Note on the Slope Parameter of the Baryonic#3;b → #3;c Isgur-Wise Function Postprint

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Full Text

Note on the Slope Parameter of the Baryonic Λc Isgur-Wise Function

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Abstract

Using the framework of Heavy Quark Effective Theory, we have re-analyzed the Isgur-Wise function describing semileptonic Λb → Λc decays in the QCD sum rule approach. The slope parameter of the Isgur-Wise function is found to be ^{2} = 1.35 ± 0.12, which is consistent with experimental measurements and lattice

calculations. The integrated Λb decay width is used to extract the CKM matrix element Vcb, for which we obtain a value of |Vcb| = 0.041 ± 0.004 in excellent agreement with the value determined from semileptonic B → D* decays.

The study of b → c semileptonic weak decays has been the subject of considerable interest in recent years, both as a source of information on Vcb and as a laboratory for understanding strong interaction effects and developing nonperturbative QCD methods. While considerable work has been carried out in the meson sector, where Heavy Quark Effective Theory (HQET) [1] was first developed, the baryon sector presents a particularly interesting case through the semileptonic decay Λb → Λc ¯ . In view of the fact that a recent experiment from DELPHI [2] shows a discrepancy with previous QCD sum rule calculations [3, 4] of the baryonic Λb → Λc form factor, an updated sum rule analysis is called for.

In the heavy quark limit, the hadronic matrix element of the Λb → Λc transition can be simply expressed in terms of a single Isgur-Wise (IW) function defined as follows [5, 6, 7]:

Λc(v)|J|Λb(v) = ( )¯uΛc(v)ΓuΛb(v),

where = v · v is the velocity transfer variable and Γ is an arbitrary gamma matrix. When the velocity of the heavy quark changes from v to v due to the weak decay, the light degrees of freedom undergo a corresponding transition due to their strong interactions with the heavy quark. The Isgur-Wise function ( ) measures the transition amplitude of the light degrees of freedom and is normalized to 1 at the zero recoil point = 1. This value is reduced by a few percent after taking into account radiative QCD correction effects [8, 9].

To obtain a theoretical description of the complete IW function, one must use nonperturbative methods which, at the current stage, are beset with large uncertainties. In the decay Λb → Λc ¯ , the physical region of lies in the range 1 to 1.43. Usually the IW function is expanded to first order in (- 1) as ( ) = 1 - ²(- 1) + ···, where the slope parameter ² at zero recoil must be calculated using nonperturbative methods.

The QCD sum rule approach [11, 12] has proven to be a reliable tool for dealing with many problems in the realm of nonperturbative QCD and has been used successfully to calculate the properties of various hadrons. For instance, besides light mesons and baryons, heavy meson properties were systematically analyzed in the sum rule approach within the framework of HQET [8]. In the heavy baryon sector, the masses of heavy baryons and the IW functions describing their weak transitions were calculated in Refs. [3, 4, 13, 14] and [15, 16, 17, 18, 19], respectively. In Ref. [20], the calculation for heavy baryons began with the full theory and the results were expanded in terms of inverse powers of the heavy quark masses. In the HQET sum rule approach, the baryonic Λb → Λc IW function was calculated in [3, 4], and the slope parameter ² was fitted to lie in the range 0.5-0.8. However, such low slope values would predict exclusive Λb

→ Λc semileptonic decay rates dangerously close to the inclusive semileptonic rate [21, 22].

A first measurement of the IW function for semileptonic Λb → Λc transitions has recently been reported by the DELPHI Collaboration [2]. The errors on this measurement are quite large. Using an exponential parametrization, they quote a value of ^{2} = 1.59 ± 1.10(stat). When the observed event rates were included in the fit, they obtained ^{2} = 0.46(stat) + 0.72(syst) [2]. Within the large error bars, the experimental slope value is compatible with the HQET sum rule results of [3, 4], although the experimental central values are considerably higher than the theoretical sum rule results.

Theoretically, the above-mentioned HQET sum rule results for the slope parameter ^{2} = 0.5-0.8 appear rather small. Because the number of light quark transitions is larger in the heavy baryon case than in the heavy meson case, one expects that the slope of the baryonic IW function is larger than that of the mesonic IW function. In fact, in the large Nc limit, ^{2} will be infinitely large [23]. In the spectator quark model approach [6, 7, 24], one finds ^{2}_baryon = 2 ^{2}_meson when the interaction between the light quarks is turned on [25], which turns into an upper bound ^{2}_baryon 2 ^{2}_meson - 1/2. Since ^{2}_meson 1/4 according to the Bjorken sum rule, one then recovers the infinite slope result of [23] in the large Nc limit.

Concerning the slope parameter of the mesonic IW function, one finds theoretical values of about 1 from sum rule calculations [8]. Experimental numbers for the mesonic slope parameter also scatter around 1 [26, 27, 28]. Using the spectator quark model estimate, one thus expects baryonic values of the slope parameter ^{2} in the vicinity of 1.5 or slightly below that number. The Skyrme model predicts ^{2} 1.3 [29]. In the infinite momentum frame model, one has ^{2} = 1.44 [18] and in the relativistic three quark model, one finds ^{2} = 1.35 [19]. For the baryonic sum rule results, radiative corrections to ^{2} and 1/mQ corrections to the form factors are not expected to be large enough to solve the discrepancy between the large experimental central value for ^{2} in [2] and the small QCD sum rule results [3, 4]. We therefore concentrate only on the leading order results in our analysis.

The purpose of this work is to present a new QCD sum rule analysis for the leading-order Isgur-Wise function describing the Λb → Λc transition. In particular, we concentrate on the sum rule prediction directly for the slope parameter ^{2}. To start with, we first review the sum rule analysis of the two-point Green' s function involving two heavy baryon currents relevant for the determination of the heavy baryon decay constant and its mass.

A possible choice of the heavy baryon current with the correct quantum numbers of the heavy baryon ΛQ is given by:

j = q C q h ,

where C is the charge conjugation matrix, is an antisymmetric flavor matrix, and h and q are the heavy and light quark fields. Note that there is an alternative choice for the heavy baryon current j which is obtained by the replacement → /v in the equation. The two baryon currents give the same diagonal sum rule for the IW function.

From the correlator:

Γ( ) = i d x e^{ik · x} 0|T{j (x)j †(0)}|0 ,

one can obtain the two-point sum rule [3, 4, 13, 14]:

2f²e^{-Λ̄/T} = (10/) d   e^{- /T} + h̄h (8T²) +   G² (16 ³/T²),

where the baryonic decay constant is defined as 0|j |ΛQ(v) = fu_{ΛQ}, and Λ̄ = m_{ΛQ} - mQ is the binding energy of the ΛQ-baryon in HQET. T is the Borel parameter. According to the duality assumption, the higher resonance and continuum contributions to the Green' s function are approximated by the perturbative contribution above a given threshold s .

Note that the factor 2 on the left-hand side of the equation is missing in the calculation of Ref. [15]. The missing factor 2 is, however, of no relevance for the calculation of Λ̄ and the IW function.

To obtain the IW function, one considers the three-point correlator:

Ξ(, ,) = i² d x d x e^{i(k · x - k · x)} 0|T{j (x)h̄ (0)Γh (0)j †(x)}|0 ,

where = v · k and = v · k . In hadronic language, the three-point function can be expressed in terms of insertions of hadronic states. The lowest contribution involves the IW function:

Ξ(, ,) = f² ( )/(Λ̄ +)(Λ̄ +) + resonances.

On the other hand, the three-point correlator Ξ can be calculated using the Operator Product Expansion. The perturbative contribution can be written in terms of a double dispersion relation:

Ξ^{pert}(, ,) = d̃ d̃ (̃, ̃ ,)/((̃ -)(̃ -)),

with the following spectral density:

 (, ,) = (1/20 ²)(+) /²(² + ² - 2)² tr(†) Θ( )Θ( )Θ(2   - ² - ²).

The condensate contributions will be included as a series of vacuum expectation values of operators ordered by their dimension.

A systematic uncertainty of QCD sum rule calculations lies in the treatment of higher state contributions to the hadronic side of Ξ. Generally, the quark-hadron duality assumption is adopted, which simulates the higher state contribution by the perturbative part above some threshold energy. For three-point Green' s functions, this assumption is more ambiguous than for the two-point case because there are two energy variables ̃ and ̃ . It was argued by Blok and Shifman in [30] that the perturbative and hadronic spectral densities cannot be locally dual to each other. The necessary way to restore duality is to integrate the spectral densities over the "off-diagonal" variable ˜ = (˜ - ˜)/2, keeping the "diagonal" variable ˜ = (˜ + ˜)/2 fixed. It is with respect to ˜ that quark-hadron duality is assumed for the integrated spectral densities. With this procedure, the sum rule for the IW function yields [16]:

2f² e^{-Λ̄/T} = (1/(+ 1)³) ^{s̃_c} d˜ ˜ e^{-˜ /T} + h̄h (16T²/(+ 1)) + G² (12 ³/T²)(2 + 1)/(+ 1)²,

where s̃_c is the threshold energy for the sum rule.

It should be noted that the Borel parameter has been chosen such that (= 1) = 1. The dependence in the gluon condensate term and the coefficient of the exponential in the quark condensate term are different from Ref. [15] but are consistent with Refs. [17, 20]. From the equation above, one obtains the sum rule for the slope parameter of the IW function, ² = -d /d |_{ =1}, given by:

2f² ²e^{-Λ̄/T} = (20/) ^{s̃_c} d˜ ˜ e^{-˜ /T} + h̄h (48T²) + G² (48 ³/T²).

The baryonic decay constant f can be obtained from the sum rule in the first equation. Note that we have not included perturbative O( ) corrections in the sum rule, which are expected to be largely canceled in the ratio of the equations, as happened in the case of heavy meson form factors [8].

For the numerical analysis of the sum rule, we use the two-point sum rule to eliminate the explicit dependence on f and Λ̄. This procedure reduces the uncertainties in the calculation. For the condensate contributions, we take the standard values: h̄h = (0.24 ± 0.01)³ GeV³, G² = (0.012 ± 0.004) GeV , and m ² = 0.8 GeV². The threshold energies are taken to be equal, i.e., s_c = s̃_c. Imposing the usual criterion on the ratio of contributions from higher-order power corrections and the continuum, and using the central values of the condensates, there is an acceptable window of stability in the range T = 0.4-0.7 GeV. The calculated results do not change appreciably if the threshold parameter s_c lies in the range $1.7 < s_c < 2.1$ GeV.

In Figure 1, the sum rule for the slope parameter ² is plotted as a function of the Borel parameter T for various choices of the continuum threshold in the range $1.7 < s_c < 2.1$ GeV. One can see that the variation is very moderate for the Borel parameter in the range $0.4 < T < 0.7$ GeV. Our prediction for the slope parameter is:

² = 1.35 ± 0.12,

where the errors reflect the uncertainty due to the sum rule window. This value is in agreement with the recent experimental result [2] and the value obtained in a lattice determination [31].

It is meaningful to ask why the present sum rule result is so different from those obtained previously. First, this may be due to the systematic uncertainty of QCD sum rules. The systematic error resulting from the use of quark-hadron duality above s_c is difficult to estimate; conservatively speaking, there is a 10-30% systematic error. Second, the previous linear fit to the IW function [16, 15] may be a rather poor fit in the range: 1 to 1.43. In the following decay rate calculation, we assume an exponential form to parametrize the baryonic IW function, as the experiment did [2]:

$$( ) = (1) \exp[- ^2(- 1)].$$

Once we have computed the IW function, we are now in a position to calculate the rate for semileptonic Λb → Λc ⁻ transitions. Neglecting the lepton mass, the differential decay rate can be written as (see e.g. [4, 7]):

$$d\Gamma/d = (G_F^2|V_{cb}|^2 m_{\Lambda b})/(192 ^3) \sqrt{[(^2 - 1)(1 + r^2 - 2r)]} \times [F ^2 + 3F ^2 + \cdots],$$

where $r = m_{\Lambda c}/m_{\Lambda b}$ and $= 1 + r^2 - 2r$. The form factors F_i and G_i can be expressed by a set of Isgur-Wise functions at each order in $1/m_Q$ in HQET. Taking into account the $1/m_Q$ corrections, they are (see e.g. [4, 7]):

$$F = [1 + (_c + _b)\bar{\Lambda}] ( ),$$
$$F = G = [1 + (_c + _b)\bar{\Lambda}/(+ 1)] ( ),$$
$$G = ( ),$$
$$F = (2 _c\bar{\Lambda}/(+ 1)) ( ),$$

where $_Q = 1/(2m_Q)$. Notice that the subleading Isgur-Wise function associated with the insertion of the Λ_{QCD}/m_c kinetic operator of the HQET Lagrangian has been neglected since it is negligibly small [16, 18]. In the numerical calculation, we take the heavy quark masses to be $m_b = 4.8$ GeV, $m_c = 1.4$ GeV, and $\bar{\Lambda} = 0.79$ GeV [4]. With the masses of Λ_b and Λ_c given by the Particle Data Group (PDG) [32], the upper limit of is $_{max} = (1 + r^2)/2r = 1.433$. The decay rate can then be calculated to be:

$$\Gamma(\Lambda_b \to \Lambda_c ^-) = (2.12 \pm 0.30) \times 10 ^{11} |V_{cb}|^2 \text{ GeV}.$$

The contribution of the $1/m_Q$ corrections amounts to about 10%.

Let us compare our theoretical prediction to experimental data. Recently, the DELPHI Collaboration measured the $\Lambda_b \to \Lambda_c ^-$ branching ratio as (5.0 ¹ • ¹ • (sys))%. On the other hand, the branching ratio given by the PDG is (9.2 ¹ • • (stat))% [32]. An error-weighted average value was given by Albertus et al. [33]: $Br(\Lambda_b \to \Lambda_c ^-) = (6.8 \pm 1.3)\%$. The total Λ_b decay width is determined by the inverse lifetime $_{\Lambda_b}{}^{1}$, where we take $_{\Lambda_b} = 1.229 \pm 0.080$ ps [32]. The value of the CKM matrix element can then be extracted to be:

$$|V_{cb}| = 0.041 \pm 0.004,$$

where the error is due to experimental uncertainties. This value is in excellent agreement with the recent experimental determination by the DELPHI Collaboration $|V_{cb}| = 0.0414 \pm 0.0012(\text{stat}) \pm 0.0021(\text{syst}) \pm 0.0018(\text{theory})$, obtained from semileptonic $B \to D^* ^-$ decays [26]. It is also in good agreement with the value obtained from a nonrelativistic quark model calculation of the $\Lambda_b \to \Lambda_c$ transition [33].

In conclusion, we have presented an HQET sum rule analysis for the slope parameter ^{2} of the baryonic IW function, obtaining $^2 = 1.35 \pm 0.12$, which is in good agreement with the recent DELPHI measurement and the results of a lattice calculation. When combined with the error-weighted average value for the branching ratio of $\Lambda_b \to \Lambda_c$ $^-$, our integrated decay width including $1/m_Q$ corrections leads to a value for $|V_{cb}|$ in excellent agreement with recent determinations from $B \to D^*$ decays. One should bear in mind that $1/m^2_Q$ corrections can decrease the decay rate by a few percent, as estimated in [22], and should be studied within HQET sum rules in future works.

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