# [SU(3) × SU(2) × U(1)]2 and Strong Unification Postprint

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## Abstract

A supersymmetric model with gauge symmetry G1 × G2, where Gi = SU(3)i × SU(2)i × U(1)i, is constructed within the framework of gauge mediated supersymmetry breaking. At the energy scale  (10−100) TeV where the gauge symmetry breaks down to the Standard Model (SM), G1 is strong and G2 is weak. The observed gauge coupling constant unification of the SM is attributed to that of G2. The messenger fields and Higgs fields just satisfy the condition that makes G2 a realization of strong unification. The SM gauginos are predicted to be generally heavier than the sleptons and squarks.

## Full Text

## Preamble

[SU(3) × SU(2) × U(1)]² and Strong Unification

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## Abstract

A supersymmetric model with gauge symmetry $G_1 \times G_2$, where $G_i = \mathrm{SU}(3)_i \times \mathrm{SU}(2)_i \times \mathrm{U}(1)_i$, is constructed within the framework of gauge-mediated supersymmetry breaking. At the energy scale $(10-100)$ TeV where the gauge symmetry breaks down to the Standard Model (SM), $G_1$ is strong and $G_2$ is weak. The observed gauge coupling constant unification of the SM is attributed to that of $G_2$. The messenger fields and Higgs fields satisfy precisely the condition that makes $G_2$ a realization of strong unification. The SM gauginos are predicted to be generally heavier than the sleptons and squarks.

**Keywords:** gauge interaction, supersymmetry
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## 1. Introduction

Extensions of the SM aim at understanding new experimental results or unsolved theoretical problems. The most popular approach is grand unification theories (GUTs) [?], such as the SU(5) GUT. There are indirect experimental evidences for GUTs from LEP and neutrino physics. To make GUTs viable, supersymmetry (SUSY) [?, ?] is a must. One novel idea toward GUT is the so-called strong unification [?, ?]. In strong GUT, the SM gauge coupling constants just reach their common Landau pole at the unification energy scale.

Strong GUT is interesting not only due to its novelty, but also because of its usefulness. There is a discrepancy between the measured value of the QCD strong coupling constant at $M_Z$, which is $\alpha_s^{\exp}(M_Z) = 0.118 \pm 0.002$ [?], and that predicted by the minimal SUSY SM, $\alpha_s^{\mathrm{MSSM}}(M_Z) = 0.126$. The discrepancy is reduced if extra matter fields are added into the MSSM. To keep the unification, the additional states should be in complete representations of GUT gauge groups. To the two-loop level, it has been shown [?], for example, that $\alpha_s(M_Z) \simeq 0.1163$ if there are additional six multiplets in $\mathbf{5} + \bar{\mathbf{5}}$ under SU(5) with masses of order $10^{2.14}$ TeV. However, the model would be artificial if these additional matters are naively added. We will illustrate that a SUSY model with the gauge symmetry $\mathrm{SU}(3)_1 \times \mathrm{SU}(2)_1 \times \mathrm{U}(1)_1 \times \mathrm{SU}(3)_2 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_2$ can be a nontrivial realization of strong GUT.

There are multiple motivations to consider such an extension of the SM [?, ?, ?]. In Ref. [?], such models were proposed as GUT generalizations of SUSY topcolor [?]. They provide a solution to the SUSY flavor-changing neutral current problem. However, the gauge coupling constant behavior was rather problematic at high energies because of the introduction of too many extra matter fields which made the gauge interactions excessively strong. This situation leads us to further consider their connection with the idea of strong GUT. In this paper, after naturally modifying the Higgs and messenger contents of the model, we note that the extra matters additional to the MSSM can make the SM-like gauge interaction $\mathrm{SU}(3)_2 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_2$ a strong GUT.

## 2. The Model

We consider a SUSY theory with the gauge group $G_1 \times G_2$ in the framework of gauge-mediated SUSY breaking (GMSB) [?], where $G_i = \mathrm{SU}(3)_i \times \mathrm{SU}(2)_i \times \mathrm{U}(1)_i$ $(i = 1, 2)$. The three coupling constants of $G_1$ are large, and those of $G_2$ are small at the TeV scale. The three generations of matter carry nontrivial quantum numbers of $G_2$ only. These numbers are assigned in the same way as they are under the SM gauge group. One gauge singlet chiral superfield $X$ is introduced for SUSY breaking, with scalar and auxiliary components $X_s$ and $F_X$. The vacuum expectation values are taken to be real: $\langle X_s \rangle = M$ and $\langle F_X \rangle = F$.

### 2.1 Messenger Sector

For the SUSY breaking messengers and gauge symmetry breaking Higgs fields, it is convenient to consider them by embedding $G_i$ into a global $\mathrm{SU}(5)_i$ symmetry. The messengers with their quantum numbers under $\mathrm{SU}(5)_1 \times \mathrm{SU}(5)_2$ are:

$$T_1(\mathbf{5},\mathbf{1}), \quad \bar{T}_1(\bar{\mathbf{5}},\mathbf{1}), \quad T_2(\mathbf{1},\mathbf{5}), \quad \bar{T}_2(\mathbf{1},\bar{\mathbf{5}})$$

The relevant superpotential is:

$$\mathcal{W}_1 = c_1 X T_1 \bar{T}_1 + c_2 X T_2 \bar{T}_2,$$

where $c_1$ and $c_2$ are coupling constants of order one. The fields $T_i$ and $\bar{T}_i$ are massive at tree level. Their fermionic components compose a Dirac fermion with mass $c_i M$, while the scalar components have a squared-mass matrix:

$$\begin{pmatrix} |c_i M|^2 & c_i^* F \\ c_i F^* & |c_i M|^2 \end{pmatrix}$$

The mass eigenstates are $(T_s \pm \bar{T}_s)/\sqrt{2}$ with squared-mass eigenvalues $m_{i1}^2 = |c_i M|^2 + |c_i F|$ and $m_{i2}^2 = |c_i M|^2 - |c_i F|$. It is assumed that $|F| \ll |M|^2$. Because $\langle F_X \rangle \neq 0$, SUSY breaking occurs in the fields $T_i$ and $\bar{T}_i$ at tree level. $G_1$ and $G_2$ sectors obtain soft SUSY breaking via the messengers at loop level. Because $G_2$ is weak at the TeV scale, its SUSY breaking effects can be calculated perturbatively. For example, $G_2$ gaugino soft masses are:

$$M_{\lambda'_r} \simeq \frac{\alpha'_r}{4\pi} \frac{F}{M},$$

where $\alpha'_r = g_r'^2/4\pi$ with $g'_r$ being the gauge coupling constants of $G_2$, and $r = 1, 2, 3$ corresponding to U(1), SU(2), and SU(3), respectively. However, $G_1$ is strong, so we can only estimate its gaugino masses:

$$M_{\lambda_r} \sim \frac{\alpha_r}{4\pi} \frac{F}{M}.$$

Numerically, the messenger masses are about $(10 - 100)$ TeV.

### 2.2 Gauge Symmetry Breaking Sector

A pair of Higgs fields $\Phi_1(\mathbf{5},\bar{\mathbf{5}})$ and $\Phi_2(\bar{\mathbf{5}},\mathbf{5})$ breaks the $G_1 \times G_2$ gauge symmetry down to that of the SM. One gauge singlet superfield $Y$ is introduced for the gauge symmetry breaking. The superpotential is:

$$\mathcal{W}_2 = c' Y \left[ \mathrm{Tr}(\Phi_1 \Phi_2) - \mu'^2 \right],$$

where the trace is taken with regard to both $\mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2$ and $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$, $\mu'$ is the energy scale relevant to gauge symmetry breaking, and $c'$ is a coupling constant. The Higgs fields get soft masses similar to those given in

Eq. (7). However, the above superpotential is not sufficient to guarantee all the $\Phi_i$ fermion components to be massive. Their masses become nonvanishing when a superfield $A$ in the adjoint representation of $\mathrm{SU}(5)_1$ is introduced with the superpotential [?]:

$$\mathcal{W}_2' = c_2' \mathrm{Tr}(\Phi_2 A \Phi_1),$$

with $c_2'$ being the coupling constant. The details of gauge symmetry breaking proceed as in Ref. [?] (its Eqs. (10-17)). The VEVs of the $\Phi_i$ fields are:

$$\langle \Phi_1 \rangle = vI_3, \quad \langle \Phi_2 \rangle = vI_2,$$

where $I_3$ and $I_2$ are unit matrices in the space of $\mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2$ and $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$, respectively. The coupling constants of the SM $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ are $g_2' = g_2$ and $g_1' = g_1$. Numerically, the gauge symmetry breaking scale $v$ is about $(10 - 100)$ TeV.

Electroweak symmetry breaking is achieved via a pair of Higgs superfields $H_u$ and $H_d$ which are nontrivial only under $G_2$ [?].

## 3. Strong Unification

Around $10 - 100$ TeV, there are many matter fields which cause the gauge coupling constants to become large at high energies. The matter fields introduced in addition to the MSSM are complete SU(5) multiplets. Therefore, the unification scale $\sim 10^{16}$ GeV is still the same as that of the MSSM, but the values of the coupling constants are significantly different. This model is a candidate for strong GUT.

Below the scale $v$, $G_1 \times G_2$ breaks spontaneously down to the MSSM. From Eq. (11), it is easy to see that the gauge coupling constants of the MSSM are almost fully determined by those of $G_2$, because $g_i \ll g_i'$. Therefore the observed unification of the MSSM is attributed to the unification of $G_2$.

Above the $(10 - 100)$ TeV scale, the theory is $G_1 \times G_2$. As far as the $G_2$ sector is concerned, the new matter fields in addition to the MSSM are the messengers $T_1$ and $\bar{T}_1$, and Higgs fields $\Phi_1$ and $\Phi_2$. The messenger fields compose one $\mathbf{5} + \bar{\mathbf{5}}$ multiplet with mass $c_1 M$, and the Higgs fields contribute five $\mathbf{5} + \bar{\mathbf{5}}$ multiplets with masses $c'v$ and $c_2'v$. We have the freedom to adjust all the masses of these six $\mathbf{5} + \bar{\mathbf{5}}$ multiplets to be about $10^{2.14}$ TeV. As shown in Ref. [?], the gauge couplings reach their common Landau pole at the GUT scale $\sim 10^{16}$ GeV. Namely, in this case, $G_2$ is a realization of strong GUT.

## 4. Remarks

Some remarks are necessary. (1) The perturbative calculation in Ref. [?] was not reliable around the GUT scale because of the large coupling constants, but around 100 TeV where the perturbative domain lies, its reliability was under control. It is in this latter low-energy region where we have made use of Ref.

[?]. (2) On the other hand, the $G_1$ sector is also expected to be a GUT. Since $(10-100)$ TeV is already its non-perturbative region, we have no reliable method yet to perform detailed analysis. (3) The unification simply means that the gauge coupling constants are equal at certain scales. We have not introduced any unified gauge group. Such a model does not have proton decay and does not suffer from the doublet-triplet splitting problem. (4) It should be noted that only because $G_2$ is SM-like can the breaking to the SM at $(10-100)$ TeV occur. Any breaking of SU(5) × SU(5) × SU(5) [?] would have occurred above $10^{16}$ GeV. (5) Some of the matter contents of $G_2$, such as the third generation, can be moved into $G_1$. Due to GMSB, the superpartners in this sector are very heavy ($\sim$ 100 TeV). They decouple at $(1-10)$ TeV energy scale. At this low energy scale the fermions, on the other hand, can form condensates due to the strong gauge interactions. Dynamical fermion masses might be generated [?]. In order to keep the strong GUT, it is possible to either introduce one more $\mathbf{5}+\bar{\mathbf{5}}$ multiplet of $G_2$, which may play a role as SUSY breaking messengers [?], or lower the SUSY breaking and messenger scales to be around 10 TeV. These possibilities should be studied further and are beyond the scope of this work. (6) If the $\mathrm{SU}(3)_1$ interaction is switched off, the model becomes a kind of top-flavor model [?].

## 5. Phenomenology

This model has interesting phenomenology. Besides the new gauge bosons, gauginos and Higgs particles with masses around $(10-100)$ TeV, the SM gaugino masses are predicted to be as heavy as $\sim 1$ TeV. Let us analyze the gaugino spectrum in more detail.

The full gaugino masses have two origins: SUSY breaking (soft masses) and spontaneous gauge symmetry breaking. It has been obtained in Ref. [?] that the relevant mass matrix in the basis of $(\lambda_r, \lambda_r', \psi_1, \psi_2)/\sqrt{2}$ is:

$$\begin{pmatrix} M_{\lambda_r} & 0 & \sqrt{2}g_r v & 0 \\ 0 & M_{\lambda_r'} & 0 & \sqrt{2}g_r' v \\ \sqrt{2}g_r v & 0 & 0 & 0 \\ 0 & \sqrt{2}g_r' v & 0 & 0 \end{pmatrix}$$

where $\psi_1$ and $\psi_2$ stand for the fermion components of $\Phi_1$ and $\Phi_2$, respectively. Numerically, at the scale $v \sim (10-100)$ TeV, $g_r' \sim 0.1$, $g_r \sim 1$. The mass matrix determines two heavy states with masses $\sim (10-100)$ TeV, and one lighter state $M_{\lambda_r}^{\text{light}} \sim (2g_r v)^2/M_{\lambda_r} \sim 1$ TeV. This lighter state is a mixture of the $G_2$ gaugino with the higgsino. It is regarded as the MSSM gaugino in this model. On the other hand, the soft masses of the three generation matters are about 100 GeV. Therefore in this model the SM gauginos are generally heavier than the sleptons and squarks. Such a mass pattern can be tested in future colliders.

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*Note: Figure translations are in progress. See original paper for figures.*