A Supersymmetry Model of Leptons (Postprint)

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Abstract

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Full Text

A Supersymmetry Model of Leptons

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Abstract

If supersymmetry (SUSY) is not for stabilizing the electroweak energy scale, what is it used for in particle physics? We propose that it is for flavor problems. A cyclic family symmetry is introduced. Under the family symmetry, only the -lepton is massive due to the vacuum expectation value (VEV) of the Higgs field. This symmetry is broken by a sneutrino VEV which results in the muon mass. The comparatively large sneutrino VEV does not result in a large neutrino mass due to requiring heavy gauginos. SUSY breaks at a high scale of 10^{13} GeV. The electroweak energy scale is unnaturally small. No additional global symmetry, like the R-parity, is imposed. Other aspects of the model are discussed.

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In elementary particle physics, SUSY [?, ?, ?] was proposed for stabilizing the electroweak (EW) scale [?, ?] which is otherwise unnaturally small compared to the grand unification scale [?, ?]. The study of the cosmological constant [?], however, suggests that unnaturalness of 10^{120} or 10^{55} fine tuning might be just so from the anthropic point of view. It was argued that string theory even supports the emergence of the anthropic landscape [?, ?]. This led to a consideration of giving up naturalness of the EW scale [?, ?]. If SUSY is not for stabilizing the EW scale, what else does it do in particle physics? Refs. [?, ?] maintained its roles in grand unification and dark matter.

In this paper, we advocate that SUSY is for flavor physics. The flavor puzzle —namely the fermion masses, mixing and CP violation—in the Standard Model (SM) requires new physics to be understood. The empirical fermion mass pattern is that the third generation is much heavier than the second generation, which is also much heavier than the first. This may imply a family symmetry [?, ?, ?, ?]. Let us consider the charged leptons. By assuming a Z_3 cyclic symmetry among the $SU(2)$ doublets L_i $(i = 1, 2, 3)$ of the three generations [?, ?], the Yukawa interactions result in a democratic mass matrix which is of rank 1. Therefore only the tau lepton gets mass; the muon and electron remain massless.

The essential point is how the family symmetry breaks. Naively the symmetry breaking can be achieved by introducing family-dependent Higgs fields. We consider this problem within SUSY. We observe that SUSY naturally provides such Higgs-like fields, which are the scalar neutrinos. If the VEVs of the sneutrinos are non-vanishing, $v_i \neq 0$, violating interactions $L_i L_j E_k^c$ (denoting the anti-particle superfields of the $SU(2)$ singlet leptons) contribute to the fermion masses, in addition to the Yukawa interactions [?, ?, ?, ?]. We think that this is the origin of family symmetry breaking.

The above idea has been proposed for some time [?, ?]. Because SUSY was used to stabilize the EW scale, that idea suffers from severe constraints. For example, the τ-neutrino should be 10 MeV heavy [?]. It is a liberation if SUSY has nothing to do with the EW scale. While the τ-lepton mass is from the Higgs VEV $\sim$ 100 GeV, the μ mass is due to λv_i with λ standing for the trilinear R-parity violation couplings. It is natural like the Yukawa couplings for the τ mass. The muon mass tells us then $v_i \sim 10$ GeV. Ten GeV v_i' s could induce a large lepton number violating effect, namely a large neutrino mass if the neutralinos are not heavy, due to $m_\nu \simeq (g_2 v_i)^2/M_{\tilde{Z}}$, where g_2 is the $SU(2)_L$ gauge coupling constant, and $M_{\tilde{Z}}$ is the gaugino mass. When we get the freedom to take $M_{\tilde{Z}}$ arbitrarily high, the above formula can produce a neutrino mass in the safe range.

In this model the Z_{3L} family symmetry mentioned above is assumed, which however is softly broken. The gauge symmetries and the matter contents in the full theory are the same as those in the SUSY SM. Under the family symmetry, the relevant kinetic terms generally include

$$\big[H_1^\dagger H_1 + H_2^\dagger H_2 + \alpha L_i^\dagger L_i + \beta(L_1^\dagger L_2 + L_2^\dagger L_3 + L_3^\dagger L_1 + \text{h.c.}) \\ +\gamma(L_i H_1 + \text{h.c.})\big]_{\theta\theta\bar\theta\bar\theta}$$

where H_1 and H_2 are the two Higgs doublets, and α, β, γ are $O(1)$ coefficients. The case of $\alpha = 1$ and $\beta = \gamma = 0$ is a special one of the above expression. Note that the gauge field e^V is not explicitly written, which does not affect our discussion on flavor physics. The superpotential is

$$\tilde W = \tilde y_j(L_1L_2+L_2L_3+L_3L_1)H_2E_j^c+\tilde\lambda_j(L_1L_2+L_2L_3+L_3L_1)E_j^c+\tilde\mu H_1H_2+\tilde\mu' H_1L_i$$

where $\tilde y_j$' s and $\tilde\lambda_j$' s are the coupling constants, and $\tilde\mu$ and $\tilde\mu'$ are mass terms. It is natural that they are about the scale of soft SUSY breaking masses. The Lagrangian of soft SUSY breaking masses is

$$\mathcal{L}_{\text{soft1}} = M_{\tilde W}\tilde W\tilde W+M_{\tilde Z}\tilde Z\tilde Z+m_1^2h_1^\dagger h_1+m_2^2h_2^\dagger h_2+m_{\tilde lij}^2\tilde l_i^\dagger\tilde l_j+m_{lRij}^2\tilde e_i^*\tilde e_j+(B\tilde\mu h_1h_2+B\tilde\mu_i h_1\tilde l_i+m_{\tilde li}'^2\tilde l_i\tilde l_i+\text{h.c.}),$$

where $\tilde W$ and $\tilde Z$ stand for the charged and neutral gauginos, respectively, and $h_1, h_2, \tilde l_i$ and $\tilde e_i$ are the scalar components of H_1, H_2, L_i and E_i^c respectively. Note that explicit breaking of Z_{3L} is introduced in the soft mass terms. The soft masses are assumed to be very large around a typical mass m_S. The trilinear soft terms should be also included,

$$\mathcal{L}_{\text{soft2}} = \tilde m_{ij}\tilde l_i h_2\tilde e_j + \tilde m_{ijk}\tilde l_i\tilde l_j\tilde e_k + \text{h.c.}$$

The mass coefficients which we denote generally as $\tilde m_S$ can be close to m_S.

The expression of the kinetic terms is not yet in the normalized standard form. The standard form

$$H_u^\dagger H_u + H_d'^\dagger H_d' + L_e^\dagger L_e + L_\mu^\dagger L_\mu + L_\tau'^\dagger L_\tau'$$

is achieved by the field re-definition:

$$
\begin{aligned}
H_u &= c_1\left(H_1 - \frac{\gamma}{\alpha+2\beta}L_1 - \frac{\gamma}{\alpha-\beta}L_2 + \frac{2\gamma}{\alpha-\beta}L_3\right)\\
H_d' &= c_2\left(H_2 + \frac{\gamma}{\alpha+2\beta}(L_1+L_2+L_3)\right)\\
L_e &= \frac{1}{\sqrt{3}}(L_1+L_2+L_3)\\
L_\mu &= \frac{1}{\sqrt{2}}(L_1-L_2)\\
L_\tau' &= c_3\left(L_1+L_2-2L_3-\frac{3\beta}{\alpha+2\beta}(L_1+L_2+L_3)\right)
\end{aligned}
$$

where $c_{1,2,3}$ are normalization constants and θ cannot be determined until the muon mass basis is fixed.

The superpotential is then

$$
\tilde{W} = y_\tau L_\tau H_d E_\tau^c + y_\mu L_\tau H_d E_\mu^c + \mu H_u H_d' + \mu' H_u L_\tau' + L_e L_\mu(\lambda_\tau E_\tau^c + \lambda_\mu E_\mu^c)
$$

where E_τ^c is defined as $(\tilde{y}_j/c_2)E_j^c$, E_μ^c is orthogonal to E_τ^c, and $\lambda_{\tau,\mu}$ are combinations of $\tilde{y}_j$' s and $\tilde{\lambda}_j$' s. Because of the Z_{3L} symmetry, the superpotential is without the field E_e^c which is orthogonal to both E_τ^c and E_μ^c. To look at the fermion masses, we simply rotate the bilinear R-parity violating term away via the field re-definition,

$$
\begin{pmatrix} H_d \\ L_\tau \end{pmatrix} = \frac{1}{\sqrt{\mu^2+\mu'^2}}\begin{pmatrix} \mu & \mu' \\ -\mu' & \mu \end{pmatrix}\begin{pmatrix} H_d' \\ L_\tau' \end{pmatrix}.
$$

It is trivial to see that the kinetic terms are diagonal in terms of H_d and L_τ. The superpotential is

$$
\tilde{W} = y_\tau L_\tau H_d E_\tau^c + y_\mu L_\tau H_d E_\mu^c + \sqrt{\mu^2+\mu'^2}H_u H_d + L_e L_\mu(\lambda_\tau E_\tau^c + \lambda_\mu E_\mu^c).
$$

The Z_{3L} family symmetry keeps the trilinear R-parity violating terms invariant. As we have expected, the Higgs field H_d contributes to the tau mass only and the sneutrinos in L_e and L_μ contribute to the muon mass after they get VEVs. The VEVs of L_e and L_μ imply the breaking of the Z_{3L} symmetry as can be seen explicitly from Eq. (6). The electron remains massless because of the absence of the E_e^c field in $\tilde{W}$. A hierarchy among charged leptons is obtained. Without losing our essential points, we could take $\lambda_\tau = 0$. In that case, L_τ is in the mass eigenstate and the tau number is conserved. The tau number conservation justifies the field rotation Eq. (11).

The breaking of the family symmetry originates from the soft SUSY masses. For simplicity and without losing generality, we assume that the soft terms in Eqs. (3) and (4) are rewritten as

$$\mathcal{L}_{\rm soft} = M_{\tilde{W}}\tilde{W}\tilde{W}+M_{\tilde{Z}}\tilde{Z}\tilde{Z}+m_{hu}^2 h_u^\dagger h_u+m_{hd}^2 h_d^\dagger h_d+m_{\tilde{l}\alpha}^2 \tilde{l}_\alpha^\dagger\tilde{l}_\alpha+m_{lR\alpha\beta}^2\tilde{e}_\alpha^*\tilde{e}_\beta+(B\mu h_u h_d+B\mu_e h_u\tilde{l}_e+\tilde{m}_{\alpha\beta}\tilde{l}_\alpha h_d\tilde{e}_\beta+\tilde{m}_{\alpha\beta}$$

where $\alpha = e, \mu, \tau$. Most of the squared masses are expected to be positive, except m_{hu}^2. The key point of the form of the soft masses lies in the $(h_u, h_d^\dagger, \tilde{l}_e^\dagger)$ mass-squared matrix, of which the eigenvalues are

$$\begin{aligned} M_1^2 &= \bar{m}^2 - \sqrt{\Delta^4 + (B\mu)^2 + (B\mu_e)^2}, \\ M_2^2 &= \bar{m}^2 + \sqrt{\Delta^4 + (B\mu)^2 + (B\mu_e)^2}, \\ M_3^2 &= m_{\tilde{l}e}^2, \end{aligned}$$

where $\bar{m}^2 = m_{hu}^2+m_{hd}^2$ and $\Delta^2 = m_{hu}^2-m_{hd}^2$. The analysis proceeds in a similar way as in Ref. [?]. By fine-tuning, $M_1^2 \sim -m_{EW}^2$, namely the EW symmetry breaking is achieved. The fine-tuning is at the order of m_S^2/m_{EW}^2. In our case, in addition to the Higgs doublets, the $\tilde{l}_e$ field also gets a VEV,

$$\langle h_u \rangle = v_u \neq 0, \quad \langle h_d \rangle = v_d \neq 0, \quad \langle \tilde{l}_e \rangle = v_{le} \neq 0.$$

The relative size of these values is determined by the soft mass parameters. It is natural to expect the Z_{3L} symmetry breaking is not large, so a hierarchy between $v_{u,d}$ and v_{le} is possible. In the extreme case where $B\mu_e \ll B\mu$, the pattern $v_u > v_d > v_{le}$ is expected if v_{le} vanishes. As an illustration, a preferred VEV pattern $v_u > v_d > v_{le}$ is assumed if $B\mu_e < B\mu$. Note that the lepton number breaks explicitly in the soft mass terms, so v_{le} does not result in any massless scalar. Because there is only one light Higgs doublet, tree-level flavor-changing neutral currents (FCNC) do not appear. Therefore, a vanishing λ_τ keeps the generality of the model.

The fact $v_{le} \neq 0$ just corresponds to the mass eigenstate of the muon. The hierarchical charged lepton mass pattern is obtained from Eq. (12) explicitly,

$$m_\tau = y_\tau v_d, \quad m_\mu = \lambda_\mu v_{le}, \quad m_e = 0.$$

Numerically it is required that $v_d \sim 100$ GeV and $v_{le} \sim 10$ GeV. Whether a large v_{le} is safe or not should be studied.

In addition, it should also be considered that a huge $B\mu_e$ induces a large lepton-Higgsino mixing. The inducement happens at the loop-level through gaugino exchange, as shown in Ref. [?]. By denoting $\tilde{h}$ as Higgsinos, the mass matrix of ν_e and the other neutralinos is given as

$$\begin{pmatrix} \nu_e & \tilde{h}^0 \end{pmatrix} \begin{pmatrix} m_{eh} & -\mu' \\ -\mu' & av_d \end{pmatrix} \begin{pmatrix} \nu_e \\ \tilde{h}^0 \end{pmatrix},$$

where m_{eh} is about $10^{-3}m_S$ and $a = (g_1^2 + g_2^2)^{1/2}$ with g_1 being the SM $U(1)_Y$ coupling constant. We simply obtain the mass eigenvalues (denoted as $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4$) of the above mass matrix by reasonably taking $v_{le} \ll v_d < v_u \ll \mu \ll M_{\tilde{Z}}$,

$$\Lambda_1 \simeq M_{\tilde{Z}}, \quad \Lambda_2 \simeq -\frac{(av_{le})^2}{M_{\tilde{Z}}}, \quad \Lambda_3 \simeq -\mu + \frac{m_{eh}^2}{2\mu}, \quad \Lambda_4 \simeq \mu + \frac{m_{eh}^2}{2\mu}.$$

Therefore the ν_e mass $m_{\nu_e} \simeq (av_{le})^2/M_{\tilde{Z}}$. It is very small, 10^{-3} eV when $M_{\tilde{Z}} \sim 10^{13}$ GeV.

To accommodate the neutrino oscillation data, the neutrino sector should be extended. Three right-handed neutrinos N_i $(i = 1, 2, 3)$ which are singlet under the SM gauge groups are introduced. The following terms should be included in the Z_{3L} symmetric superpotential Eq. (2),

$$y'_j L_i H_1 N_j + M_{ij} N_i N_j + \tilde{c}_j H_1 H_2 N_j,$$

with y'_j's and $\tilde{c}_j$'s being the coupling constants of $O(10^{-2})$, and M_{ij} the Majorana masses. The superpotential does not include purely linear terms of N_i' s with large mass-squared coefficients, because N_i's are supposed to be charged under a larger gauge group beyond the SM. The soft masses of N_i' s are simply assumed to be large enough that N_i' s do not develop non-vanishing VEVs.

The trilinear soft terms associated with N_i' s can be written explicitly, which however play little role in the analysis. Through the previous field redefinition, Eq. (12) then includes

$$y'_\tau H_u L_\tau N_\tau + M_{\alpha\beta} N_\alpha N_\beta + H_u H_d(\tilde{c}_\tau N_\tau + \tilde{c}_\mu N_\mu + \tilde{c}_e N_e),$$

where N_α' s are combinations of N_i' s with N_τ being that which couples to $H_u L_\tau$. y'_τ and $\tilde{c}_\alpha$ are combinations of y'_i' s, $\tilde{c}_i$' s, $c_{1,2}$ and μ'/μ. The ν_τ mass is determined by the see-saw mechanism from Eq. (21),

$$m_{\nu_\tau} \simeq \frac{(y'_\tau v_d)^2}{M_{\alpha\beta}} \sim 10^{-2} \text{ eV}$$

by taking $M_{\alpha\beta} \sim 10^{11}$ GeV. The Dirac neutrino mass matrix is diagonal in the e-μ-τ basis. A bi-large neutrino mixing originates from the mass matrix $M_{\alpha\beta}$.

The electron mass comes from the soft trilinear R-parity violating terms in Eq. (4). Their soft breaking of Z_{3L} generates non-vanishing masses for the charged leptons through the one-loop diagram with gaugino exchange. The mixing of the scalar leptons associated with different chiralities is due to the soft trilinear terms, which is then about $\tilde{m}_S v_d$. The one-loop contribution to the charged lepton masses is about

$$\delta M^l_{\alpha\beta} \sim \frac{1}{16\pi^2}\frac{\tilde{m}_S v_d}{M_{\tilde{Z}}}.$$

Taking $\tilde{m}_S/m_S \simeq 0.1$, $\delta M^l_{\alpha\beta} \sim O(\text{MeV})$, which determines the electron mass.

The lepton mixing mainly depends on the neutrino mass matrix. In the charged lepton mass matrix, m_μ and m_τ are at the diagonal positions; the non-diagonal elements are $\delta M^l_{\alpha\beta}$. The mixing from the charged leptons is then basically small, $U_{\mu\tau} \simeq \delta M^l_{\mu\tau}/m_\tau$, $U_{e\mu} \simeq \delta M^l_{e\mu}/m_\mu$. If the mixing due to the neutrino mass matrix is bi-large, the lepton mixing required by the neutrino oscillation data can be obtained.

Let us briefly comment on the quark masses. Like those of the charged leptons, the quark masses also have three origins: the Higgs VEVs, the sneutrino VEV, and soft trilinear R-parity violating terms. However, the roles of the sneutrino VEV and the soft trilinear terms are switched [**?**]. The sneutrino VEV contributes to the first generation quark masses, and the soft trilinear R-parity violating terms to the charm and strange quark masses. More details will be in a separate work [**?**]. One important merit of this framework is that we do not need to introduce baryon number conservation. Because the sparticles are very heavy, they suppress baryon number violating processes to be unobservable [**?**]. An essentially same observation was pointed out in split SUSY [**?**].

In summary, we have proposed that SUSY is for flavor problems in particle physics. A family symmetry Z_{3L}, which is the cyclic symmetry among the three generation $SU(2)_L$ doublets, is introduced. No R-parity is imposed. SUSY breaks at a high scale of 10^{13} GeV. The electroweak energy scale is unnaturally small. Under the family symmetry, only the τ-lepton gets its mass. This symmetry is broken by a sneutrino VEV which results in the muon mass. A hierarchical pattern of the charged lepton masses is obtained. The comparatively large sneutrino VEV does not result in a large neutrino mass because the gaugino masses are very heavy. The quark masses and other aspects of the model have also been discussed.

At low energies, the model is basically the same as the SM. One essential feature of this model is that the unnaturally light Higgs has a component of a slepton. Related to this point, the model allows for relatively long-lived Higgsinos. We may consider a case where their masses are lower than m_S. If they are loop-induced, the Higgsino masses are thousand times smaller than m_S. A Higgsino decays to a Higgs and a virtual gaugino which further goes into a lepton and a virtual slepton; the slepton decays to a lepton pair via R-parity violating interaction. Because this four-body decay is suppressed by the R-parity violating coupling and doubly suppressed by m_S, a 10^{10} GeV heavy Higgsino has a lifetime of 10^{-12} sec. The cosmological and astrophysical implications should be studied in future works.

Acknowledgments

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Note: Figure translations are in progress. See original paper for figures.