EXCITED HEAVY BARYON MASSES FROM THE 1/Nc EXPANSION OF HQET (Postprint)

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Abstract

The mass spectra of the L = 1 orbitally excited heavy baryons with light quarks in both the spin-flavor symmetric and the mixed representations are studied by the 1/Nc expansion method in the framework of the heavy quark effective theory. The mixing effect between the baryons in the two representations is also considered. The general pattern of the spectrum is predicted which will be verified by the experiments in the near future.

Full Text

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The mass spectra of the $L = 1$ orbitally excited heavy baryons with light quarks in both the spin-flavor symmetric and the mixed representations are studied by the $1/N_c$ expansion method in the framework of the heavy quark effective theory. The mixing effect between the baryons in the two representations is also considered. The general pattern of the spectrum is predicted, which will be verified by experiments in the near future.

1 Introduction

Experimentally, a substantial amount of data for orbitally excited heavy baryons has been accumulating. The following charmed baryon states have been found: $\Lambda_c(2593)^+$ with $I(J^P) = 0(1/2^+)$, $\Lambda_c(2625)^+$ with $I(J^P) = 0(3/2^+)$ denoted as $\Lambda_{c1}(1/2^+)$ and $\Lambda_{c1}(3/2^+)$ from the quark model, and their strange analogues $\Xi_{c1}(1/2^+)$ and $\Xi_{c1}(3/2^+)$. Theoretical understanding of these baryons is necessary.

The heavy quark effective theory (HQET) provides a systematic way to investigate hadrons containing a single heavy quark. To obtain detailed predictions, however, some non-perturbative QCD methods must be employed, such as lattice simulation, $1/N_c$ expansion, chiral Lagrangian, and QCD sum rules. In this work, we report the application of the $1/N_c$ expansion method.

Let us first briefly review HQET. It is an effective field theory of QCD for heavy hadrons. In the limit $m_Q/\Lambda_{QCD} \to \infty$, the heavy quark spin-flavor symmetry (HQS) becomes explicit. The 4-velocity v of the heavy quark becomes a good quantum number. Because the heavy quark mass m_Q is unusable as an energy scale, it can be removed by redefining the heavy quark field: $h_v = e^{im_Q v\cdot x}Q$. The effective Lagrangian is then $\mathcal{L}_{eff} = \bar{h}_v v \cdot D h_v + O(1/m_Q)$. The hadron mass is expanded as $M_H = m_Q + \bar{\Lambda}_H + O(1/m_Q)$ in the effective theory, where $\bar{\Lambda}_H$ represents the contributions from the light degrees of freedom.

Second, let us consider the $1/N_c$ expansion. This is a non-perturbative method in QCD. The idea is to extract non-perturbative information about $SU(N_c)$ gauge theory by taking $N_c \to \infty$. The N_c counting rules are as follows: the interaction vertex scales as $g_s/\sqrt{N_c}$; the quark propagator remains unchanged; and the gluon propagator is represented by double lines, one for quarks and the other for anti-quarks.

For mesons, non-perturbative properties can be understood from the analysis of planar diagrams. The large N_c limit is quite successful. Because the meson decay amplitude scales as $1/\sqrt{N_c}$, mesons and glue states are free and stable. This agrees qualitatively with color confinement. Another example is the explanation of Zweig's rule.

For baryons, the diagrammatic method does not work. Instead, the Hartree approximation can be adopted. The key observation is that in the $N_c \to \infty$ limit, the interaction between any pair of quarks is negligible ($O(1/N_c)$), while the total potential on an individual quark, which is $O(1)$, is a sum of many small terms. Therefore, it can be regarded as a background potential or a c-number potential. For ground state baryons, the many-body wave function can be written as $\Psi(x_1, \dots, x_{N_c}, t) = \prod_{i=1}^{N_c} \phi(x_i, t)$, where $\phi(x, t)$ is a one-body state. Interesting results for large N_c baryons can be obtained: the baryon-(anti-)baryon interaction is $O(N_c)$, while the baryon-meson interaction is $O(1)$. However, the Hartree potential is not known, as two-body, three-body, and many-body interactions are equally important. One conjecture is that baryons are solitons of the mesonic theory—skyrmions.

More can be said about large N_c baryons. For ground state baryons, there is a contracted $SU(2N_f)$ light quark spin-flavor symmetry (LQS) in the large N_c limit. This was first obtained from the chiral perturbation theory of baryon-pion interactions when deriving consistency conditions for the coupling constants in the large N_c limit. It can also be understood in the Hartree picture. This makes a $1/N_c$ expansion based on the spin-flavor structure practically possible for baryons, and many quantitative predictions and further extensions have been made.

In fact, this expansion represents another scheme of the $1/N_c$ expansion. This can be seen simply by considering the masses of non-strange baryons:

$$M_H = N_c\Lambda_{QCD} + O(1) + O(1/N_c) + ...$$
$$= N_c\tilde{\Lambda}_{QCD} + c_1(S_l^2)/N_c + ...$$

where the first line represents the ordinary $1/N_c$ expansion, and the second line shows the expansion based on the spin-flavor structure. Of course, in the $N_c \to \infty$ limit, $M_H = \bar{\Lambda}_H = N_c\Lambda_{QCD} = N_c\tilde{\Lambda}_{QCD}$, which is not particularly useful.

2 Excited Heavy Baryons in the 1/Nc Expansion

For charmed baryons such as $\Lambda_{c1}(1/2^+)$ and $\Lambda_{c1}(3/2^+)$, we analyze their masses $\bar{\Lambda}_H$ in the $1/N_c$ expansion. The classification is according to the angular momentum J, the isospin I, and the total angular momentum of the light degrees of freedom J_l, which becomes a good quantum number due to HQS. In this case, the excited hadron spectrum shows degeneracy between pairs of states related by HQS.

Constituently, there are two ways to achieve $L = 1$ excitation: either the heavy quark is excited, or one light quark is excited. Correspondingly, under the LQS, the N_c light quarks are in the symmetric and mixed representations, respectively.

2.1 Symmetric Representation

In the symmetric representation, the picture for the light quarks is essentially the same as that for ground state heavy baryons. The spin-flavor decomposition rule is $I = S_l$ for non-strange baryons, where S_l is the total spin of the light quark system. Note that the light quark system as a whole has orbital angular momentum $L = 1$. All possible states of excited heavy baryons are listed in Table 1 .

In the Hartree-Fock picture of baryons, the N_c counting rules require us to include many-body interactions in the analysis. However, a large portion of these interactions are spin-flavor irrelevant, contributing universally at order $N_c\Lambda_{QCD}$ to all baryons with different spin-flavor structures in Table 1. The mass splittings among baryons can be obtained from the remaining interactions.

For the purely light quark contribution to $\bar{\Lambda}_H$, the $1/N_c$ analysis proceeds similarly to that for ground state heavy baryons. There is LQS at the leading order of the $1/N_c$ expansion, with mass splittings due to LQS violation starting at $O(1/N_c)$. However, unlike ground state baryons, the orbital angular momentum of the heavy quark formally gives a more dominant contribution to $\bar{\Lambda}_H$ than $O(1/N_c)$ due to orbital-light-quark-spin interactions.

After summing all relevant many-body interactions, this $O(1)$ contribution can be expanded. The mass $\bar{\Lambda}_H$ can be written simply as:

$$\bar{\Lambda}_H = N_c\tilde{c}_0 + \tilde{c}_1\vec{L}\cdot\vec{S}_l + O(1/N_c)$$

where the coefficients $\tilde{c}_i \sim \Lambda_{QCD}$ $(i = 0, 1)$. There should also be a term proportional to L^2 in the above equation, which gives a constant contribution to $\bar{\Lambda}_0$ for a given light quark representation and has therefore been absorbed into $\tilde{c}_0$. With $\vec{J}_l$ defined as $\vec{J}_l = \vec{L} + \vec{S}_l$, the term $\vec{L}\cdot\vec{S}_l$ can be rewritten as $(J_l^2 - L^2 - S_l^2)/2$. Since $L = 1$ and S_l is fixed for a given representation, the mass formula becomes:

$$\bar{\Lambda}_H = N_c c_0 + c_1(J_l^2) + O(1/N_c)$$

where the coefficients $c_i \sim \Lambda_{QCD}$.

2.2 Mixed Representation

In the mixed representation, all states are listed in Table 2 . Again, $\bar{\Lambda}'_H$ is trivially $N_c\Lambda_{QCD}$ at the leading order of the $1/N_c$ expansion. However, the spin-flavor dependence is more complicated. For the spectrum of excited light baryons, see ref. 8.

The many-body Hamiltonians related to the spin-flavor structure that involve orbital angular momentum L give $O(1)$ contributions. We use the following operators, which were employed in ref. 8, to analyze $\bar{\Lambda}_H$:

$$H_{LS} = \sum_a \hat{a}^\dagger\vec{\sigma}\hat{a}\cdot\vec{L}\tau^a\hat{a}^\dagger\hat{a}$$
$$H_T = \sum_{a,i} G_i^a G_i^a$$
$$H_1 = \sum_{a,i} \hat{a}^\dagger\sigma_i\tau^a\hat{a}L_i$$
$$H_2 = \sum_{a,i,j} G_i^a L_i L_j \tau^a G_j^a$$

The first one, H_{LS}, is a one-body Hamiltonian, while the others are two-body Hamiltonians. G_i^a are the generators of the spin-flavor symmetry group $SU(4)$, given by $G_i^a = \hat{a}^\dagger\sigma_i\tau^a\hat{a}$, with σ_i and τ^a being the spin and isospin matrices, respectively. Such structure gives coherent addition over $N_c - 2$ core quarks. The first G_i^a in H_T acts on the excited quark, while the other G_i^a's act on the $N_c - 2$ unexcited light quarks, namely the core quarks. In our case, all operators must be understood as acting on the light degrees of freedom. Note that higher-order many-body Hamiltonians containing more factors of G_i^a can be reduced to those given in the equation above.

The contributions to baryon masses from these Hamiltonians are obtained by calculating baryonic matrix elements. The matrix elements of these operators between the states of light quarks that specify the states of excited heavy baryons are given as follows:

$$\langle N_c-1; II_3; S_l' S_{l3}' | H_T | N_c-1; II_3; S_l S_{l3}\rangle = 2c_T \delta_{S_l',S_l}\delta_{S_{l3}',S_{l3}}\delta_{m,m'}(-1)^{1-S_l-I} \times (2I+1)(2S_l+1)\left\{\begin{smallmatrix} I & 1 & I \\ S_l & 1/2 & 1/2 \end{smallmatrix}\right\}\left\{\begin{smallmatrix} S_l & 1 & S_l \\ 1/2 & 1/2 & 1/2 \end{smallmatrix}\right\}$$

$$\langle N_c-1; II_3; l=1, S_l', J_l J_{l3} | H_{LS} | N_c-1; II_3; l=1, S_l, J_l J_{l3}\rangle = c_{LS}(-1)^{S_l-S_l'} \times \sqrt{(2S_l+1)(2S_l'+1)}\sum_{j=1/2,3/2}(2j+1)\left\{\begin{smallmatrix} 1 & 1/2 & j \\ 1/2 & S_l & 1 \end{smallmatrix}\right\}\left\{\begin{smallmatrix} 1 & 1/2 & j \\ 1/2 & S_l' & 1 \end{smallmatrix}\right\}$$

$$\langle N_c-1; II_3; l=1, S_l', J_l J_{l3} | H_1 | N_c-1; II_3; l=1, S_l, J_l J_{l3}\rangle = 6\bar{c}_1(-1)^{I-J_l+S_l-S_l'} \times \sqrt{(2S_l+1)(2S_l'+1)}\left\{\begin{smallmatrix} I & 1 & I_c \\ 1/2 & 1/2 & 1/2 \end{smallmatrix}\right\}\left\{\begin{smallmatrix} S_l & 1 & S_l' \\ J_l & I & I_c \end{smallmatrix}\right\}$$

$$\langle N_c-1; II_3; l=1, S_l', J_l J_{l3} | H_2 | N_c-1; II_3; l=1, S_l, J_l J_{l3}\rangle = 3\bar{c}_2(-1)^{1+J_l+I+S_l'} \times \sqrt{(2S_l+1)(2S_l'+1)}\left\{\begin{smallmatrix} I & 1 & I_c \\ 1/2 & 1/2 & 1/2 \end{smallmatrix}\right\}\left\{\begin{smallmatrix} S_l & 2 & S_l' \\ J_l & 1 & J_l \end{smallmatrix}\right\}$$

where I_c is the isospin of the core quarks. In the real world ($N_c = 3$), there is only one quark in the core, so I_c always equals 1/2. With these matrix elements, we can express the excited heavy baryon mass up to zeroth order in $1/N_c$:

$$\bar{\Lambda}_H' = N_c\bar{c}_0 + c_{LS}\langle H_{LS}\rangle + c_T\langle H_T\rangle + \bar{c}_1\langle H_1\rangle + \bar{c}_2\langle H_2\rangle$$

where c_{LS}, c_T, and $\bar{c}_i$'s are coefficients $\sim \Lambda_{QCD}$.

2.3 Mixing

It is necessary to consider mixing between baryons with light quarks in the spin-flavor symmetric and mixed representations. When they share the same quantum numbers (J, I, J_l), there is no physical way to distinguish them. This consideration yields the physical spectrum. Because of the light quark spin-flavor symmetry at the leading order of the $1/N_c$ expansion, baryons with the same (J, I, J_l) quantum numbers but in different representations do not mix (S_l is a good quantum number in $N_c \to \infty$). The mixing occurs at sub-leading order. The classification of baryons by spin-flavor symmetry is therefore physical at leading order.

For the physical spectrum, the mixing results in a deviation from $\bar{\Lambda}_H^0$. Denoting the mixing mass as $\tilde{m}$, which is $O(1)$, the mass matrix for baryons with the same (J, I, J_l) is written as:

$$\begin{pmatrix} \bar{\Lambda}_H^0 & \tilde{m} \\ \tilde{m} & \bar{\Lambda}_H'^0 \end{pmatrix}$$

The mass difference $\bar{\Lambda}_H'^0 - \bar{\Lambda}_H^0$ is $O(1)$. Taking $\tilde{m} < \bar{\Lambda}_H'^0 - \bar{\Lambda}_H^0$ for illustration, the physical masses are corrected to be:

$\bar{\Lambda}_H^0 + \tilde{m}^2/(\bar{\Lambda}_H'^0 - \bar{\Lambda}_H^0)$ and $\bar{\Lambda}_H'^0 - \tilde{m}^2/(\bar{\Lambda}_H'^0 - \bar{\Lambda}_H^0)$

The $1/N_c$ expansion of $\tilde{m}$ is parameterized as $\tilde{m} = \tilde{m}_0 + O(1/N_c)$, where $\tilde{m}_0$ is universal due to LQS. To $O(1)$, the spectrum is:

$$\bar{\Lambda}(1/2,0,1) = N_c c_0 + 2c_1 - 2c_1^2/k$$
$$\bar{\Lambda}(3/2,0,1) = N_c c_0 - 6\bar{c}_1 - 4\bar{c}_2 - 18c_T - 18c_T^2/k$$
$$\bar{\Lambda}(1/2,1,0) = N_c c_0 - 2\bar{c}_1 + 4\bar{c}_2$$
$$\bar{\Lambda}(3/2,1,1) = N_c c_0 + 4c_1 - 4c_1^2/k$$
$$\bar{\Lambda}(5/2,1,2) = N_c c_0 + 4c_1$$

where k is an $O(1)$ constant representing what remains after cancellation between $\bar{\Lambda}_H^0$ and $\bar{\Lambda}_H'^0$. Note that the masses of states $(1/2,1,0)$ and $(3/2,1,2)$ are not affected by mixing, as there are no physical states with the same quantum numbers in the mixed representation.

From the above spectrum, we see that $c_1 > 0$. The states $(5/2,1,2)$ are always the highest states. They are heavier than the other states by at least $4c_1$, which follows from requiring the states $(1/2,0,1)$ to be the lowest. If $2c_1 > 6\bar{c}_1 - 4\bar{c}_2 - 18c_T$, the requirement implies $6\bar{c}_1 - 4\bar{c}_2 - 18c_T > 4c_1$, giving the spectrum pattern:

$$M(1/2,0,1) < M(3/2,1,0) < M(1/2,1,1) < M(3/2,1,2) < M(5/2,1,2)$$

On the other hand, if $2c_1 < 6\bar{c}_1 - 4\bar{c}_2 - 18c_T$, the requirement is:

$$6\bar{c}_1 - 4\bar{c}_2 - 18c_T - 2c_1 - k > 2c_1$$

which gives the spectrum:

$$M(1/2,0,1) < M(1/2,1,0) < M(3/2,1,0) < M(1/2,1,1) < M(3/2,1,2) < M(5/2,1,2)$$

The experimentally found baryons $\Lambda_{c1}(1/2^+)$ and $\Lambda_{c1}(3/2^+)$ correspond to the $(1/2,0,1)$ and $(3/2,0,1)$ states, respectively. More data are needed to determine the unknown parameters c_i's, $\bar{c}_i$'s, k, and c_{LS}. In the near future, experiments will test the predicted spectrum. Hopefully, one of the above mass patterns will be selected. This will serve as a check of our method's validity if the parameters are in a reasonable range (Λ_{QCD}) and satisfy the relations given above.

3 Summary

In summary, we have applied the $1/N_c$ expansion method to study the spectra of $L = 1$ orbitally excited heavy baryons within the framework of HQET. The analysis is remarkably simple for baryons with light quarks in the spin-flavor symmetric representation, compared to that for heavy baryons with light quarks in the mixed representation. This simplicity is a unique feature that arises because the light quark system remains in its ground state while the heavy quark is orbitally excited. However, the mixing effect from baryon states in the mixed representation corrects the spectrum pattern at sub-leading order in the $1/N_c$ expansion. This effect is important for obtaining realistic spectra at this order. The general pattern of the baryon spectrum has been presented, which will be verified by experiments in the near future. The $1/m_Q$ and $SU(3)$ corrections have been considered in ref. 5, and certain mass relations for the baryons $\Lambda_{c1}^{(*)}$, $\Sigma_{c1}^{(*)}$, $\Xi_{c1}^{(*)}$, and $\Omega_{c1}^{(*)}$ have been derived. The same analysis can be applied to bottom baryons.

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