1/m corrections to heavy baryon masses in the heavy quark effective theory sum rules (postprint)

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Full Text

Preamble

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Abstract

The 1/m corrections to heavy baryon masses are calculated from QCD sum rules within the framework of heavy quark effective theory. Numerical results for heavy baryons are obtained, and the implications of these results are discussed.

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Introduction

Heavy baryons provide a testing ground for the Standard Model, particularly for QCD in several aspects. With the accumulation of experimental data on heavy baryons, more reliable theoretical calculations are needed, although some of these calculations are rather complicated. Within the framework of heavy quark effective theory (HQET), which is a model-independent method, the theoretical analysis of heavy baryons containing a single heavy quark is comparatively simple due to heavy quark symmetry. However, there remain quantities in this framework that need to be determined from nonperturbative QCD.

QCD sum rules, regarded as a nonperturbative method rooted in QCD itself, have been used successfully to calculate the properties of various hadrons. For instance, besides light mesons, light baryons were first considered in Ref. [**?**]. Heavy meson properties were systematically analyzed within HQET in Refs. [**?**, **?**]. Heavy baryons were first discussed in Ref. [**?**], and masses and Isgur-Wise functions for heavy baryons were calculated in HQET to leading order in the heavy quark expansion in Refs. [**?**, **?**]. In Ref. [**?**], the calculation for heavy baryons began with the full theory and results were expanded in inverse powers of heavy quark masses. In this paper, within the framework of HQET, we study heavy baryonic two-point correlators to subleading order in the heavy quark expansion using QCD sum rules and obtain results for heavy baryon masses to that order.

In HQET, the heavy quark mass m_Q, defined perturbatively as the pole mass, has been removed by field redefinition. The heavy quark field h_v is defined as $P_+Q(x) = \exp(-im_{Qv} \cdot x)h_v(x)$, where $P_+ = \frac{1}{2}(1 + \not{v})$. To order $1/m_Q$, the effective Lagrangian for the heavy quark is [**?**]

$$\mathcal{L}_{\text{eff}} = \bar{h}_v iv \cdot Dh_v + \frac{1}{2m_Q}\bar{h}_v(iD)^{2h}_v - \frac{1}{2m_Q}\bar{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v.$$

Among the $1/m_Q$ terms, the first one still respects heavy quark spin symmetry, while the last term violates spin symmetry. The heavy baryon mass M is expanded as [**?**]

$$M = m_Q + \bar{\Lambda} + \frac{\Delta}{2m_Q}\langle\vec{s}_Q \cdot \vec{j}_\ell\rangle + \mathcal{O}(1/m_Q^2),$$

where $\bar{\Lambda}$ is the heavy baryon mass in the heavy quark limit, which has been calculated in Ref. [**?**]. $\delta\Lambda_K$ and $\delta\Lambda_G$ parameterize the spin-conserved and spin-violated $1/m_Q$ corrections respectively. All these quantities characterize the properties of the light degrees of freedom. $\vec{s}_Q$ denotes the heavy quark spin, and $\vec{j}_\ell$ stands for the total angular momentum of the light degrees of freedom. For Λ_Q baryons, the $\delta\Lambda_G$ term vanishes. For Σ_Q baryons, both $\delta\Lambda_K$ and $\delta\Lambda_G$ terms are nonvanishing, with $\langle\vec{s}_Q \cdot \vec{j}_\ell\rangle = -\frac{1}{2}$ for Σ_Q and $+\frac{1}{2}$ for Σ_Q^*.

The heavy baryonic currents $\tilde{j}_v$ have been given in Refs. [?, ?] in the rest frame of the heavy baryons. Generally they can be expressed as

$$\tilde{j}_v = \epsilon^{abc}(q_a^T C\Gamma\tau q_b)\Gamma' h_c,$$

where C is the charge conjugation matrix, τ is a flavor matrix, Γ and Γ' are gamma matrices, and a, b, c denote color indices. Γ and Γ' can be chosen covariantly as

$$\Gamma_\Lambda = \gamma_5, \quad \Gamma'_\Lambda = 1,$$

for Λ_Q baryons;

$$\Gamma_\Sigma = \gamma^\mu, \quad \Gamma'_\Sigma = (v^\mu + \gamma^\mu)\gamma_5,$$

for Σ_Q baryons;

$$\Gamma_{\Sigma^*} = \gamma^\nu, \quad \Gamma'_{\Sigma^*} = -g_{\mu\nu} + \gamma_\mu\gamma_\nu - (\gamma_\mu v_\nu - \gamma_\nu v_\mu) + v_\mu v_\nu,$$

for Σ_Q^* baryons.

The choice of Γ is not unique. Another kind of baryonic current can be obtained by inserting a factor $\not{v}$ before the Γ in Eqs. (5-7). The currents given by Eqs. (5-7) are denoted as $\tilde{j}_v^1$, and those with $\not{v}$ insertion as $\tilde{j}_v^2$. We define the "baryonic decay constant" f in HQET as follows:

$$\langle 0|\tilde{j}_v|\Lambda_Q\rangle = f_\Lambda u,$$

$$\langle 0|\tilde{j}_v|\Sigma_Q\rangle = f_\Sigma u,$$

$$\langle 0|\tilde{j}_v^\mu|\Sigma_Q^*\rangle = \frac{1}{\sqrt{3}} f_{\Sigma^*} u^\mu,$$

where u is the spinor and u^μ is the Rarita-Schwinger spinor in HQET respectively. f_{Σ^*} is the same as f_Σ in the heavy quark limit. As in the mass expansion (3), the square of f can be expanded similarly:

$$f^2 = \bar{f}^2 + \frac{\delta f_K^2}{2m_Q} + \frac{\delta f_G^2}{2m_Q}\langle \vec{s}_Q \cdot \vec{j}_\ell\rangle + \mathcal{O}(1/m_Q^2),$$

where $\bar{f}^2$ denotes the leading order result and δf_K^2 and δf_G^2 represent the spin-conserved and spin-violated $1/m_Q$ corrections respectively.

The two-point correlator $\Gamma(\omega)$ that we choose for sum rule analysis in HQET is

$$\Gamma_{ij}(\omega) = i \int d^{4x} e^{ikx} \langle 0|T\{\tilde{j}_v^i(x) \bar{\tilde{j}}_v^j(0)\}|0\rangle, \quad i,j = 1,2,$$

where $\omega = 2v \cdot k$. The hadronic representation of this correlator is

$$\Gamma_{ij}(\omega) = \frac{4\bar{f}^2 \delta_{ij}}{(2\bar{\Lambda} - \omega)^2} + \frac{2\bar{f}^2}{(2\bar{\Lambda} - \omega)^2} \left[\frac{\delta\Lambda}{2m_Q} + \frac{\delta f^2}{2m_Q} \frac{1 + \not{v}}{2} \right] + \text{res.},$$

where $\delta\Lambda$ and δf^2 stand for the $1/m_Q$ corrections in Eqs. (3) and (9). On the other hand, $\Gamma_{ij}(\omega)$ can be calculated in terms of quark and gluon degrees of freedom with vacuum condensates. This establishes the sum rule. We use the commonly adopted quark-hadron duality for the resonance part of Eq. (11),

$$\text{res.} = \frac{1}{\pi} \int_{\omega_c}^{\infty} d\omega' \frac{\text{Im}\Gamma_{\text{pert}}(\omega')}{\omega' - \omega},$$

where $\Gamma_{\text{pert}}(\omega)$ denotes the perturbative contribution, and ω_c is the continuum threshold. In this work, we shall consider only the diagonal correlators ($i = j$).

The calculations of $\Gamma(\omega)$ are straightforward. The fixed-point gauge is used [?]. All condensates with dimensions lower than 6 are retained. We also include the dimension-6 condensate $\langle \bar{q}(0)q(x)\rangle^2$ in our analysis, which gives a main contribution. We use the Gaussian ansatz for the spacetime distribution of this condensate [?].

In the heavy quark limit, we have double-checked the analysis of Ref. [?]. We use the following values for the condensates:

$$\langle \bar{q}q\rangle \simeq -(0.23\,\text{GeV})^3, \quad \langle \alpha_s GG\rangle \simeq 0.04\,\text{GeV}^4, \quad \langle g\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle \equiv m_0^2\langle \bar{q}q\rangle, \quad m_0^2 \simeq 0.8\,\text{GeV}^2.$$

When ω_c lies between $2.1 - 2.7$ GeV for Λ_Q and between $2.3 - 2.9$ GeV for Σ_Q, a stability window exists. We obtain

$$\bar{\Lambda}_\Lambda = 0.79 \pm 0.05 \text{ GeV}, \quad \Lambda = (0.3 \pm 0.1) \times 10^{-3} \text{ GeV}^6,$$

for Λ_Q baryons; and

$$\bar{\Lambda}_\Sigma = 0.96 \pm 0.05 \text{ GeV}, \quad \Sigma = (1.7 \pm 0.5) \times 10^{-3} \text{ GeV}^6,$$

for Σ_Q baryons. The normalization $\text{Tr}\tau^\dagger\tau = 1$ has been used in the analysis. The errors quoted in Eqs. (14) and (15) contain only those from the stability of the sum rule windows. We still do not know the α_s corrections for baryons. Taking the meson system as a reference [?], the α_s correction is very small for $\bar{\Lambda}$, but very large (30%) for $\bar{f}$. The numerical results are in agreement with those of Ref. [?], and the range of the Borel parameter is the same: $T = 0.4 - 0.7$.

The $1/m_Q$ corrections to the two-point correlator $\Gamma(\omega)$ can be calculated by including insertions of the $1/m_Q$ operators from the Lagrangian (2) using the standard method shown in Fig. 1 [Figure 1: see original paper]. The insertions of spin-conserved and spin-violated operators are calculated separately. The final form of the sum rules is obtained by performing Borel transformation. With some simple tricks [?], the sum rules for the mass and f can be separated.

The results for the mass of Λ_Q baryons come from the spin-conserved operators only ($\delta\Lambda_\Lambda = \delta\Lambda_K$):

$$\delta\Lambda_\Lambda^1 = -\frac{1}{16\bar{f}^2} I_{\Lambda 1}, \quad \delta\Lambda_\Lambda^2 = -\frac{1}{16\bar{f}^2} I_{\Lambda 2},$$

where

$$I_{\Lambda 1} = \left(\frac{27\pi^4}{5} \int d\omega\, \omega^6 e^{-\omega/T} + \frac{0\langle\bar{q}q\rangle^2}{2T^2} + \frac{43\langle\alpha_s GG\rangle}{25\pi^3} 9T^3 \right) e^{2\bar{\Lambda}/T},$$

$$I_{\Lambda 2} = \left(\frac{27\pi^4}{5} \int d\omega\, \omega^6 e^{-\omega/T} + \frac{0\langle\bar{q}q\rangle^2}{2T^2} + \frac{61\langle\alpha_s GG\rangle}{25\pi^3} 9T^3 \right) e^{2\bar{\Lambda}/T},$$

and the superscripts 1 and 2 denote $\tilde{j}_v^1$ and $\tilde{j}_v^2$ respectively. The sum rule for f of Λ_Q is

$$\delta f_{\Lambda i}^2 = -\frac{d}{d(1/T)} \left(\frac{1}{T} I_{\Lambda i} \right).$$

The masses of baryons Σ_Q and Σ_Q^* are given in terms of $\delta\Lambda_K$ and $\delta\Lambda_G$. They are determined by the following sum rules:

$$\delta\Lambda_K^{\Sigma 1} = -\frac{1}{16\bar{f}^2} \Sigma_1, \quad \delta\Lambda_K^{\Sigma 2} = -\frac{1}{16\bar{f}^2} \Sigma_2,$$

$$\delta\Lambda_G^{\Sigma 1} = \frac{1}{16\bar{f}^2} \Sigma_1, \quad \delta\Lambda_G^{\Sigma 2} = -\frac{1}{16\bar{f}^2} \Sigma_2,$$

where

$$\Sigma_1 = \left(\frac{27\pi^4}{5} \int d\omega\, \omega^6 e^{-\omega/T} + \frac{0\langle\bar{q}q\rangle^2}{2T^2} + \frac{13\langle\alpha_s GG\rangle}{25\pi^3} 3T^3 \right) e^{2\bar{\Lambda}/T},$$

$$\Sigma_2 = \left(\frac{27\pi^4}{5} \int d\omega\, \omega^6 e^{-\omega/T} + \frac{0\langle\bar{q}q\rangle^2}{2T^2} - \frac{5\langle\alpha_s GG\rangle}{25\pi^3} 3T^3 \right) e^{2\bar{\Lambda}/T}.$$

The sum rules for f are given by

$$\delta f^{\Sigma i}_{K,G} = -\frac{d}{d(1/T)} \left(\frac{1}{T} I^{\Sigma i}_{K,G} \right).$$

It can be seen that while the two diagonal sum rules coincided with each other at leading order, they are no longer the same for the spin-conserved $1/m_Q$ corrections.

The numerical sum rule results for the $1/m_Q$ corrections—$\delta\Lambda$ and δf^2 in Eqs. (3) and (9)—are given in Tables 1 and 2 and Figs. 2-4. The numerical differences resulting from the different choices of $\tilde{j}_v$ are not significant. The values of ω_c are generally smaller than the leading order results, but still lie within the allowed range of the leading order results. The lower limit of the Borel parameter $T = 0.4$ GeV is determined by requiring that the condensates in Eqs. (17) and (20) contribute less than 40%. The upper limit $T = 0.6$ GeV is obtained by requiring that the pole contribution exceeds 70%. This window is narrower than the leading order one. In the window $T = 0.4 - 0.6$ GeV, the results for $\delta\Lambda_\Lambda$ and $\delta\Lambda^\Sigma_K$ are comparatively stable. However, from Fig. 4 [Figure 4: see original paper], we see that $\delta\Lambda^\Sigma_G$ has no good stability in this window. This is because we have not included Feynman diagrams with internal gluon lines, which are expected to be important for the spin-violated terms. Therefore, the value of $\delta\Lambda^\Sigma_G$ in Table 1 is not reliable. The errors quoted in Tables 1 and 2 again refer only to those from the stability of the sum rule windows.

From m_{Λ_c} and m_{Λ_b} [?], we determine the heavy quark masses $m_c = 1.43 \pm 0.05$ GeV and $m_b = 4.83 \pm 0.07$ GeV. These values give the following results:

$$m_{\Sigma_c} = 2.52 \pm 0.08 \text{ GeV}, \quad m_{\Sigma_c^*} = 2.55 \pm 0.08 \text{ GeV},$$

$$m_{\Sigma_b} = 5.83 \pm 0.09 \text{ GeV}, \quad m_{\Sigma_b^*} = 5.84 \pm 0.09 \text{ GeV}.$$

From the discussion above, we know that individual mass values in Eqs. (22) and (23) suffer from the inaccuracy of $\delta\Lambda^\Sigma_G$. The quantity $\frac{1}{3}(m_{\Sigma_Q} + 2m_{\Sigma_Q^*}) = m_Q + \bar{\Lambda} + (0.22 \pm 0.06 \text{ GeV}^2)/m_Q$ is independent of $\delta\Lambda^\Sigma_G$ and therefore more reliable. It is 2.54 ± 0.08 GeV for the c quark case and 5.83 ± 0.09 GeV for the b quark case. Experimentally, $m_{\Sigma_c} = 2453 \pm 0.2$ MeV [?]. There is experimental evidence for Σ_c^* at $m_{\Sigma_c^*} = 2530 \pm 7$ MeV [?]. If we take this value for $m_{\Sigma_c^*}$, we have $\frac{1}{3}(m_{\Sigma_c} + 2m_{\Sigma_c^*}) = 2504 \pm 5$ MeV, which is in reasonable agreement with the theoretical value. The corresponding quantity for the bottom quark can be checked by experiments in the near future.

In conclusion, we have calculated the $1/m_Q$ corrections to heavy baryon masses from QCD sum rules within the framework of HQET. This study refines the leading order analysis of Ref. [?]. Furthermore, within this framework, we can study three-point correlators which will give the form factors for weak transitions of heavy baryons [?] to order $1/m_Q$. It is also viable to include QCD radiative corrections in both leading order and subleading order calculations. Both of these aspects are currently under investigation.

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Figure Captions

Fig. 1. The subleading operator insertions relevant to our analysis.

Fig. 2 [Figure 2: see original paper]. Sum rules for $\delta\Lambda_\Lambda$ with (a) $\tilde{j}_v^1$ and (b) $\tilde{j}_v^2$. $\omega_c = 2.0, 2.1, 2.4$ GeV for solid, dashed, dash-dotted curves respectively. The sum rule window is $T = 0.4 - 0.6$ GeV.

Fig. 3 [Figure 3: see original paper]. Sum rules for $\delta\Lambda_K^\Sigma$ with (a) $\tilde{j}_v^1$ and (b) $\tilde{j}_v^2$. $\omega_c = 2.2, 2.4, 2.7$ GeV for solid, dashed, dash-dotted curves respectively. The sum rule window is $T = 0.4 - 0.6$ GeV.

Fig. 4. Sum rule for $\delta\Lambda_G^\Sigma$. The sum rule window is $T = 0.4 - 0.6$ GeV.

Tables

Table 1. Numerical results for $\delta\Lambda$.

	$\delta\Lambda_\Lambda$ (GeV^2)	$\delta\Lambda_K^\Sigma$ (GeV^2)	$\delta\Lambda_G^\Sigma$ (GeV^2)
$\omega_c = 2.1 \pm 0.1$ GeV	0.09 ± 0.03	0.09 ± 0.05	0.22 ± 0.06
$\omega_c = 2.4 \pm 0.2$ GeV	0.03 ± 0.02	0.21 ± 0.06	0.03 ± 0.02

Table 2 . Numerical results for δf^2.

	δf_Λ^2 (10^{-3} GeV^6)	$\delta f_K^{\Sigma 2}$ (10^{-3} GeV^6)	$\delta f_G^{\Sigma 2}$ (10^{-3} GeV^6)
$\omega_c = 2.1 \pm 0.1$ GeV	-0.20 ± 0.1	-0.3 ± 0.1	0.7 ± 0.4
$\omega_c = 2.4 \pm 0.2$ GeV	0.8 ± 0.5	-0.1 ± 0.1	-0.1 ± 0.1

Note: Figure translations are in progress. See original paper for figures.