A Phenomenological Study on Lepton Mass Matrix Textures (Postprint)

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Abstract

The three active light neutrinos are used to explain the neutrino oscillations. The inherently bi-large mixing neutrino mass matrix and the Fritzsch type, bi-small mixing charged lepton mass matrix are assumed. By requiring the maximal #23; −#23;#28; mixing for the atmospheric neutrino problem and the mass-squared difference appropriate for the almost maximal mixing solution to the solar neutrino problem, the following quantities are predicted: the #23;e−#23; mixing, Ve3, CP violation in neutrino oscillations, and the effective electron-neutrino mass relevant to neutrinoless double beta decays.

Full Text

Preamble

A Phenomenological Study on Lepton Mass Matrix Textures

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Abstract

We employ three active light neutrinos to explain neutrino oscillations, assuming an inherently bi-large mixing neutrino mass matrix and a Fritzsch-type, bi-small mixing charged lepton mass matrix. By requiring maximal $\nu_\mu - \nu_\tau$ mixing for the atmospheric neutrino problem and an appropriate mass-squared difference for the almost maximal mixing solution to the solar neutrino problem, we predict the following quantities: the ν_e mixing, V_{e3}, CP violation in neutrino oscillations, and the effective electron-neutrino mass relevant to neutrinoless double beta decays.

Keywords: lepton, neutrino oscillation, CP violation

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Introduction

Understanding the fermion mass pattern represents a great challenge in elementary particle physics. Lacking a standard theory for flavor physics, phenomenological ansätze might prove very helpful [?]. In view of recent observations of neutrino oscillations [?], this paper studies the lepton sector. The masses of charged leptons have been experimentally determined quite well [?] and are expected to have a similar origin as quarks, which exhibit small mixings among three generations. The small neutrino masses indicated by experiments can be naturally understood via the seesaw mechanism [?]. However, observations have increasingly shown evidence that leptonic mixings are bi-maximal or almost bi-maximal among the three generations. Such mixing scenarios have been considered extensively [?, ?, ?].

This paper begins from the flavor eigenstates of both charged leptons and neutrinos. We assume the charged lepton mass matrix is of the Fritzsch type [?]:

$$M_l = \begin{pmatrix} 0 & ae^{-i\alpha} & 0 \\ ae^{-i\alpha} & 0 & be^{-i\beta} \\ 0 & be^{-i\beta} & c \end{pmatrix}$$

where $a > 0$ and $a < b^2/c$. The neutrino mass matrix is of the inherently bi-large mixing type [?]:

$$M_\nu = \begin{pmatrix} \epsilon & m_1 & m_2 \\ m_1 & \epsilon & 0 \\ m_2 & 0 & \epsilon \end{pmatrix}$$

where $m_1 \sim \epsilon > 0$. Note that m_1, m_2, and ϵ are always real in the above form of M_ν. These two matrices offer simplicity in analysis, and the parameters in them are uniquely fixed. Although Eq.~(2) will be speculated upon further at the end of this paper, we still have no definite principles for them. Additional theoretical works on bi-maximal leptonic mixing have been considered in Refs.~[?, ?, ?].

Mass Relations and Diagonalization

Equation~(1) yields the charged lepton masses:

$$m_e = \frac{ab^2}{c(a+b^2/c)} - \frac{ab^2}{c(a+b^2/c)} \cdot \frac{m_\mu m_\tau}{m_\mu + m_\tau}, \quad m_\mu = \frac{ab^2}{c(a+b^2/c)} + \frac{ab^2}{c(a+b^2/c)} \cdot \frac{m_\tau m_e}{m_\mu + m_\tau}, \quad m_\tau = c - \frac{ab^2}{c(a+b^2/c)}$$

Equation~(2) gives the neutrino masses:

$$m_{\nu_1} = \epsilon + \sqrt{m_1^2 + m_2^2}, \quad m_{\nu_2} = \epsilon + \sqrt{m_1^2 + m_2^2}, \quad m_{\nu_3} = \epsilon$$

Charged leptons provide bi-small mixing among the three generations, whereas neutrinos provide bi-large mixing. The diagonalization of M_l is performed by the following unitary matrix:

$$U_l = \begin{pmatrix} U_{11}^l e^{-i\alpha} & U_{12}^l e^{-i\alpha} & U_{13}^l e^{-i\alpha} \\ U_{21}^l e^{-i(\alpha+\beta)} & U_{22}^l e^{-i(\alpha+\beta)} & U_{23}^l e^{-i(\alpha+\beta)} \\ U_{31}^l & U_{32}^l & U_{33}^l \end{pmatrix}$$

where the U_{ij}^l elements are determined by the mass ratios. The neutrino mass matrix M_ν is diagonalized by:

$$U_\nu = \begin{pmatrix} \sin\theta & \cos\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $\sin\theta = m_1/\sqrt{m_1^2 + m_2^2}$. Note that U_ν is independent of ϵ.

Physical Mixing Matrix

The physical lepton mixing is given by:

$$V = U_l^\dagger U_\nu$$

In our scenario, $\cos\theta$ deviates from $1/\sqrt{2}$ only slightly. It is the combination of the large mixing from U_ν and the small mixing from U_l that yields the maximal mixing of $\nu_\mu - \nu_\tau$. The $\nu_e - \nu_\mu$ mixing deviates from maximal mixing remarkably because the (23) component of V is mainly composed of $\cos\theta$ and U_{23}^l, which is not negligible. On the other hand, the matrix U_ν itself would give a maximal mixing $\nu_\mu - \nu_\tau$ oscillation, because the charged lepton contribution to V_{12} is only about $m_e/m_\mu \sim m_\mu/m_\tau \sim 10^{-2}$.

Numerical Results

Let us discuss the numerical results. The atmospheric neutrino problem indicates $\Delta m^2_{\text{atm}} \sim 10^{-3}$ eV^2. By requiring maximal $\nu_\mu - \nu_\tau$ mixing, we obtain:

$$\sqrt{m_1^2 + m_2^2} \simeq 0.05 \text{ eV}, \quad m_2 \simeq 10^{-2} \text{ eV}$$

The solar neutrino problem is solved by the energy-independent solution [?], which requires $\Delta m_{\odot}^2 \sim 10^{-4}$ eV^2 or $10^{-6} - 10^{-8}$ eV^2. With the above results, we get:

$$\sin^2 2\theta_{e\mu} \simeq 0.98$$

The $\nu_e - \nu_\tau$ mixing is predicted as:

$$|V_{e3}| \simeq 0.06$$

CP violation in neutrino oscillations is determined by the rephasing-invariant parameter J [?]:

$$J = \mathrm{Im}(V_{i\lambda} V_{j\rho} V_{i\rho}^* V_{j\lambda}^*) = \mathrm{Im}(V_{11} V_{22} V_{12}^* V_{21}^*)$$

In our case, Eqs.~(5)-(8) give:

$$J = \frac{(U_{11}^l U_{22}^l - U_{12}^l U_{21}^l) \sin\theta \cos\theta \sin\alpha}{1 + m_2^2/m_1^2}$$

Numerically, choosing $\alpha = \beta = \pi/2$, we obtain $J \approx 0.008$; choosing $\alpha = 0$ and $\beta = \pi/2$ yields $J \approx 0.004$.

The neutrinoless double beta decay experiments will measure the effective electron-neutrino mass, which in our case (keeping ϵ terms to leading order) is:

$$m_{\nu_e}^{\mathrm{eff}} \equiv \left| \sum_\lambda V_{e\lambda}^2 m_{\nu_\lambda} \right| \simeq \frac{\epsilon}{\sqrt{1 + m_2^2/m_1^2}} \left[(U_{21}^l \sin\theta + U_{31}^l \cos\theta \cos\beta)^2 + (U_{31}^l \cos\theta \sin\beta)^2 \right]^{1/2} \simeq 0.006 \text{ eV}$$

Experimental Tests

Experiments in the near future will test the lepton mass matrices studied in this paper. In addition to SNO, Borexino and KamLAND will check the result of Eq.~(11) for the $\nu_e - \nu_\mu$ mixing [?]. Long-baseline neutrino experiments [?] and neutrino factories will measure V_{e3} and CP violation in neutrino oscillations. GENIUS will be able to test the effective electron-neutrino mass down to 10^{-3} eV.

Underlying Theory

Finally, let us examine the underlying reasons for the neutrino mass matrix in Eq.~(2). These Majorana masses are thought to be generated by the seesaw mechanism. It is natural to assume that the Dirac neutrino mass matrix has a similar form to that of charged leptons, given by:

$$M_D = \begin{pmatrix} 0 & \tilde{a} & 0 \\ \tilde{a} & 0 & \tilde{b} \\ 0 & \tilde{b} & \tilde{c} \end{pmatrix}$$

where possible phases are not considered because M_ν in Eq.~(2) is real and we are concerned with magnitudes of right-handed neutrino masses. In this case, the texture of Eq.~(2) requires the following form of the right-handed neutrino mass matrix:

$$M_R = \begin{pmatrix} \tilde{a}^2 \cos^2\theta & \tilde{a}\tilde{b}\sin\theta\cos\theta & \tilde{a}\tilde{c}\cos\theta \\ \tilde{a}\tilde{b}\sin\theta\cos\theta & \tilde{b}^2\sin^2\theta & \tilde{b}\tilde{c}\sin\theta \\ \tilde{a}\tilde{c}\cos\theta & \tilde{b}\tilde{c}\sin\theta & \tilde{c}^2 \end{pmatrix}^{-1} \cdot \frac{v^2}{1 + m_2^2/m_1^2}$$

Note that in the above equation, the first matrix is the leading one, but it is rank one. Only with the second matrix, which serves as a perturbation to the first, is M_R nonsingular. In the right-handed neutrino spectrum, there is one heavy neutrino with mass around 10^{15} GeV, and two relatively light neutrinos about two orders of magnitude smaller than the first, if we take $\epsilon \sim 10^{-4}$ eV. It appears that some tuning is needed to maintain the texture assumed in Eq.~(2). We wonder whether there exists a natural way to produce it, for instance from some flavor symmetry.

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Note: Figure translations are in progress. See original paper for figures.