Baryonic Isgur-Wise Functions in Large Nc HQET Postprint

Authors: Ming-Kai Du, Chun Liu

Date: 2017-09-17T00:00:00+00:00

Abstract

Large Nc relations among baryonic Isgur-Wise functions appearing at the order of 1/mQ areanalyzed. An application to b → c weak decays is given.

Full Text

Baryonic Isgur-Wise Functions in Large Nc HQET

Ming-Kai Du and Chun Liu
State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China

Abstract

We analyze the large Nc relations among baryonic Isgur-Wise functions appearing at order 1/mQ and present an application to Ωb → Ωc weak decays.

PACS numbers: 12.39.Hg, 11.15.Pg, 14.20.Mr, 13.30.Ce

Introduction

Weak decays of heavy baryons are of considerable interest both experimentally and theoretically. They are currently under investigation at the LHC experiments, as well as in previous Tevatron and LEP experiments, and provide an important testing ground for nonperturbative QCD methods. Heavy baryons containing a single heavy quark are described by Heavy Quark Effective Theory (HQET) \cite{1-4}, where relevant physical quantities can be factorized into a calculable perturbative part and universal hadronic quantities. To compute the latter, nonperturbative methods such as the large Nc approach [?] are required.

Consider the heavy baryon weak transitions Λb → Λc and Σ()*b → Σ()*c. The matrix elements of vector and axial currents (Vμ = c̄ μb and Aμ = c̄ μ 5b) between Λb and Λc can be parametrized as:

Λc(v , s)|Vμ|Λb(v, s) = ūΛc(v , s)(F1( ) μ + F2( )vμ + F3( )v μ)uΛb(v, s),

Λc(v , s)|Aμ|Λb(v, s) = ūΛc(v , s)(G1( ) μ + G2( )vμ + G3( )v μ) 5uΛb(v, s),

and those between Σb and Σ(*)c are:

Σc(v , s)|Vμ|Σb(v, s) = ūΣc(v , s)(F 1 μ + F 2vμ + F 3v μ)uΣb(v, s),

Σc(v , s)|Aμ|Σb(v, s) = ūΣc(v , s)(G 1 μ + G 2vμ + G 3v μ) 5uΣb(v, s),

Σc(v , s)|Vμ|Σ*b(v, s) = ūΣc(v , s)(N 1v μ + N 2v vμ + N 3v v μ + N 4g μ) 5uΣb(v, s),

Σc(v , s)|Aμ|Σ*b(v, s) = ūΣc(v , s)(K 1v μ + K 2v vμ + K 3v v μ + K 4g μ)uΣb(v, s),

where = v · v and Fi( )( ), Gi( )( ), Ni( ) and Ki( ) are general form factors. As is well known, in HQET these form factors can be expressed in terms of several independent universal form factors known as Isgur-Wise functions.

The Isgur-Wise functions are defined as follows. Note that in HQET, heavy quark fields and baryon fields have their own definitions, despite using the same symbols as in full QCD. For the Λb → Λc transition at leading order of heavy quark expansion, there is only one Isgur-Wise function ( ):

Λc(v , s)|c̄v Γbv|Λb(v, s) = ( )ūΛc(v , s)ΓuΛb(v, s),

where Γ stands for general matrices, and ( ) is normalized at zero recoil, namely (1) = 1. For Σ(*) transitions, two Isgur-Wise functions 1( ) and 2( ) appear at leading order [?]:

Σ()*c(v , s)/c̄v Γbv/Σ()*b(v, s) = [gμ 1( ) + vμv 2( )] ūμ(v , s)Γu (v, s),

where 1(1) = 1, and u is the Rarita-Schwinger spinor for a spin-3/2 particle defined as:

u ΣQ(v, s) = (+ v) 5uΣQ(v, s).

Isgur-Wise functions at 1/mQ order will be discussed in detail in the next section.

In the heavy quark limit, the general form factors in Eqs. (1) and (2) are simplified. For the Λb → Λc transition:

F1 = G1 = C(μ) ( ),
F2 = G2 = F3 = G3 = 0,

where C(μ) is a perturbatively calculable coefficient. For Σ(*) transitions, the formulas are more complex and can be found in [?]. Note that Isgur-Wise functions are independent of the weak currents.

At this stage, the Isgur-Wise functions remain unknown and require nonperturbative methods for their calculation. In the large Nc limit, interesting information about baryonic Isgur-Wise functions can be obtained. In large Nc baryons, there exists a spin-flavor symmetry of light quarks [?] that not only gives mass degeneracy between Σ(*)Q and ΛQ, but also yields the following relations among the Isgur-Wise functions [?, ?]:

1( ) = ( ),
2( ) = (1 +) ( ).

This large Nc result is also consistent with that obtained from the large Nc constituent quark model [?]. Furthermore, in the heavy baryon Skyrme model [?], ( ) is calculated to be [?]:

( ) = 0.99 exp[1.3(- 1)].

In fact, in the true large Nc limit, ( ) is actually a -function [?].

From Eqs. (3), (4) and (7), we observe that to obtain the Σ(*)Q matrix elements of weak currents in the large Nc approximation, we need only multiply the ΛQ matrix element by the following Lorentz tensor:

gμ + (1 +)vμv .

This is because the two kinds of decays are essentially the same, except for Lorentz structures (a kinetic result of the light degrees of freedom). With this observation, we will extend the large Nc relations in Eq. (7) to O(1/mQ) in the following sections.

Isgur-Wise Functions at O(1/mQ)

More Isgur-Wise functions appear at the 1/mQ order. Before considering large Nc relations among these subleading Isgur-Wise functions, it is useful to start from their definitions.

A. A Mini-Review

The 1/mQ corrections arise from two sources. One is due to the HQET Lagrangian at O(1/mQ), and the other comes from the 1/mQ expansion of heavy quark currents in full QCD.

First, let us consider the Lagrangian corrections. To order O(1/mQ) the effective Lagrangian is [?, ?]:

L = h̄viv · Dh̄v + (1/2mQ)h̄v[D² + gs μ Gμ]hv.

For the hadronic matrix element of a heavy quark current, the correction due to the heavy quark kinetic energy is:

Hc|c̄v Γ(1/2mQ)D²bv|Hb .

For HQ being ΛQ, Eq. (10) is parametrized as:

Λc(v , s)|c̄v Γ(1/2mQ)D²bv|Λb(v, s) = -¯( )ūΛc(v , s)ΓuΛb(v, s),

where ( ) is the ΛQ Isgur-Wise function at order 1/mQ. It satisfies (1) = 0. In the true large Nc limit, ( ) = 0 [?]. In the case of HQ being Σ(*)Q, Eq. (10) is parametrized via two additional Isgur-Wise functions:

Σ()*c(v , s)/c̄v Γ(1/2mQ)D²bv/Σ()*b(v, s) = -[gμ 1( ) + vμv 2( )] ūμ(v , s)Γu (v, s),

with 1(1) = 2(1) = 0.

The correction due to the heavy quark chromomagnetic interaction is:

HQ|c̄v Γ(1/2mQ)gs μ Gμ bv|Hb .

For HQ being Λb, this is zero [?]. The situation is more complicated for HQ being Σ(*)Q. According to Ref. [?], the chromomagnetic 1/mQ correction can be parametrized as:

Σ()*c(v , s)/c̄v Γ(1/2mQ)gs μ Gμ bv/Σ()*b(v, s) = 1( )gμ + 2( )gμ v v + 3( )g vμv ,

where the i( ) are additional Isgur-Wise functions.

Now consider the 1/mQ corrections of the current operator. The relation between QCD currents and HQET operators is:

c̄Γb = c̄v [1 + (i←D /2mc + i→D /2mb) + ···]Γbv.

For the Λb → Λc decay [?, ?]:

Λc(v , s)|c̄v i←D Γbv|Λb(v, s) = (1 +)ūΛc(v , s)ΓuΛb(v, s)(v - v) ( ) + (v + v)¯( ),

where ¯( ) = ( ) + (Λ̄/2)(1 +) ( ) and Λ̄ = mΛQ - mQ. For the Σ()*b → Σ()*c transition, the 1/mQ correction is:

Σ()*c(v , s)/c̄v i←D Γbv/Σ()*b(v, s) = ūμ(v , s)Γu (v, s)Pμ ,

where Pμ = 1v vμv + 2v vμv + 3gμ v + 4gμ v + 5gμ + 6g μ.

Actually, only two of these Isgur-Wise functions are independent [?]:

1 = - 1/(1 +) + (2 - 5),
3 = - 1/(1 +) + (4 - 6),
5 = 1/(1 +) - (1 + 2),
6 = 1/(1 +) - (1 + 2).

The expressions of form factors in Eqs. (1) and (2) in terms of all these Isgur-Wise functions can be found in [?].

B. Large Nc Relations

Now considering the large Nc limit, there exist relations among the subleading order Isgur-Wise functions. The key observation is that the insight from

Sec. I remains applicable. The only difference here is that the heavy quark currents have different forms, which is irrelevant due to heavy quark symmetry. Therefore, in the large Nc limit, Eq. (12) for the charm quark kinetic energy correction should be:

gμ + (1 +)vμv .

This results in the following relations:

1( ) = ( ),
2( ) = (1 +) ( ).

Using the same method, we obtain a remarkable result: in the large Nc limit, to order 1/mQ in HQET, there are no chromomagnetic corrections in Σ(*)b → *Σ()*c decays:

1( ) = 2( ) = 3( ) = 0.

This can be easily understood since in the large Nc limit, the spins and isospins of light degrees of freedom in Σ(*)b and Σ()*c have decoupled. This decoupling makes Σ(*)Q baryons different from ΛQ baryons. Consequently, in the large Nc limit, the time-ordered product of 1/mQ terms in the Lagrangian with the heavy quark current merely produces a trivial correction for Σ()*Q decays: a redefinition of the leading order Isgur-Wise functions, similar to the case of ΛQ decays.

Examining the 1/mc correction of the current operator, for Σ(*)Q we have:

Σ(*)c(v , s)|c̄v i←D Γbv|Σ()*b(v, s) = (1 +)ūμ(v , s)Γu (v, s)gμ + (1 +)vμv ( ) + (v + v)¯( ),

where¯( ) is defined as¯( ) = ( ) + (Λ̄Σ/2)(1 +) ( ), and Λ̄Σ = mΣb - mΣc - mc. Note that Λ̄Σ = Λ̄ in the large Nc limit. From this we obtain new relations:

1 = -(1 +)² ,
2 = -(1 +)² ,
3 = -(1 +) ,
4 = -(1 +) ,
5 = 6 = 0.

We observe that after taking the large Nc approximation, the relations obtained in HQET, such as Eq. (20), still hold.

C. General Form Factors

Up to this point, we have derived all large Nc relations for the 1/mc corrections. The 1/mb corrections can be obtained similarly. Including all 1/mQ corrections, in the large Nc limit the general form factors in Eqs. (1) and (2) are expressed as:

F1 = ( ) + ( )/2mb,
G1 = ( ) - ( )/2mb,
F 1 = ( ) + ( )/2mb,

G 1 = ( ) - ( )/2mb,
N 1 = -2 ( )/3(1 +) + ( )/2mb,
K 1 = -2 ( )/3(1 +) - ( )/2mb,
F 2 = - ( ) + ( )/2mb,
G 2 = - ( ) - ( )/2mb,
N 2 = -4 ( )/3(1 +) + ( )/2mb,
K 2 = -4 ( )/3(1 +) - ( )/2mb,
F 3 = G 3 = 0,
N 3 = -2 ( )/3(1 +),
K 3 = -2 ( )/3(1 +),
N 4 = -2 ( )/3(1 +),
K 4 = 0,

where ( ) = ( ) + ( )/mb, and all form factors should be multiplied by C(μ) from Eq. (3). We have verified that all results are consistent with [?], where all 1/mQ form factors are listed. After taking the large Nc limit, all relations of 1/mQ form factors shown in [?] still hold, particularly the following normalization relations at the zero recoil point:

F1(1) + F2(1) + F3(1) = C(μ),
F 1(1) + F 2(1) + F 3(1) = C(μ),
G1(1) = C(μ);
G 1(1) = C(μ),
K 4(1) = C(μ).

In fact, through our analysis it is easy to see that the large Nc limit and HQET are commutative; in other words, the large Nc approximation preserves all relations obtained in HQET.

The Weak Decays

As an application, we now calculate the Ωb → Ωc weak decay rates [?]. Since Σ()b *has strong interaction decay modes, we mainly consider the semileptonic decays of Ωb. In the SU(3) light quark flavor symmetry limit, Ω()*b(c) baryons are identical to Σ()*b(c) baryons. Therefore, the same results for the Isgur-Wise functions can be obtained for Ω()*b(c).

Neglecting lepton masses, for the decay Ωb → Ωc Γ the differential decay rate can be expressed [?, ?] in terms of the general form factors in Eq. (1) as:

dΓ1( )/d = (G^{2}F|Vcb|^{2}m^{3}Ωb/96 ^{3}) ^{2}(+ 1) × [F ^{2}1 + (+ 1) ^{2}F ^{2}2 + 2(+ 1) ^{2}G ^{2}1 + (+ 1)(1 + r2)F 1G 1 + (+ 1)(1 - r2)F 2G 1 + (+ 1)r2F 1G 2],

where r2 = mΩc/mΩb and ^{2} = 1 + r2^2 - 2r2 .

For the decay Ωb → Ω*c Γ, we have [?]:

dΓ2( )/d = (G^{2}F|Vcb|^{2}m^{3}Ωb/96 ^{3}) ^{3} × [2(+ 1)N ^{2}1 + (+ 1) ^{2}N ^{2}2 + 2(+ 1) ^{2}K ^{2}1 + (+ 1)(r3 + 1)N 1K 1 + (+ 1)(r3 - 1)N 2K 1 + (+ 1)K ^{2}3 + (^{2} - 1)(r3 + 1)K 1K 2 + (^{2} - 1)(r3 - 1)K 3K 2 + (^{2} - 1)K ^{2}4], where r3 = mΩ*c/mΩb and ³ = 1 + r3² - 2r3 .

The form factors have been expanded in Eq. (26) to order 1/mc and 1/mb. There is only one Isgur-Wise function ( ) at leading order, one ( ) at subleading order, and the mass parameter ΛΩ. With Eq. (8) and ( ) = 0 [?], we obtain the decay widths:

$\Gamma(\Omega b \to \Omega c\ \Gamma) = 3.38 \times 10^{1}$ GeV,
$\Gamma(\Omega b \to \Omega^{*} c\ \Gamma) = 3.34 \times 10^{1}$ GeV.

In our calculations, we have taken the following parameters [?]: mΩb = 6.07 GeV, mΩc = 2.70 GeV, mΩ*c = 2.77 GeV, and |Vcb| = 40.9 × 10^{3}. The pole masses of heavy quarks have been taken as mb = 4.83 GeV and mc = 1.43 GeV.

Discussions

In this paper, we have studied O(1/mQ) universal baryonic Isgur-Wise functions in the large Nc limit, with our results explicitly listed in Eqs. (22), (23) and (25). As an application, we have calculated the semileptonic decays of Ωb. Actually, the same results could be obtained using the leading order large Nc analysis in \cite{9-11}, while our method is considerably simpler.

Let us now consider the uncertainties of our results. Since the 1/mQ corrections have been included, the uncertainties inherent in HQET have been suppressed to O(Λ²QCD/m²Q) 1/25. The remaining uncertainties arise from two approximations: flavor SU(3) symmetry and the large Nc limit. Effects of flavor SU(3) violation might not be substantial, especially near the zero recoil point. Since Ωb contains s-quarks, we could expect the effects to be of order (ms/ΛQCD) 30%, which would be small by choosing an appropriate renormalization scale μ. The main uncertainties are therefore produced by the large Nc approximation, about which we have limited knowledge. Sometimes they can be as large as 30%. In our case, however, such pessimism is unnecessary. As a general rule of thumb, the large N limit is a good approximation for baryons and also works well for Isgur-Wise functions. Because we replaced ΛΣ with ΛΩ in the decay calculation, we have already taken into account part of the flavor SU(3) violation effects and part of the corrections to the large Nc limit.

Finally, it is important to note that regardless of whether the large Nc limit can be treated as a good approximation, at least in the vicinity of the zero recoil point the uncertainties produced by the large Nc limit should not be substantial, since it preserves the normalizations of Isgur-Wise functions as in HQET, as we emphasized in Sec. III. This will be tested at the LHC or the proposed Z-factory in the near future.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under nos. 11075193 and 10821504, and by the National Basic Research

Program of China under Grant No. 2010CB833000.

References

[1] N. Isgur and M.B. Wise, Phys. Lett. B232, 113 (1989); 237, 527 (1990); M.B. Voloshin and M.A. Shifman, Sov.J.Nucl.Phys. 45 (1987) 292, Yad. Fiz. 45, 463 (1987); M.B. Voloshin and M.A. Shifman, Sov.J.Nucl.Phys. 47 (1988) 511, Yad. Fiz. 47, 801 (1988); E. Eichten, B.R. Hill, Phys. Lett. B234, 511(1990); E.V. Shuryak, Phys. Lett. B93, 134 (1980).

[2] H. Georgi, Phys. Lett. B240, 447 (1990); A.F. Falk, H. Georgi, B. Grinstein, and M.B. Wise, Nucl. Phys. B343, 1 (1990).

[3] N. Isgur and M.B. Wise, Nucl. Phys. B348, 276 (1991); H. Georgi, Nucl. Phys. B348, 293 (1991); T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B355, 38 (1991).

[4] For reviews, see A.V. Manohar and M.B. Wise, Heavy Quark Physics, (Cambridge University Press, Cambridge, England 2000); A. Grozin, Heavy Quark Effective Theory, (Springer, Newyork, 2004).

[5] G. ' t Hooft, Nucl. Phys. B72, 461 (1974); E. Witten, Nucl. Phys. B160, 57 (1979).

[6] N. Isgur and M. B. Wise, Nucl. Phys. B348, 276(1991). H. Georgi, Nucl. Phys. B348, 293(1991). T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B355, 38(1991).

[7] C. G. Boyd and D. E. Brahm, Phys. Lett. B254, 468 (1991).

[8] R. F. Dashen and A.V. Manohar, Phys. Lett. B315,425(1993); 315,438(1993); E. Jenkins, ibid. Phys. Lett. B315, 431(1993); 315, 441(1993); 315, 447(1993); R. F. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D49, 4713(1994); 51, 3697(1995); C. Carone, H. Georgi and S. Osofsky, Phys. Lett. B322,227(1994); M. A. Luty and J. March-Russell, Nucl. Phys. B426, 71(1994); J.-L. Gervais and B. Sakita, Phys. Rev. D30, 1795(1984).

[9] C. K. Chow, Phys. Rev. D51, 1224 (1995); C. K. Chow, Phys. Rev. D54, 873 (1996).

[10] F. Hussain and G. Thompson, Phys. Lett. B335, 205 (1994).

[11] F. Hussain, J. G. Körner, M. Krämer, and G. Thompson, Z. Phys. C51, 321 (1991); J. G. Körner, M. Krämer and D.Pirjol, Prog. Part. Nucl. Phys. 33, 787 (1994); J. G. Körner, Nucl. Phys. Proc. Suppl. 50, 140 (1996); F. Hussain, J. G. Körner, J. Landgraf and S. Tawfiq, Z. Phys. C69, 655 (1996).

[12] E. Jenkins, A. V. Manohar, and M. B. Wise, Nucl. Phys. B396, 27(1993); C. G. Callan and I. Klebanov, ibid. Nucl. Phys. B262, 365(1985); M. Rho, D. O. Riska, and N. N. Scoccola, Phys. Lett. B251, 597(1990); Z. Phys. A

341, 343(1992); Y. Oh, D.-P. Min, M. Rho, and N. N. Scoccola, Nucl. Phys. A 534,493(1991).

[13] E. Jenkins, A.V. Manohar, and M. B. Wise, Nucl. Phys. B396, 38 (1993).

[14] J.-P. Lee, C. Liu and H.S. Song, Phys. Rev. D58, 014013 (1998); C. Liu, Phys. Rev. D57, 1991 (1998).

[15] H. Georgi, B. Grinstein and M. Wise, Phys. Lett. B252, 456 (1990).

[16] M. Luke, Phys. Lett. B252, 447 (1990).

[17] Y.-B. Dai, C.-S. Huang, C. Liu, and C.-D. Lü, Phys. Lett. B371, 99 (1996).

[18] J. G. Körner, M. Krämer, and D. Pirjol, Prog. Part. Nucl. Phys.33, 787(1994).

[19] M.-k. Du and C. Liu, Phys. Rev. D 84 (2011) 056007.

[20] A. K. Leibovich and I. W. Stewart, Phys. Rev. D57, 5620(1998).

[21] J. Beringer et al. (Particle Data Group). Phys. Rev. D86, 010001 (2012).

Note: Figure translations are in progress. See original paper for figures.