Understanding the D_{sJ}(2317)^+ and D_{sJ}(2460)^+ with Sum Rules in HQET Postprint

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Full Text

Preamble

Understanding the $D_s^+J(2317)$ and $D_s^+J(2460)$ with Sum Rules in HQET

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Abstract

In the framework of heavy quark effective theory we use QCD sum rules to calculate the masses of the $\bar{c}s$ $(0^+, 1^+)$ and $(1^+, 2^+)$ excited states. The results are consistent with the interpretation that the states $D_{sJ}(2317)$ and $D_{sJ}(2460)$ observed by BABAR and CLEO are the 0^+ and 1^+ states in the $j_\ell = \frac{1}{2}$ doublet.

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1. Introduction

Recently the BaBar Collaboration announced the discovery of a positive-parity narrow state with a mass of approximately 2317 MeV in the $D_s^+(1969)\pi$ channel [?], which was subsequently confirmed by CLEO [?] and BELLE [?]. Based on its low mass and decay angular distribution, its J^P is believed to be 0^+.

In the same experiment, CLEO [?] observed a state at 2460 MeV with possible spin-parity $J^P = 1^+$ in the $D_s^*\pi$ channel; BaBar [?] also found a signal in this region. Since these two states lie below the DK and D^*K thresholds respectively, the potentially dominant s-wave decay modes such as $D_{sJ}(2317) \to D_s K$ are kinematically forbidden. Consequently, radiative decays and isospin-violating strong decays become the favored decay modes. The latter decay proceeds in two steps with the help of virtual $D_s\eta$ intermediate states, $D_{sJ}(2317) \to D_s\pi^0$, where the second step arises from the tiny η-π^0 isospin-violating mixing due to $m_u \neq m_d$.

The experimental discovery of these two states has triggered heated debate in the literature regarding their nature. The key issue is understanding their low masses. The $D_{sJ}(2317)$ mass is significantly lower than the 0^+ mass values in the range of 2.4 GeV calculated in quark models [?]. A model using the heavy quark mass expansion of the relativistic Bethe-Salpeter equation [?] predicted a lower value of 2.369 GeV for the 0^+ mass, which is still 50 MeV higher than the experimental data. Bardeen, Eichten, and Hill interpreted these states as the $\bar{c}s$ $(0^+, 1^+)$ spin doublet serving as parity conjugate states of the $(0^-, 1^-)$ doublet in the framework of chiral symmetry [?] (see also Ref. [?]). A quark-antiquark picture was also advocated by Colangelo and Fazio [?], Cahn and Jackson [?], and Godfrey [?]. Based on this "conventional" picture, various decay modes were discussed in Refs. [?, ?, ?].

Apart from the quark-antiquark interpretation, $D_{sJ}(2317)$ was suggested to be a four-quark state by Cheng and Hou [?] and by Barnes, Close, and Lipkin [?]. Szczepaniak even argued that it could be a strong $D\pi$ atom [?]. However, we consider it would be very exceptional for a molecule or atom to have a binding energy as large as 40 MeV. Van Beveren and Rupp [?] argued from experience with $a_0/f_0(980)$ that the low mass of $D_{sJ}(2317)$ could arise from mixing between the DK continuum and the lowest scalar nonet. In this way, the 0^+ $\bar{c}s$ state is artificially pushed much lower than expected from quark models.

Recent lattice calculations suggest a value around 2.57 GeV for the 0^+ state mass [?], much larger than the experimentally observed $D_{sJ}(2317)$ and compatible with quark model predictions. The conclusion in Ref. [?] is that $D_{sJ}(2317)$ might contain a large DK component, with physics resembling $a_0/f_0(980)$. Such a large DK component makes lattice simulation very difficult.

In this paper we shall use QCD sum rules [?] in the framework of heavy quark effective theory (HQET) [?] to extract the masses, since HQET provides a systematic method to compute properties of heavy hadrons containing a single heavy quark via the $1/m_Q$ expansion, where m_Q is the heavy quark mass. The masses of ground-state heavy mesons have been studied with QCD sum rules in HQET in [?, ?, ?]. In [?, ?, ?], masses of the lowest excited non-strange heavy meson doublets $(0^+, 1^+)$ and $(1^+, 2^+)$ were studied with sum rules in HQET up to order $(1/m_Q)$. These masses were also analyzed in earlier works [?] with sum rules in full QCD. In this short note we extend the same formalism from [?, ?] to include the light quark mass in calculating $\bar{c}s$ mesons. We shall also use more stringent criteria for the stability windows of the sum rules in the numerical analysis.

The proper interpolating current $J^{\alpha_1\cdots\alpha_j}_{j,P,j_\ell}$ for a state with quantum numbers (j,P,j_ℓ) in HQET was given in [**?**]. These currents satisfy the following conditions:

$$\langle 0|J^{\alpha_1\cdots\alpha_j}_{j,P,j_\ell}(0)|j',P',j'_\ell\rangle = f_{P,j_\ell}\delta_{jj'}\delta_{PP'}\delta_{j_\ell j'_\ell}\eta^{\alpha_1\cdots\alpha_j}, \tag{1}$$

$$\begin{aligned}\Pi^{\alpha_1\cdots\alpha_j,\beta_1\cdots\beta_j}_{P,j_\ell}(x) &\equiv \langle 0|T\{J^{\alpha_1\cdots\alpha_j}_{j,P,j_\ell}(x)J^{\dagger\beta_1\cdots\beta_j}_{j',P',j'_\ell}(0)\}|0\rangle \\ &= \delta_{jj'}\delta_{PP'}\delta_{j_\ell j'_\ell}g_t^{\alpha_1\beta_1}\cdots g_t^{\alpha_j\beta_j}\int dt\,\delta(x-vt)\Pi_{P,j_\ell}(x),\end{aligned} \tag{2}$$

where in the $m_Q\to\infty$ limit v is the velocity of the heavy quark, $g_t^{\alpha\beta}=g^{\alpha\beta}-v^\alpha v^\beta$ is the transverse metric tensor, $\eta^{\alpha_1\cdots\alpha_j}$ is the polarization tensor for the spin-j state, and $(\alpha_1\cdots\alpha_j)$ denotes symmetrizing the indices and subtracting the trace terms separately in the sets $(\alpha_1\cdots\alpha_j)$ and $(\beta_1\cdots\beta_j)$. The quantities f_{P,j_ℓ} and Π_{P,j_ℓ} are a constant and a function of x respectively, which depend only on P and j_ℓ.

We consider the correlator

$$\Pi(\omega) = i\int d^4x\, e^{ik\cdot x}\langle 0|T\{J^{\alpha_1\cdots\alpha_j}_{j,P,j_\ell}(x)J^{\dagger\beta_1\cdots\beta_j}_{j',P',j'_\ell}(0)\}|0\rangle, \tag{3}$$

where $\omega=2v\cdot k$. It can be written as

$$\Pi(\omega) = \frac{f^2_{P,j_\ell}}{2\bar\Lambda_{j,P,j_\ell}-\omega} + \text{higher states}, \tag{4}$$

where $\bar\Lambda_{j,P,j_\ell} = \lim_{m_Q\to\infty}(m_{M_{j,P,j_\ell}}-m_Q)$. On the other hand, it will be calculated in terms of quarks and gluons. Invoking Borel transformation to Eq. (3) we get

$$f^2_{P,j_\ell}e^{-2\bar\Lambda_{j,P,j_\ell}/T} = \int_0^{\omega_c} d\omega\,\rho(\omega)e^{-\omega/T} + \text{condensates}, \tag{5}$$

where m_q is the light quark mass, $\rho(\omega)$ is the perturbative spectral density, and ω_c is the threshold parameter used to subtract the higher-state contribution with the help of quark-hadron duality assumption.

We shall study the low-lying $(0^+,1^+)$ and $(1^+,2^+)$ $\bar cs$ states. The values of various QCD condensates are

$$\langle\bar{q}q\rangle = -(0.24\,\mathrm{GeV})^3, \quad \langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle, \quad \langle\alpha_s G^2\rangle = 0.038\,\mathrm{GeV}^4, \quad \langle g_s\bar{q}\sigma Gq\rangle = (0.8\,\mathrm{GeV}^2)\langle\bar{q}q\rangle. \tag{6}$$

We use $m_s(1\mathrm{GeV}) = 0.15$ GeV for the strange quark mass in the $\overline{\mathrm{MS}}$ scheme. We use $\Lambda_{\mathrm{QCD}} = 375$ MeV for three active flavors and $\Lambda_{\mathrm{QCD}} = 220$ MeV for four active flavors.

The sum rules with massless light quarks have been obtained in [**?**, **?**]. In the numerical analysis of the QCD sum rules we require that the high-order power corrections be less than 30% of the perturbation term. This condition yields the minimum value $T_{\min}$ of the allowed Borel parameter. We also require that the pole term, which is equal to the sum of the cut-off perturbative term and the condensation terms, be larger than 60% of the perturbative term, which leads to the maximum value $T_{\max}$ of the allowed T. Thus we have the working interval $T_{\min} < T < T_{\max}$ for a fixed ω_c. If $T_{\min} \geq T_{\max}$, we are unable to extract useful information from such a sum rule.

In the ideal case, the difference between the meson mass m_M and the heavy quark mass m_Q (or other observables) is almost independent of T for certain values of ω_c, meaning the dependence on both the Borel parameter and continuum threshold is minimal. In realistic cases, the variation of a sum rule with both T and ω_c will contribute to the errors of the extracted value, together with the truncation of the operator product expansion and the uncertainty of vacuum condensate values.

In subsections 2.1 and 2.2 we present sum rules for the $(0^+, 1^+)$ and $(1^+, 2^+)$ doublets using interpolating currents with derivatives, respectively. For comparison we discuss sum rules for $(0^+, 1^+)$ using interpolating currents without derivatives in subsection 2.3.

2.1 $(0^+, 1^+)$ Doublet With Derivative Currents

We employ the following interpolating currents [**?**]:

$$J_{0,+,1/2} = \bar{h}_v \gamma_5 \frac{i\not{D}_t}{\tau} s, \tag{7}$$

$$J^{\alpha}_{1,+,1/2} = \bar{h}_v \gamma_5 \gamma^{\alpha} \frac{i\not{D}_t}{\tau} s, \tag{8}$$

where $D_\mu = \partial_\mu + igA_\mu$ is the gauge-covariant derivative and $D_t^\mu = D^\mu - v^\mu v\cdot D$.

We obtain the sum rule relevant to $\bar{\Lambda}$:

$$f^2_{+,1/2} e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega\, \frac{\omega^4 + 2m_s\omega^3}{16\pi^2} e^{-\omega/T} + \langle\bar{s}s\rangle \int_0^{\omega_c} d\omega\, \omega e^{-\omega/T}. \tag{9}$$

Taking the derivative of the logarithm of the above equation with respect to $(1/T)$ yields the sum rule for $\bar{\Lambda}$. Substituting the obtained value of $\bar{\Lambda}$ into (8) gives the sum rule for $f_{+,1/2}$. In Figure 1 [Figure 1: see original paper], the variation of $\bar{\Lambda}(1/2^+)$ with T and ω_c is shown. According to the criteria stated above, the working range is $0.38 < T < 0.58$ GeV. We obtain

$$\bar{\Lambda}(1/2^+) = (0.86 \pm 0.10)\ \text{GeV}, \quad f_{+,1/2} = (0.31 \pm 0.05)\ \text{GeV}^{5/2}, \tag{10}$$

where the central value corresponds to $T = 0.52$ GeV and $\omega_c = 2.9$ GeV.

2.2 $(1^+, 2^+)$ Doublet

For the $(1^+, 2^+)$ doublet, using the following interpolating currents [**?**]:

$$J^{\alpha_1}_{1,+,3/2} = \bar{h}_v \gamma_5 \left(\frac{iD_t^{\alpha_1}}{\tau} - \frac{1}{3}\gamma^{\alpha_1} \frac{i\not{D}_t}{\tau} \right) s, \tag{11}$$

$$J^{\alpha_1\alpha_2}_{2,+,3/2} = \bar{h}_v \gamma_5 \frac{iD_t^{\alpha_1}}{\tau} \gamma^{\alpha_2} s + \text{symmetrization}, \tag{12}$$

we obtain the sum rule

$$f^2_{+,3/2} e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega\, \frac{\omega^4 + 2m_s\omega^3}{48\pi^2} e^{-\omega/T} + \langle \bar{s}s \rangle \int_0^{\omega_c} d\omega\, \omega e^{-\omega/T}. \tag{13}$$

From this sum rule, the variation of $\bar{\Lambda}(3/2^+)$ with T and ω_c is plotted in Figure 2 [Figure 2: see original paper]. The working range is $0.55 < T < 0.65$ GeV. We find

$$\bar{\Lambda}(3/2^+) = (0.83 \pm 0.10)\ \text{GeV}, \quad f_{+,3/2} = (0.19 \pm 0.03)\ \text{GeV}^{5/2}, \tag{14}$$

where the central value corresponds to $T = 0.62$ GeV and $\omega_c = 3.0$ GeV.

2.3 $(0^+, 1^+)$ Doublet With Non-Derivative Currents

Possible alternative currents for the $(0^+, 1^+)$ doublet are the non-derivative currents:

$$J_{0,+,1/2} = \bar{h}_v s, \tag{15}$$

$$J^{\alpha}_{1,+,1/2} = \bar{h}_v \gamma_5 \gamma_t^{\alpha} s. \tag{16}$$

With these non-derivative currents the sum rule reads

$$f_{+,1/2}^2 e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega \, \frac{\omega^2}{8\pi^2} e^{-\omega/T} + \langle \bar{s}s \rangle \int_0^{\omega_c} d\omega \, e^{-\omega/T}. \tag{17}$$

Requiring that the condensate contribution be less than 30% of the perturbative term gives $T_{\rm min} = 0.75$ GeV. In Figure 3 [Figure 3: see original paper], we show the ratio of the pole term (the sum of the cut-off perturbative term and condensation terms) to the perturbative term for the sum rule (14). In the entire range $T > T_{\rm min}$, this pole contribution is less than 40%. Hence, there is no stability window for this sum rule satisfying our stated criteria.

If we arbitrarily loosen the analysis criteria and require only that the pole contribution exceed 30%, we obtain the working range $0.75 < T < 1.2$ GeV. In Figure 4 [Figure 4: see original paper], the variation of $\bar{\Lambda}(1/2^+)$ with T and ω_c is shown. Numerically,

$$\bar{\Lambda}(1/2^+) = (1.30 \pm 0.15) \text{ GeV}, \quad f_{+,1/2} = (0.39 \pm 0.05) \text{ GeV}^{3/2}, \tag{18}$$

where the central value corresponds to $T = 1.0$ GeV and $\omega_c = 2.9$ GeV. Because of the weaker criteria used, these results are less reliable and will not be used in the final numerical results.

We would like to make some remarks here. Usually, currents with the least number of derivatives are used in QCD sum rule approaches. The sum rules then are less sensitive to the threshold energy ω_c. However, it was pointed out in [**?**] that in the non-relativistic limit the coupling constant of these currents to P-wave states vanishes. If this coupling constant is suppressed for this reason, the relative importance of contributions from the DK continuum and other states (which are usually neglected) would be enhanced. Furthermore, it was shown in [**?**, **?**] using the soft pion theorem that the contribution of the $D\pi$ continuum is large in the sum rule with the non-derivative current for the 0^+ state of the non-strange D system, significantly decreasing the value of $\bar{\Lambda}$. A similar calculation is not reliable in the present case, but it indicates that the contribution of the DK continuum with the non-derivative current may also be large.

3. Sum Rules at Order $1/m_Q$

To order $1/m_Q$, the Lagrangian of HQET is

$$\mathcal{L}_{\rm eff} = \bar{h}_v iv \cdot D h_v + \frac{1}{2m_Q} \mathcal{K} - \frac{1}{2m_Q} \mathcal{S} + \mathcal{O}(1/m_Q^2), \tag{19}$$

where $h_v(x)$ is the velocity-dependent effective field related to the original heavy-quark field $Q(x)$ by $h_v(x) = e^{im_Q v\cdot x} \frac{1+\not{v}}{2} Q(x)$. The kinetic operator is defined as $\mathcal{K} = \bar{h}_v (iD_t)^2 h_v$, and the chromo-magnetic operator is $\mathcal{S} =$

$C_{\rm mag}(m_Q/\mu)\bar{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v$, where $C_{\rm mag} = \left[\alpha_s(m_Q)/\alpha_s(\mu)\right]^{3/\beta_0}$ with $\beta_0 = 11 - 2n_f/3$.

Considering $1/m_Q$ corrections, the pole term of the correlator on the hadron side becomes

$$\Pi(\omega)_{\rm pole} = \frac{(f+\delta f)^2}{2(\bar{\Lambda}+\delta m)-\omega} = \frac{f^2}{2\bar{\Lambda}-\omega} + \frac{2\delta m f^2}{(2\bar{\Lambda}-\omega)^2} + \frac{2f\delta f}{2\bar{\Lambda}-\omega}, \qquad (20)$$

where δm and δf are of order $(1/m_Q)$.

To extract δm in (20) we follow the approach of [?] and consider the three-point correlation functions

$$\delta_O\Pi^{\alpha_1\cdots\alpha_j,\beta_1\cdots\beta_j}_{j,P,i}(\omega,\omega') = i^2\int d^4x\, d^4y\, e^{ik\cdot x - ik'\cdot y}\langle 0|T\{J^{\alpha_1\cdots\alpha_j}_{j,P,i}(x)O(0)J^{\dagger\beta_1\cdots\beta_j}_{j,P,i}(y)\}|0\rangle, \qquad (21)$$

where $O = \mathcal{K}$ or $\mathcal{S}$. The scalar function corresponding to (21) can be represented as the double dispersion integral

$$\delta_O\Pi(\omega,\omega') = \int ds\, ds'\, \frac{\rho_O(s,s')}{(s-\omega)(s'-\omega')}. \qquad (22)$$

The pole parts of $\delta_O\Pi(\omega,\omega')$ are

$$\delta_{\mathcal{K}}\Pi(\omega,\omega')_{\rm pole} = \frac{f^2 G_{\mathcal{K}}(\omega')}{(2\bar{\Lambda}-\omega)(2\bar{\Lambda}-\omega')} + \frac{f^2 G_{\mathcal{K}}(\omega)}{(2\bar{\Lambda}-\omega)(2\bar{\Lambda}-\omega')} + \frac{d_M\Sigma f^2}{(2\bar{\Lambda}-\omega)(2\bar{\Lambda}-\omega')}, \qquad (23)$$

$$\delta_{\mathcal{S}}\Pi(\omega,\omega')_{\rm pole} = \frac{f^2 G_{\mathcal{S}}(\omega')}{(2\bar{\Lambda}-\omega)(2\bar{\Lambda}-\omega')} + \frac{f^2 G_{\mathcal{S}}(\omega)}{(2\bar{\Lambda}-\omega)(2\bar{\Lambda}-\omega')}, \qquad (24)$$

where

$$\langle j,P,j_\ell|\bar{h}_v(iD_\perp)^2h_v|j,P,j_\ell\rangle = 2d_M\Sigma_{j,P,j_\ell}, \qquad (25)$$

$$\langle j,P,j_\ell|\bar{h}_v\frac{g}{2}\sigma_{\mu\nu}G^{\mu\nu}h_v|j,P,j_\ell\rangle = d_M\Sigma_{j,P,j_\ell}, \qquad (26)$$

$$d_M = d_{j,j_\ell}, \quad d_{j_\ell-1/2,j_\ell} = 2j_\ell + 2, \quad d_{j_\ell+1/2,j_\ell} = -2j_\ell. \qquad (27)$$

Setting $\omega = \omega'$ in eqs. (22) and (23) and comparing with (20), one obtains [?]

$$\delta m = -\frac{1}{2m_Q}(\mathcal{K} + d_M C_{\rm mag}\Sigma). \tag{28}$$

The single-pole terms in (22) and (23) come from the region where $s(s') = 2\bar{\Lambda}$ and $s'(s)$ is at the pole for a radial excited state or in the continuum. They are suppressed by making a double Borel transformation in both ω and ω'. The Borel parameters corresponding to ω and ω' are taken to be equal. One thus obtains the sum rules for $\mathcal{K}$ and Σ:

$$f^2\mathcal{K}e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega \int_0^{\omega_c} d\omega'\, e^{-(\omega+\omega')/2T}\rho_{\mathcal{K}}(\omega,\omega'), \tag{29}$$

$$f^2\Sigma e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega \int_0^{\omega_c} ds'\, \rho_S(\omega,s')e^{-\frac{\omega+s'}{2T}}. \tag{30}$$

In this section we shall neglect the m_s corrections to $\mathcal{K}$ and Σ. For the doublet with derivative currents we obtain

$$f^2_{+,1/2}\mathcal{K}e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega\, \frac{\omega^6}{24\pi^2}e^{-\omega/T} + \frac{2m_s\langle\bar{s}s\rangle}{T^3}, \tag{31}$$

$$f^2_{+,1/2}\Sigma e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega \int_0^{\omega_c} ds'\, \rho(s,s')e^{-\frac{s+s'}{2T}} + \frac{2m_s\langle\bar{s}s\rangle}{T^3}, \tag{32}$$

with

$$\rho(s,s') = \frac{\alpha_s(2T)}{\pi^3}C_{\rm mag}ss'\left[s'^2(3s-s')\theta(s-s') + (s\leftrightarrow s')\right]. \tag{33}$$

We can eliminate the dependence of $\mathcal{K}$ and Σ on $f_{+,1/2}$ and $\bar{\Lambda}$ by dividing the above sum rules (27) by the sum rule (8). We use the same working windows as those of the two-point sum rules for $\bar{\Lambda}$ in evaluating the $(1/m_Q)$ corrections. Numerically we have

$$\mathcal{K}_{1/2} = (0.28\pm 0.30)\ \text{GeV}^2, \quad \Sigma_{1/2} = (0.40\pm 0.05)\ \text{GeV}^2. \tag{34}$$

The variations of $\mathcal{K}$ and Σ with T and ω_c for the $j^P = 1/2^+$ doublet are shown in Figures 5 [Figure 5: see original paper] and 6 [Figure 6: see original paper], respectively.

For the $j^P = 3/2^+$ doublet, we have

$$f_{+,3/2}^2 \mathcal{K} e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega \, \frac{\omega^6}{72\pi^2} e^{-\omega/T} + \frac{2m_s\langle\bar{s}s\rangle}{T^3}, \tag{35}$$

$$f_{+,3/2}^2 \Sigma e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega \int_0^{\omega_c} ds' \, \rho(s,s') e^{-\frac{s+s'}{2T}} + \frac{2m_s\langle\bar{s}s\rangle}{T^3}, \tag{36}$$

with

$$\rho(s,s') = \frac{\alpha_s(2T)}{\pi^3} C_{\text{mag}} \left\{ s'^4 (s-s')\theta(s-s') + (s \leftrightarrow s') \right\}. \tag{37}$$

The variations of $\mathcal{K}$ and Σ with T and ω_c for the $j^P = 3/2^+$ doublet are plotted in Figures 7 [Figure 7: see original paper] and 8 [Figure 8: see original paper], respectively. Numerically we obtain

$$\mathcal{K}_{3/2} = (0.058 \pm 0.40) \text{ GeV}^2, \quad \Sigma_{3/2} = (0.01 \pm 0.06) \text{ GeV}^2. \tag{38}$$

The spin-symmetry violating term not only causes splitting of masses within the same doublet but also causes mixing of states with the same j, P but different j_ℓ. In Ref. [?] corrections from mixing were found to be negligible, so we omit this effect here.

4. Numerical Results and Discussions

We present our results for the $\bar{c}s$ system assuming HQET is sufficiently accurate for excited D mesons. For the $(0^+, 1^+)$ doublet with the derivative current, the weighted average mass is

$$\frac{m_{D_{s0}} + 3m_{D_{s1}}}{4} = m_c + (0.86 \pm 0.10) + \frac{1}{2m_c} \left[(0.40 \pm 0.08) \text{ GeV}^2 \right]. \tag{39}$$

The mass splitting is

$$m_{D_{s1}} - m_{D_{s0}} = \frac{1}{2m_c} \left[(0.28 \pm 0.05) \text{ GeV}^2 \right]. \tag{40}$$

For the $(1^+, 2^+)$ doublet we have

$$\frac{3m_{D_{s1}} + 5m_{D_{s2}}}{8} = m_c + (0.83 \pm 0.10) + \frac{1}{2m_c} \left[(0.41 \pm 0.10) \text{ GeV}^2 \right]. \tag{41}$$

The 1^+, 2^+ mass splitting is

$$m_{D_{s2}} - m_{D_{s1}} = \frac{1}{2m_c}\left[(0.116 \pm 0.06)\ \mathrm{GeV}^2\right]. \tag{42}$$

The results for the $\bar{b}s$ system are obtained by replacing m_c with m_b and multiplying Σ by 0.8 (since $C_{\mathrm{mag}} \approx 0.8$ for the B system, using the values of Λ_{QCD} given in Section 2) in the above equations.

Choosing m_c to fit the experimental value $(3m_{D_{s1}} + 5m_{D_{s2}})/8 = 2.56$ GeV, where we again neglect mixing between the two 1^+ states, we obtain $m_c = 1.44$ GeV. Using this m_c value we obtain the following numerical results. The 1^+, 2^+ mass splitting is

$$m_{D_{s2}} - m_{D_{s1}} = (0.080 \pm 0.042)\ \mathrm{GeV}, \tag{43}$$

which is consistent with the experimental value of 37 MeV within the large theoretical uncertainty. Experimentally, the mass splitting in the $(1^+, 2^+)$ doublet in the D_s system is almost equal to that in the D system. This justifies neglecting the m_s correction to the Σ term in our calculation. For the $(0^+, 1^+)$ doublet, the weighted average mass is

$$\frac{m_{D_{s0}} + 3m_{D_{s1}}}{4} = (2.57 \pm 0.12)\ \mathrm{GeV}. \tag{44}$$

The mass splitting is

$$m_{D_{s1}} - m_{D_{s0}} = (0.19 \pm 0.04)\ \mathrm{GeV}. \tag{45}$$

Therefore, the 0^+ mass is predicted to be

$$m_{0^+} = (2.42 \pm 0.13)\ \mathrm{GeV}. \tag{46}$$

This is consistent with the experimental value of 2.317 GeV, though the central value is 100 MeV larger than the data. If the 1^+ D_s state of mass 2.460 GeV found by CLEO [**?**, **?**] is assigned as the other member of the $(0^+, 1^+)$ doublet, then the observed mass splitting of 0.143 GeV is consistent with (32). Notice that the $\Sigma_{1/2}$ value in Fig. 6 changes very slowly between $T = 0.35$ GeV, so the result (32) is insensitive to the stability window used.

We would like to note that the stability of the sum rules (in particular those for $\mathcal{K}$ and Σ) obtained by us is not as good as those for the ground states, as can be seen from the figures. Furthermore, for charm-flavor hadrons, the $1/m_c$ corrections are significant. Consequently, the mass predictions have relatively large uncertainties, which are estimated by the given errors.

If we replace the strange quark condensate by the up/down quark condensate and set m_s to zero, we can extract the non-strange excited D meson masses. For the $(1^+, 2^+)$ doublet, the sum rule window is $0.57 < T < 0.67$ GeV for $\omega_c = 2.8$ GeV. The results are

$$\bar{\Lambda}_{3/2} = (0.73 \pm 0.08) \text{ GeV}, \tag{47}$$

$$\mathcal{K}_{3/2} = (0.4 \pm 0.4) \text{ GeV}^2, \tag{48}$$

$$\Sigma_{3/2} = (0.057 \pm 0.01) \text{ GeV}^2, \tag{49}$$

with the central value at $T = 0.62$ GeV. For the $(0^+, 1^+)$ doublet, the sum rule window is $0.46 < T < 0.6$ GeV for $\omega_c = 2.7$ GeV. The results are

$$\bar{\Lambda}_{1/2} = (0.79 \pm 0.08) \text{ GeV}, \tag{50}$$

$$\mathcal{K}_{1/2} = (1.57 \pm 0.4) \text{ GeV}^2, \tag{51}$$

$$\Sigma_{1/2} = (0.286 \pm 0.05) \text{ GeV}^2, \tag{52}$$

with the central value at $T = 0.53$ GeV. The effect of the strange quark is to make the mass of D_s mesons slightly larger, as expected. Experimentally, the Belle collaboration found $m_{D_0} = (2290 \pm 20)$ MeV and $m_{D_1^*} = (2400 \pm 40)$ MeV recently [?], while the CLEO collaboration found $m_{D_1^*} = 2400^{+48}_{-42}$ MeV [?]. The $j = 3/2$ non-strange doublet masses are known precisely [?]: $m_{D_2} = 2460$ MeV and $m_{D_1} = 2420$ MeV. Our results (33)-(34) are consistent with the experimental data within theoretical uncertainties.

In summary, we have calculated the masses of the excited $(0^+, 1^+)$ and $(1^+, 2^+)$ doublets for the $\bar{c}s$ system to order $1/m_Q$ in HQET sum rules. The numerical results imply that the $D_{sJ}(2317)$ and $D_{sJ}(2460)$ observed by BABAR and CLEO can be consistently identified as the $(0^+, 1^+)$ doublet with $j_\ell = \frac{1}{2}$ within theoretical uncertainty. In particular, the mass splitting in the $(0^+, 1^+)$ doublet is reproduced quite well. The repulsion between these states and the DK and D^*K continuum may help to lower their masses. In the framework of the sum rule, this effect should arise from the contribution of the DK and D^*K continuum to the dispersion integral.

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