. As a consequence, today' s particle horizon and Hubble horizon are roughly of the same order [?]. Then for Li' s model to share the merit of Hubble-scale cutoff in reproducing the correct magnitude of the cosmological constant, the following question emerges: why is the future event horizon R_e comparable to the particle horizon and the Hubble horizon at present? This may be regarded as a holographic variation of the cosmological coincidence problem (CCP), which concerns the mysterious approximate equality of matter and DE density today [?].

Those suggestions on holographic DE have been extensively discussed in the past few years [?, ?, ?, ?]. It was found that after considering the interaction between dark matter and DE, the choice of H^{-1} as the IR cutoff of the holographic DE may be compatible with the desired $w_{DE} = -1$ [?]. Nevertheless, with that interaction included, the scaling behavior of the cosmological constant which is in favor of the CCP may also be obtained. After loosening the constraint on $w_{DE} = -1$, Ref. [?] has also realized acceleration and a scaling solution. There, the transition from deceleration to acceleration requires a varying coefficient d. Lately, a model which uses R_e instead of H^{-1} as IR cutoff and leaves w_{DE} undetermined has also recovered the acceleration transition and moreover an EOS transition from $w_{DE} > -1$ to $w_{DE} < -1$ phantom regimes [?] (see also [?]).

It can be seen that studies up to now have shown the scaling solution in the Hubble horizon criterion of holographic DE, no matter whether the EOS is fixed to be -1. However, the future event horizon cutoff which is used to scale the holographic DE clearly leads to a finale of $\Omega_{DE} = 1$ [?], where Ω_{DE} is the DE fraction of the total energy density. This is unable to address the CCP.

2. The Model

In this Letter, we study a particular holographic DE model [?] and exhibit the theoretical possibility to drive an R_e version of holographic DE to understand the CCP. The basic ingredients are as follows: the holographic DE scales as $\rho_\Lambda \propto R_e^{-2}$, but different from previous models it is assumed to possess a constant EOS $w_\Lambda = -1$, i.e., we here deal with a varying but "true" cosmological constant.

The holographic DE density is

$$\rho_\Lambda = 3d^2 M_P^2 R_e^{-2}.$$

Here we keep d as a free positive dimensionless parameter and R_e is the proper size of the future event horizon,

$$R_e(t) = a(t)\int_t^{\infty}\frac{dt'}{a(t')} = a(t)\int_x^{\infty}\frac{dx'}{H(x')},$$

where a is the scale factor of the universe. For a spatially flat, isotropic and homogeneous universe with ordinary matter and dark energy, the Friedmann equation can be written as

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda)$$

or equivalently $\Omega_\Lambda + \Omega_m = 1$, where ρ_m (ρ_Λ) is the energy density of matter (dark energy) and the critical density $\rho_{cr} = 3M_P^2H^2$. Because of the conservation of the energy-momentum tensor, the evolution of the energy of matter and DE are governed by

$$\dot{\rho}_m + 3H\rho_m = Q,$$

$$\dot{\rho}_\Lambda = -Q,$$

respectively. Here we have used the requirement $\dot{\rho}_\Lambda + \dot{\rho}_m + 3H(\rho_m + p_\Lambda) = 0$ and Q represents the undetermined interaction between matter and DE.

By definition Eq. (1), we have $\rho_\Lambda = 3d^2M_P^2H^2\Omega_\Lambda$. According to the definition of the future event horizon (2), a straightforward calculation gives

$$\dot{R}_e = HR_e - 1.$$

Hereafter the superscript dot denotes the derivative with respect to cosmic time t. Then the rate of change of both energy components may be expressed as

$$\dot{\rho}_\Lambda = 6M_P^2H^3\Omega_\Lambda\left(1 - \frac{\sqrt{\Omega_\Lambda}}{d}\right),$$

$$\dot{\rho}_m = 6M_P^2H^3\Omega_\Lambda\left(\frac{\sqrt{\Omega_\Lambda}}{d} - \Omega_\Lambda\right),$$

where Eq. (3) has been recalled. It' s clear that the energy density of matter and DE cannot be conserved separately in our model. There is energy transfer between these two components and the coupling term Q is just of the form $H\rho_\Lambda$ multiplied by a variable coefficient.

In the present framework of our model, the truly independent continuity equation is Eq. (9) and it may be employed to produce the evolution equation for Ω_Λ. By means of Eq. (3), Eq. (9) can be cast into

$$\Omega'_\Lambda = \frac{2\Omega_\Lambda}{d}\left(\sqrt{\Omega_\Lambda} - d\Omega_\Lambda\right),$$

where the prime denotes the derivative with respect to $x = \ln a$.

To study the scaling behavior of the cosmological evolution, it' s convenient to introduce an auxiliary quantity $r = \rho_m/\rho_\Lambda = (1-\Omega_\Lambda)/\Omega_\Lambda$ and then $\dot{\Omega}_\Lambda = H\Omega'_\Lambda$. The rate $\dot{r}$ of the energy density ratio r of matter and dark energy can be written as

$$\dot{r} = 3Hr\left[\frac{1}{3d}\sqrt{\frac{1+r}{r}} - 1\right].$$

Then Eq. (10) becomes

$$\dot{r} = 3Hr\left[\frac{1}{3d}\sqrt{\frac{1+r}{r}} - 1\right],$$

where $\dot{r} = 0$ gives the possible cosmological scaling behavior

$$\Omega^*_\Lambda = \frac{1+\sqrt{1+3d^2}}{3d}.$$

When the parameter d is greater than 1, the positive root Ω^+_Λ is smaller than 1, leading to a meaningful scaling solution. Moreover, the larger d is, the smaller the value of Ω^+_Λ. For example, if we choose $\Omega^0_\Lambda = 0.7$, then the parameter d should not be larger than 1.5. People might worry whether the parameter d can take values greater than 1 since the original bound $L^3\rho_\Lambda \leq M_P^3$ proposed by Cohen [**?**] would be violated. Careful analysis suggests this may not be the case. The model we propose is only a phenomenological framework, and it' s unclear whether it' s appropriate to tightly constrain the value of d by analogy to black hole physics. In fact, the possibility of $d > 1$ has been seriously considered, and a modest value of d larger than one could be favored in the literature [**?**].

When d equals 1, the positive root $\Omega^+_\Lambda = 1$ and the cosmic expansion approaches a de Sitter phase asymptotically. What about $d < 1$? Unlike the original holographic dark energy model [**?**, **?**] with R_e as the IR cutoff where the universe approaches a phantom phase for $d < 1$, numerical simulations indicate that there exists no consistent physical solution. In fact, $d < 1$ always makes Ω'_Λ positive and $\dot{r} < 0$, even when the increasing DE fraction has reached 1 with the matter component vanishing. This would then result in an unacceptable negative matter density. We should note that some previous fits to observational SN Ia data [**?**, **?**] in the context of holographic dark energy were based on assumptions different from our model, and the best fits suggesting a free parameter d smaller than 1 may be irrelevant to the present model.

To study the stability of the critical point of Eq. (10) which corresponds to the scaling solution Ω^+_Λ, we substitute a linear perturbation $\Omega_\Lambda = \Omega^+_\Lambda + \delta$ into Eq. (10). To first order in the perturbation, this gives

$$\delta' = -\frac{3}{2}\left(\sqrt{1+3d^2} - 1\right)\delta.$$

It is easy to check that the scaling solution Ω^+_Λ is always the late-time stable attractor solution.

Now let' s turn to more details of this model that are relevant to other observational quantities. The transition from deceleration to acceleration occurs when $(\rho_\Lambda + 3p_\Lambda + \rho_m) = 0$. Using Eqs. (3) and (5) as well as the above formula, we find that the transition emerges at $\Omega_\Lambda^T = 1/3$, which is independent of the parameter d. This result coincides with that of the standard ΛCDM scenario since both models have the same EOS of -1.

The deceleration parameter q is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(1 + 3w_\Lambda\Omega_\Lambda).$$

Through Eqs. (2) and (6), we have

$$HR_e = \frac{1}{\sqrt{1-\Omega_\Lambda}},$$

and taking the derivative with respect to x on both sides of the above equation we get

$$q = \frac{1}{2} - \frac{3}{2}\Omega_\Lambda.$$

Using the information extracted from Eq. (10), the deceleration parameter q may be determined. To exhibit the evolution of this model in redshift, we note that $1 + z = 1/a$ (by convention we have chosen the present scale factor $a_0 = 1$) and then there is a relation $x = -\ln(1 + z)$.

After specifying the value of the parameter d, the behavior of the DE evolution may be obtained through Eq. (10). The dependence of the evolution of DE on the constant d is shown in Fig. 1. We see that for different values of the parameter $d > 1$, the evolution of the universe will approach different tracker solutions. The solution Ω_Λ^+ is illustrated as the plateau of a particular curve in Fig. 1. What' s more, the larger d is, the more gently Ω_Λ climbs up to its final value, and the smaller that value is. For $d = 1$, the evolution approaches a de Sitter universe and the matter component becomes infinitely diluted. Once again, we note that in our model $d < 1$ yields an unphysical solution and is therefore not allowed.

[Figure 1: see original paper]

Fig. 2 shows the evolution of the deceleration parameter q where for definiteness the value of Ω_Λ^0 is set to be 0.7. The discussion above has shown that in this case $d \in [1, 1.5)$. From this figure, we can easily see that $q_0 = -0.5$ is independent of the parameter d and there is a transition from deceleration to accelerating expansion. Fig. 3 shows the relation between the redshift of the turning point z_T at which the transition from deceleration to acceleration occurs and the value of the parameter d. It' s obvious that the evaluation of the present DE density fraction imposes a practical constraint on the parameter d, as indicated by the rapidly ascending curve.

[Figure 2: see original paper]

[Figure 3: see original paper]

3. Conclusion

In conclusion, we have studied a holographic DE model in which the future event horizon is chosen as the IR cutoff and the equation of state is fixed to be -1. In this model, an interaction between matter and dark energy naturally appears. We find that the accelerating expansion as well as the transition from deceleration to acceleration is well recovered. There exists a stable tracker solution for the dimensionless parameter $d > 1$. This model provides one possible phenomenological framework to alleviate the cosmological coincidence problem with holographic motivation. We show that by means of only one cosmological parameter—the DE density fraction—the constant d obtains an observational upper bound. Specifically, if today' s universe is DE-dominated, there is a practical constraint on the numerical factor d which cannot deviate much from 1. It would be interesting to further examine this model with current observational data and determine whether other strategies such as a varying Newton' s constant [**?**, **?**, **?**, **?**] are necessary to extend our framework while still ameliorating the cosmological coincidence problem.

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Note: Figure translations are in progress. See original paper for figures.