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# QCD Sum Rule Analysis for the #3;b → #3;c Semileptonic Decay Postprint

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## Abstract

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## Full Text

## Preamble

**QCD Sum Rule Analysis for the $\Lambda_b \to \Lambda_c$ Semileptonic Decay**

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## Abstract

The $1/m_c$ and $1/m_b$ corrections to the $\Lambda_b \to \Lambda_c$ semileptonic decay are analyzed by QCD sum rules. Within the framework of heavy quark effective theory, the subleading baryonic Isgur-Wise function for the $\Lambda_b \to \Lambda_c$ transition has been calculated. It is shown that the corrections due to the $1/m_Q$ Lagrangian insertion are negligibly small. The sizable $1/m_Q$ effect to the decay lies only in the weak current. The decay spectrum and the branching ratio are given.

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**Keywords:** heavy baryon, weak decay, heavy quark effective theory, QCD sum rule, $1/m_Q$ correction

## 1. Introduction

The weak decays of heavy baryons provide an important testing ground for the Standard Model. They reveal crucial features of heavy quark physics. From the study of heavy quark physics, fundamental parameters of the Standard Model, for instance, the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{cb}$, can be extracted by comparing experimental data with theoretical calculations for the decay mode $\Lambda_b \to \Lambda_c l^- \bar{\nu}$.

The main difficulties in Standard Model calculations arise from the poor understanding of nonperturbative aspects of strong interactions (QCD). Besides numerical lattice methods, several analytic, model-independent nonperturbative QCD methods have been developed. For heavy hadrons containing a single heavy quark, an effective theory of QCD based on heavy quark symmetry in the heavy quark limit [1], the so-called heavy quark effective theory (HQET), has been proposed [2].

The classification of weak decay form factors for heavy baryons has been greatly simplified in HQET [3]. To increase the precision of the analysis, subleading corrections [4] to the heavy quark limit results have also been considered for baryons [5]. However, for a complete analysis of heavy baryons, we still need to employ additional nonperturbative methods.

Combining the QCD sum rule method [6], a complete analysis for heavy baryons can be performed within HQET. As a nonperturbative method rooted in QCD itself, QCD sum rules have been successfully applied to calculate properties of various hadrons [6, 7]. For heavy mesons, this method has been used in the HQET framework at leading order in the heavy quark expansion to calculate masses, decay constants, and the Isgur-Wise function [8], and $1/m_Q$ corrections have also been computed [9, 10]. Heavy baryons were first studied using QCD sum rules in Ref. [11]. The heavy baryon masses and baryonic Isgur-Wise functions have been calculated using HQET sum rules at leading order in Refs. [12] and [13], respectively. We [14] and another group [15] have calculated the $1/m_Q$ corrections to heavy baryon masses building on the results of Ref. [12]. In this paper, we extend the analysis to calculate the subleading Isgur-Wise function for the weak transition $\Lambda_b \to \Lambda_c$ using HQET sum rules.

The hadronic matrix element of the weak current for $\Lambda_b \to \Lambda_c$ is generally parameterized by six form factors $F_i$ and $G_i$ $(i = 1, 2, 3)$:

$$\langle \Lambda_c(v')|\bar{c}\gamma_\mu(1-\gamma_5)b|\Lambda_b(v)\rangle = \bar{u}_{\Lambda_c}(v')[F_1\gamma_\mu + F_2 v_\mu + F_3 v'_\mu]u_{\Lambda_b}(v) + \bar{u}_{\Lambda_c}(v')[G_1\gamma_\mu + G_2 v_\mu + G_3 v'_\mu]\gamma_5 u_{\Lambda_b}(v),$$

where $v$ and $v'$ denote the four-velocities of $\Lambda_b$ and $\Lambda_c$, respectively. These form factors must be determined by some nonperturbative QCD method. Within HQET, their classification is greatly simplified. To order $1/m_Q$, the effective Lagrangian for the heavy quark $h_v$ is:

$$\mathcal{L}_{eff} = \bar{h}_v iv \cdot D h_v + \frac{1}{2m_Q}[\bar{h}_v (iD)^2 h_v + \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v].$$

In the heavy quark limit, the form factors are determined by a single universal function $\xi(y)$:

$$\langle\Lambda_c(v')|\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}|\Lambda_b(v)\rangle = \xi(y)\bar{u}_{\Lambda_c}(v')\Gamma u_{\Lambda_b}(v),$$

where $y = v\cdot v'$ and $\Gamma$ is some gamma matrix structure. To order $1/m_Q$, they are determined by one mass parameter $\bar{\Lambda}$ and an additional function $\chi(y)$, defined as:

$$\bar{\Lambda} = m_{\Lambda_Q} - m_Q,\ \langle\Lambda_c(v')|\bar{h}_{v'}^{(c)}\Gamma(iD)^2 h_v^{(b)}|\Lambda_b(v)\rangle = -2\chi(y)\bar{u}_{\Lambda_c}(v')\Gamma u_{\Lambda_b}(v).$$

Both the leading-order universal function $\xi$ and the subleading function $\chi$ are called Isgur-Wise functions. While $\xi$ and $\bar{\Lambda}$ have been calculated using QCD sum rules, we will calculate the subleading Isgur-Wise function $\chi$.

## 2. QCD Sum Rule Analysis

QCD sum rules provide a method for calculating nonperturbative physical quantities [8]. The Green's function from which the Isgur-Wise function can be extracted is the three-point correlator of heavy baryonic currents $\tilde{j}$ and the weak current in HQET.

Generally, the current for heavy $\Lambda$-baryons is:

$$\tilde{j}_v = \epsilon^{abc}(q^{aT}C\tilde{\Gamma}\tau q^b)h_v^c,$$

where $C$ is the charge conjugation matrix, $\tau$ is an antisymmetric flavor matrix, $a, b, c$ are color indices, and the choice of the gamma matrix $\tilde{\Gamma}$ is not unique. There are two common choices:

$$\tilde{\Gamma}_1 = \gamma_5, \quad \tilde{\Gamma}_2 = \not{v}\gamma_5.$$

We denote the corresponding currents as $\tilde{j}_v^1$ for $\tilde{\Gamma}_1$ and $\tilde{j}_v^2$ for $\tilde{\Gamma}_2$. Before performing the sum rule analysis for the three-point correlator needed to obtain the subleading function $\chi$, we first review some two-point correlator results from QCD sum rules [14], as they are related to the three-point correlator analysis.

In Ref. [14], we obtained heavy baryon masses and the so-called baryonic "decay constants" to order $1/m_Q$ through QCD sum rule analysis of two-point correlators. With the definition of the "decay constant" $f$ in HQET, where $u$ is the HQET spinor, the sum rules give:

$$f_1^2 e^{-2\bar{\Lambda}_1/T} = \frac{1}{5\pi^4}\int_0^{\omega_c} d\omega\omega^5 e^{-\omega/T} + \frac{1}{12}\langle\bar{q}q\rangle^2 e^{-2/T^2} + \frac{1}{27\pi^4}\langle\alpha_s GG\rangle\int_0^{\omega_c} d\omega\omega^6 e^{-\omega/T} + \frac{\langle\bar{q}q\rangle^2}{2T^2} + \frac{13\langle\alpha_s GG\rangle}{3T^3},$$

for $\tilde{j}_v^1$, and

$$f_2^2 e^{-2\bar{\Lambda}_2/T} = \frac{1}{5\pi^4}\int_0^{\omega_c} d\omega\omega^5 e^{-\omega/T} + \frac{1}{12}\langle\bar{q}q\rangle^2 e^{-2/T^2} + \frac{1}{27\pi^4}\langle\alpha_s GG\rangle\int_0^{\omega_c} d\omega\omega^6 e^{-\omega/T} + \frac{\langle\bar{q}q\rangle^2}{2T^2} + \frac{19\langle\alpha_s GG\rangle}{3T^3},$$

for $\tilde{j}_v^2$. In these equations, $T$ is the Borel parameter and $\omega_c$ is the continuum threshold. There are some errors in the coefficients of the gluon condensates in the $1/m_Q$ corrections in Ref. [14]. For $\Lambda$ baryons, the coefficients are corrected in this paper. Additionally, in Eq. (20) of Ref. [14], the coefficients 13/3 and 5/3 should be replaced by 3. However, these modifications do not affect the numerical results of Ref. [14].

The three-point correlator $\tilde{\Xi}(\omega,\omega',y)$ we choose for sum rule analysis in HQET is:

$$\tilde{\Xi}_{ij}(\omega,\omega',y) = i^2 \int d^4x' \int d^4x e^{ik'\cdot x' - ik\cdot x} \langle 0|T\{\tilde{j}^i_{v'}(x')\bar{h}^{(Q')}_{v'}(0)\Gamma h^{(Q)}_v(0)\bar{\tilde{j}}^j_v(x)\}|0\rangle,$$

with $i,j = 1,2$, where $\omega = 2v\cdot k$ and $\omega' = 2v'\cdot k'$. Due to heavy quark symmetry, we take $m_Q = m_{Q'}$ for simplicity. The hadronic representation of this correlator is:

$$\tilde{\Xi}_{ij}(\omega,\omega',y) = \frac{f_i f_j}{(2\bar{\Lambda}_i-\omega)(2\bar{\Lambda}_j-\omega')}[\xi(y) + \frac{1}{2m_Q}\chi(y) + \ldots] + \text{resonances},$$

where $\bar{\Lambda}$ and $f^2$ have been given in the sum rules above to order $1/m_Q$.

On the other hand, $\tilde{\Xi}(\omega,\omega',y)$ can be calculated in terms of quark and gluon fields with vacuum condensates, establishing the sum rule. Only diagonal correlators ($i=j$) will be considered. It should be remarked that in general one could consider a correlation function of the linear combination $\tilde{j}_v^1 + x\tilde{j}_v^2$ with mixing parameter $x$. However, with the commonly adopted quark-hadron duality, the mixed correlator $\tilde{\Xi}_{12}$ has no perturbative contribution in the sum rule. Therefore, mixing effects are expected to be small.

The calculation of $\tilde{\Xi}(\omega,\omega',y)$ is straightforward. In addition to the Feynman diagrams at leading order in the heavy quark expansion given in Ref. [13], the diagrams for the $1/m_Q$ corrections to the three-point correlator $\tilde{\Xi}(\omega,\omega',y)$ are shown in Fig. 1. These are calculated by inserting the $1/m_Q$ operators from the Lagrangian using standard methods. The chromo-magnetic operator insertion vanishes for the $\Lambda_Q \to \Lambda_{Q'}$ transition, so only kinetic energy term insertions need to be considered. We adopt the coordinate representation in our calculation rather than the momentum representation.

The heavy quark propagator takes a very simple form in coordinate representation, making calculations comparatively easy. Including the insertion of the pure kinetic energy term at order $1/m_Q$, the heavy quark propagator becomes:

$$T\{h_v(x)\bar{h}_v(0)\} = (1 + \frac{i\partial_\mu\partial^\mu}{2m_Q})\delta(x).$$

The fixed-point gauge [16] is used. All condensates with dimension lower than 6 are retained. We also include the dimension-6 condensate $\langle\bar{q}(x)q(x')\rangle^2$, which gives a major contribution. We use the Gaussian ansatz for the spacetime distribution of this condensate [17]. The condensate values used are:

$\langle\bar{q}q\rangle = -(0.23\text{ GeV})^3$, $\langle\alpha_s GG\rangle = 0.04\text{ GeV}^4$, $\langle g\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle = m_0^2\langle\bar{q}q\rangle$, $m_0^2 = 0.8\text{ GeV}^2$.

The normalization $\mathrm{Tr}(\tau^\dagger\tau) = 1$ is used in the analysis. In the fixed-point gauge, spacetime translational invariance is violated, but it is restored by adding all diagrams in Fig. 1, providing a check of our calculation.

We employ the commonly adopted quark-hadron duality for the resonance contribution. Generally, duality simulates the resonance contribution by the perturbative part above some threshold energy $\omega_c$. The perturbative contribution of the three-point correlator $\tilde{\Xi}_{pert}(\omega,\omega',y)$ can be expressed through a double dispersion relation:

$\tilde{\Xi}_{pert}(\omega,\omega',y) = \int\int d\tilde{\omega}d\tilde{\omega}' \frac{\mathrm{Im}\,\tilde{\Xi}_{pert}(\tilde{\omega},\tilde{\omega}',y)}{(\tilde{\omega}-\omega)(\tilde{\omega}'-\omega')}$.

The integration domain is a kite-like area. With the redefinition of integration variables:

$\omega_+ = (\tilde{\omega}+\tilde{\omega}'+y+1)^{1/2}$, $\omega_- = (\tilde{\omega}-\tilde{\omega}'+y+1)^{1/2}$,

the integration becomes:

$\int\int d\tilde{\omega}d\tilde{\omega}' \cdots = 2(y+1)\int d\omega_+ \int d\omega_- \ldots$

It is in $\omega_+$ that quark-hadron duality is assumed [18]:

resonance $= \frac{1}{\pi^2}\int_{\omega_c}^{\infty} d\omega_+ \frac{\mathrm{Im}\,\tilde{\Xi}_{pert}(\tilde{\omega},\tilde{\omega}',y)}{(\tilde{\omega}-\omega)(\tilde{\omega}'-\omega')}$.

In the heavy quark limit, we have double-checked the analysis of Ref. [13]. There are two sum rules for the leading-order Isgur-Wise function corresponding to the two choices of baryonic current. For $\omega_c$ between 1.8–2.5 GeV and $T = 0.3$–$0.6$ GeV, the two results for the Isgur-Wise function are consistent with each other.

For $y$ in the physical region $1 \le y \le 1.43$, a linear approximation fits the results:

$\xi(y) = 1 - \rho(y-1), \quad \rho = 0.55 \pm 0.15$,

where the uncertainty in $\rho$ accounts for variations in $\omega_c$ and $T$, as well as differences between the two sum rule results. For $y$ in the range $1 \le y \le 3$, we find that the following function fits our numerical results very well for reasonable $\omega_c$ and $T$:

$\xi(y) = (\frac{y+1}{2})^{0.5}\exp[-\rho(y-1)]$.

We note that the $y$-dependence of the Isgur-Wise function is not as steep as that predicted by the Skyrme model [19] and the quark model [20].

The sum rule for the subleading Isgur-Wise function $\chi(y)$ is:

$\chi(y) = \frac{e^{2\bar{\Lambda}/T}}{8\bar{\Lambda}f^2}[J(y) - \xi(y)J(1)]$,

where

$J_1(y) = \frac{1}{27\pi^4}\int_0^{\omega_c} d\omega\omega^6 e^{-\omega/T}[\frac{4y}{y+1} + \frac{4T^2}{y+1}(y^2-1)] + \frac{\langle\alpha_s GG\rangle}{3\pi^4}[\frac{4y}{y+1} + \frac{4T^2}{y+1}(y^2-1)] + \frac{\langle\bar{q}q\rangle^2}{2T^2}[\frac{4y}{y+1} + \frac{4T^2}{y+1}(y^2-1)]e^{-4T^2/(y+1)} + \frac{13\langle\alpha_s GG\rangle}{3T^3}[\frac{4y}{y+1} + \frac{4T^2}{y+1}(y^2-1)]$,

$J_2(y) = \frac{1}{27\pi^4}\int_0^{\omega_c} d\omega\omega^6 e^{-\omega/T}[\frac{4y}{y+1} + \frac{4T^2}{y+1}(y^2-1)] + \frac{\langle\alpha_s GG\rangle}{3\pi^4}[\frac{4y}{y+1} + \frac{4T^2}{y+1}(y^2-1)] + \frac{\langle\bar{q}q\rangle^2}{2T^2}[\frac{4y}{y+1} + \frac{4T^2}{y+1}(y^2-1)]e^{-4T^2/(y+1)} + \frac{19\langle\alpha_s GG\rangle}{3T^3}[\frac{4y}{y+1} + \frac{4T^2}{y+1}(y^2-1)]$,

with subscripts 1 and 2 denoting the two baryonic currents. Luke's theorem [4] for baryons, $\chi(1) = 0$, is satisfied automatically. The numerical results are shown in Fig. 2, where the two curves correspond to the two sum rule results. The range of $\omega_c$ is the same as in the leading-order case. The sum rule window is narrower than at leading order. In the window $T = 0.35$–$0.55$ GeV, the results for the subleading Isgur-Wise function are stable. The two results can be regarded as consistent with each other. Nevertheless, it is obvious that the subleading Isgur-Wise function is negligibly small, of order $10^{-2}$.

## 3. Decay Rate and Branching Ratio

The semileptonic decay $\Lambda_b \to \Lambda_c l^- \bar{\nu}$ can be analyzed directly after obtaining the hadronic matrix elements from QCD sum rules. Neglecting the lepton mass, the differential decay rate is:

$$\frac{d\Gamma(\Lambda_b \to \Lambda_c l^- \bar{\nu})}{dy} = \frac{G_F^2 |V_{cb}|^2 m_{\Lambda_b}^5}{192\pi^3}\sqrt{(y^2-1)(1+r^2-2ry)}[Ar^2 + 2Br + C],$$

where $r = m_{\Lambda_c}/m_{\Lambda_b}$. In the above equation:

$A = 2F_1F_2 + (y+1)F_2^2 + 2G_1G_2 + (y-1)G_2^2$, $B = F_1F_2 + F_2F_3 + F_3F_1 + yF_2F_3 + G_1G_2 - G_2G_3 + G_3G_1 + yG_2G_3$, $C = (y+1)F_1^2 + 2F_1F_3 + (y-1)G_1^2 + 2G_1G_3$.

To order $1/m_c$ and $1/m_b$, the form factors $F_i$ and $G_i$ are expressed as:

$F_1 = C(\mu)\xi(y) + (\frac{1}{m_c} + \frac{1}{m_b})[2\chi(y) + \xi(y)]$, $G_1 = C(\mu)\xi(y) + (\frac{1}{m_c} + \frac{1}{m_b})[2\chi(y) + \xi(y)]$, $F_2 = \frac{1}{m_c}\frac{\xi(y)}{y+1}$, $G_2 = \frac{1}{m_b}\frac{\xi(y)}{y+1}$, $F_3 = G_3 = 0$,

where $C(\mu)$ is the perturbative QCD coefficient. The subleading Isgur-Wise function can be safely neglected. The $1/m_Q$ corrections arise mainly from the weak current.

Using the leading-order Isgur-Wise function from Eq. (19), the differential decay rate for $\Lambda_b \to \Lambda_c l^- \bar{\nu}$ is shown in Fig. 3. In this figure, we have taken the heavy quark masses $m_b = 4.83$ GeV, $m_c = 1.44$ GeV, and $\bar{\Lambda} = 0.79$ GeV [14], the renormalization point $\mu = 470$ MeV, and the CKM matrix element $V_{cb} = 0.04$ [21]. The width and branching ratio for this decay mode are:

$\Gamma = 6.05 \times 10^{-14}$ GeV, $Br = 9.8\%$.

The $1/m_Q$ correction contributes about 10% to this branching ratio.

## 4. Conclusion

We have analyzed the $\Lambda_b \to \Lambda_c$ semileptonic decays using QCD sum rules within the HQET framework to order $1/m_c$ and $1/m_b$. In the heavy quark limit, the analysis of $\Lambda_b \to \Lambda_c$ decay depends on one independent form factor—the leading-order Isgur-Wise function—which was calculated using QCD sum rules in Ref. [13]. However, for a more precise analysis, leading-order calculations are insufficient.

In this paper, we have considered $1/m_Q$ corrections. The subleading Isgur-Wise function has been calculated using HQET sum rules and is shown to be so small that it can be neglected. The $1/m_Q$ correction to the $\Lambda_b \to \Lambda_c$ decay arises primarily from the weak current. The differential decay distribution has been presented. The branching ratio is predicted to be $Br(\Lambda_b \to \Lambda_c l^- \bar{\nu}) = 9.8\%$ for $V_{cb} = 0.04$, which will be useful for comparison with future experiments. Polarization effects for this decay have not been calculated and will be considered elsewhere.

Finally, we remark on perturbative QCD corrections in sum rule calculations. Such corrections to the baryonic Isgur-Wise function, which have not yet been included, would involve three-loop calculations. However, we expect them to be small. The Isgur-Wise function obtained from QCD sum rules is actually a ratio of three-point to two-point correlator results. While both correlators receive large perturbative QCD corrections, their ratio does not depend significantly on these corrections due to cancellation. Therefore, the results for the Isgur-Wise function are more reliable than those for heavy baryon masses, as occurs in the heavy meson case [8]. The perturbative QCD corrections to the two-point correlators, and thus to heavy baryon masses, will be calculated elsewhere.

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## References

[1] N. Isgur and M.B. Wise, Phys. Lett. B 232 (1989) 113, B 237 (1990) 527; M.B. Voloshin and M.A. Shifman, Yad. Fiz. 45 (1987) 463, 47 (1988) 801.

[2] H. Georgi, Phys. Lett. B 240 (1990) 447; A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. B343 (1990) 1.

[3] N. Isgur and M.B. Wise, Nucl. Phys. B 348 (1991) 276; H. Georgi, Nucl. Phys. B 348 (1991) 293; T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B 355 (1991) 38.

[4] M.E. Luke, Phys. Lett. B 252 (1990) 447.

[5] H. Georgi, B. Grinstein and M.B. Wise, Phys. Lett. B 252 (1990) 456; Y.B. Dai, X.H. Guo and C.S. Huang, Nucl. Phys. B412 (1994) 277.

[6] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385, 448.

[7] B.L. Ioffe, Nucl. Phys. B 188 (1981) 317, Errata B 191 (1981) 591; Y. Chung, H.G. Dosch, M. Kremer and D. Schall, Nucl. Phys. B 197 (1982) 55; K.T. Chao and J.H. Liu, in Proc. Beijing Workshop on Weak Interaction and CP Violation, (1988) p.118, World Scientific.

[8] M. Neubert, Phys. Rev. D45 (1992) 2451; E. Bagan, P. Ball, V.M. Braun and H.G. Dosch, Phys. Lett. B 278 (1992) 457; A.V. Radyushkin, Phys. Lett. B 271 (1991) 218; For a review, see M. Neubert, Phys. Rep. 245 (1994) 259.

[9] M. Neubert, Phys. Rev. D46 (1992) 1076, 3914; M. Neubert, Z. Ligeti and Y. Nir, Phys. Lett. B 301 (1993) 101; Z. Ligeti, Y. Nir and M. Neubert, Phys. Rev. D49 (1994) 1302.

[10] P. Ball and V.M. Braun, Phys. Rev. D49 (1994) 2472; P. Ball, Nucl. Phys. B 421 (1994) 593; C.W. Luo, Ph D Thesis, Inst. High Ener. Phys., Beijing (1995).

[11] E.V. Shuryak, Nucl. Phys. B 198 (1982) 83.

[12] A.G. Grozin and O.I. Yakovlev, Phys. Lett. B 285 (1992) 254; see also E. Bagan, M. Chabab, H.G. Dosch and S. Narison, Phys. Lett. B 278 (1992) 367, B 287 (1992) 176, B 301 (1993) 243.

[13] A.G. Grozin and O.I. Yakovlev, Phys. Lett. B 291 (1992) 441.

[14] Y.B. Dai, C.S. Huang, C. Liu and C.D. Lü, Phys. Lett. B 371 (1996) 99.

[15] P. Colangelo, C.A. Dominguez, G. Nardulli and N. Paver, BARI-TH/95-219, UTS-DFT-95-13, hep-ph/9512334.

[16] For a review, see V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Fortschr. Phys. 32 (1984) 585.

[17] S.V. Mikhailov and A.V. Radyushkin, Sov. J. Nucl. Phys. 49 (1989) 494.

[18] M. Neubert, Phys. Rev. D45 (1992) 2451.

[19] E. Jenkins, A.V. Manohar and M.B. Wise, Nucl. Phys. B396 (1993) 38.

[20] X.H. Guo and P. Kroll, Z. Phys. C59 (1993) 567.

[21] M. Neubert, in Proc. LP'95: Int. Symp. on Lepton Photon Interactions, (1995) Beijing.

## Figure Captions

**Fig. 1.** Feynman diagrams for the $1/m_Q$ corrections to $\tilde{\Xi}(\omega, \omega', y)$. The insertions are only the kinetic energy terms at order $1/m_Q$.

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**Fig. 2.** Subleading Isgur-Wise function $\chi(y)$. The lower and upper curves correspond to the sum rules (29) of $J = J_1$ with $\omega_c = 2.2$ GeV, $T = 0.55$ GeV and $J_2$ with $\omega_c = 2.5$ GeV, $T = 0.39$ GeV, respectively.

**Fig. 3.** The differential decay rate of $\Lambda_b \to \Lambda_c l^- \bar{\nu}$ $(y = v \cdot v')$.

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