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Abstract

Some phenomenological aspects of a supersymmetric model for fermion mass hierarchy proposed previously are discussed. It is required that the lepton universality violation is near to its current experimental bound. The lepton number violation decay modes $\tau \rightarrow 2e$ and 3μ maybe observable in the near future. The Majorana mass of electron-neutrino is predicted to be about 0.1 eV. The fine-tuning problem is discussed.

Full Text

Preamble

Phenomenology of A Supersymmetric Model for Fermion Mass Hierarchy

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Abstract

We discuss some phenomenological aspects of a supersymmetric model for fermion mass hierarchy proposed previously. The analysis requires that lepton universality violation be close to its current experimental bound. The lepton-number-violating decay modes $\tau \rightarrow 2e\mu$ and $\tau \rightarrow 3\mu$ may be observable in the near future. The Majorana mass of the electron neutrino is predicted to be about 0.1 eV. The fine-tuning problem is also discussed.

Introduction

In this paper, we discuss some phenomenological aspects of a model [?] proposed for understanding the fermion mass hierarchy within the framework of low-energy supersymmetry. In the context of R -parity-violating supersymmetry [?], we considered that, in addition to the Yukawa interactions, the trilinear lepton-number-violating interactions also contribute to fermion masses if the sneutrinos have nonvanishing vacuum expectation values (vevs). We further introduced a family symmetry among the three generations, which is broken by the sneutrino vevs, potentially yielding a realistic pattern of fermion mass hierarchy. CP violation occurs superweakly through sneutrino exchanges. Although the model is interesting, it is necessary to examine its detailed phenomenological implications. We will show that these phenomenological considerations indeed impose restrictive constraints on the model and make several specific predictions.

Let us first review the model. The particle content includes, besides all the fields of the minimal supersymmetric extension of the Standard Model, an $SU(2)$ singlet superfield X . In addition to the gauge symmetries identical to those of the Standard Model, we introduce a discrete family symmetry—a Z_3 symmetry among the $SU(2)$ doublets of the three generations. Denoting L_i and Q_i as the $SU(2)$ doublet superfields of leptons and quarks respectively, with $i = 1, 2, 3$ labeling the three families, this Z_3 symmetry requires the model to be invariant under the cyclic operation $(L_1, Q_1) \rightarrow (L_2, Q_2) \rightarrow (L_3, Q_3) \rightarrow (L_1, Q_1)$. Instead of R -parity, we adopt baryon number conservation. The superpotential is

$$W = g_l^i L_a^i E_c^j H_d^b \epsilon^{ab} + g_u^i Q_a^i U_c^j H_u^b \epsilon^{ab} + g_d^i Q_a^i D_c^j H_d^b \epsilon^{ab} + \lambda_j (L_a^j L_b^k \epsilon^{ab} + L_a^j Q_b^k \epsilon^{ab}) + \lambda' X (H_u^a H_d^b \epsilon^{ab} + \dots),$$

where a and b are $SU(2)$ indices, E_c , U_c , and D_c denote the $SU(2)$ singlet antiparticle fields of leptons, up-type quarks, and down-type quarks respectively, and H_u and H_d are the two Higgs doublets that couple to U_c and D_c fields in Yukawa interactions. The term proportional to λ' is nonvanishing to avoid a fourth massless neutrino [?] and to break the gauge symmetries; these are the reasons for introducing the field X [?].

Soft breaking terms must be added to the Lagrangian. The principle guiding their construction is gauge invariance and baryon number conservation. We do not assume exact family symmetry in the soft breaking sector. There may be explicit lepton-number-violating mass terms $\tilde{m}^2 \tilde{L}_i^\dagger \phi_d$ with \tilde{L}_i and ϕ_d being the scalar components of L_i and H_d respectively, ensuring that there are neither massless majorons nor light scalar particles. The scalar potential derived in Ref. [?] determines the vevs of the neutral scalar fields. Besides the Higgs fields, the scalar neutrinos also acquire nonzero vevs of about 30 GeV from previous discussions [?, ?].

Given the model described above, a hierarchical pattern of fermion masses can be obtained [?]. The charged leptons and quarks of the third generation receive

masses from the Yukawa interactions, which actually produce a kind of democratic mixing of fermions. The muon mass and the down quark mass originate from tree-level trilinear R -parity-violating interactions, whereas due to the soft breaking of the family symmetry, the electron, the charm quark, and the strange quark might obtain masses from loops with neutralino and gluino internal lines respectively. While the pattern of mass hierarchy arises in an interesting manner in this model, more careful discussion of the related phenomenology is needed. Some general features of R -parity-violating models have been discussed already [?]; we focus on specific aspects of this model.

1. Lepton Universality Violation

Without loss of generality, we consider the scenario where only the third sneutrino field has a nonvanishing vev, denoted as v_3 . Such a scenario has been discussed elegantly by Ross and Valle [?]. Because $v_3 \neq 0$, the fermions in superfield L_3 mix with those in the Higgs and gauge superfields. For simplicity, we first consider the case where the Yukawa and R -parity-violating interactions are turned off. For the fermion fields, we found in Ref. [?] that the following compositions, which are orthogonal to the physical Higgsinos and gauginos, remain massless:

$$\nu'_3 = N_{\nu'} \left(\nu_3 - \frac{v_d}{\mu} \tilde{\phi}_u^0 + \frac{v_u}{\mu} \tilde{\phi}_d^0 \right), \quad e'_3 = N_{e'} \left(e_3 + \frac{v_d}{\mu} \tilde{\phi}_d^- \right),$$

where $N_{\nu'}$ and $N_{e'}$ are normalization constants. In Eq. (2), (ν_3, e_3) is the fermionic component of the superfield L_3 , while $\tilde{\phi}_d^-$ and $\tilde{\phi}_u^0$ are the fermionic components of the superfields H_d and H_u with vevs v_d and v_u respectively. Besides the fermions (ν_1, e_1) and (ν_2, e_2) in the superfields L_1 and L_2 , (ν'_3, e'_3) constitutes the third lepton doublet. At this stage, all three lepton doublets are weak eigenstates.

Now we turn on the Yukawa interactions. It can be shown [?] that the charged leptons still exhibit democratic mixing [?], resulting in the left-handed τ mass eigenstate $(e_1 + e_2 + e'_3)/\sqrt{3}$. At this stage, the muon and electron are massless, so their physical eigenstates cannot be uniquely determined. They can be parameterized as follows:

$$e_\mu = (e_1 - e_2) \cos \theta + (e_1 + e_2 - 2e'_3) \sin \theta,$$

$$e_e = (e_1 - e_2) \sin \theta + (e_1 + e_2 - 2e'_3) \cos \theta,$$

where θ cannot be determined until the muon mass basis is fixed.

After the inclusion of R -parity violation, the muon acquires mass. From Eq. (1) we see that in terms of the mass eigenstates (e_L, μ_L, τ_L) , the relevant part of the superpotential for the leptons is

$$W' = g_\tau L_a^\tau H_d^b \epsilon^{ab} + L_e^\dagger \mu (\lambda_\mu E_c^\mu + \lambda_\tau E_c^\tau),$$

where g_τ is proportional to g_l^j , and g_τ , λ_μ , λ_τ are the Yukawa coupling constant and the R -parity-violating coupling constants in the mass eigenstates respectively. They are compositions of g_l^j , λ_j ($j = 1, 2, 3$) from Eq. (1). E_c^μ is a combination of E_c^j orthogonal to E_c^τ and will be shown to be the right-handed muon mass eigenstate. While it is not necessarily the mass eigenstate at this stage, we always have the freedom to write the superpotential in the above form. The assumption that only ν_3 gets a vev leads to $\cos\theta = 1$, and it is at this stage that we see E_c^μ corresponds (approximately) to the mass eigenstate of the right-handed muon.

It is because ν_3' and e_3' in Eq. (2) do not coincide in form that the leptons deviate from universality. Compared to the $e_1 \rightarrow \nu_1$ weak transition, the $e_3' \rightarrow \nu_3'$ transition amplitude is suppressed by a factor of $C = N_{\nu'} N_{e'}$, which can be effectively absorbed into the gauge interaction coupling constant g_e . For the $e \rightarrow \nu_e$ weak transition, where $\nu_e \equiv (\nu_1 + \nu_2 + \nu_3')/\sqrt{3}$, the suppression factor is $\tilde{C}_e = \frac{1}{3}(1 + 2C)$. For the $e \rightarrow \nu_\mu$ weak transition, because (ν_3', e_3') is not the composition, the corresponding factor is $\tilde{C}_\mu = 1$. For the $\tau \rightarrow \nu_\tau$ weak decay, where $\nu_\tau = (\nu_1 + \nu_2 + \nu_3')/\sqrt{3}$, the suppression factor is $\tilde{C}_\tau = \frac{1}{3}(2 + C)$, which can be effectively absorbed into the gauge interaction coupling constant g_τ . Therefore, \tilde{C}_l ($l = e, \tau$) directly measures the lepton universality violation. With a reasonable choice of $\tan\beta$, such as $\tan\beta \simeq 2.2$ [?], if v_3 were taken as 30 GeV, the violation of factor C from unity would be as large as 10%. Only if $v_3 \lesssim 10$ GeV can this violation be within 0.4%, making the violation of the factors \tilde{C}_l from unity small enough to satisfy current experiments [?]. However, v_3 cannot be too small in this model because it contributes to the muon mass. We will take v_3 as 10 GeV in the following. With this choice of parameters, we have the following lepton universality violation:

$$g_e : g_\mu : g_\tau = \tilde{C}_e : \tilde{C}_\mu : \tilde{C}_\tau = \frac{1}{3}(1+2C) : 1 : \frac{1}{3}(2+C) = 0.997 : 1 : 0.999 = 1 : 1.003 : 1.002,$$

which is still consistent with experiments [?].

It should be remarked that the above discussion of lepton universality violation differs from that in Refs. [?] and [?]. Because we have introduced the coupling λ' in this model (see Eq. (1)), there is no fourth massless neutrino, so the violation cannot be rotated away by redefining the τ -neutrino field.

2. Lepton Number Violation and FCNC

Generally, this model allows lepton number violation processes at tree level. The violation is caused by the trilinear R -parity-violating terms with slepton

exchange. From Eq. (5), it is interesting to see that the decay $\mu \rightarrow 3e$ does not occur in this model, thereby avoiding one of the most stringent restrictions on lepton number violation. Lepton number violation instead appears in rare τ decays. Considering processes with only charged leptons as final states, the decay modes $\tau \rightarrow 2e\mu$ and $\tau \rightarrow 3\mu$ can occur, as shown explicitly in Figure 1 [Figure 1: see original paper]. They have the same decay rate:

$$\Gamma(\tau \rightarrow 2e\mu) = \Gamma(\tau \rightarrow 3\mu) = \frac{|\lambda_\mu \lambda_\tau|^2}{192\pi^3} m_\tau^5 / m_{\tilde{l}}^4.$$

From the muon mass $m_\mu \simeq \lambda_\mu v_3$, we get $\lambda_\mu \sim 10^{-2}$. It is reasonable to assume that λ_τ has the same order of magnitude as λ_μ . In this case, the branching ratio of the decays $\tau \rightarrow 2e\mu$ and $\tau \rightarrow 3\mu$ is 10^{-11} if the sneutrino mass is around 100 GeV. Such a branching ratio may be observable in the near future. The modes $\tau \rightarrow e\mu\mu$ or $2\mu e$, however, cannot occur in this model.

A remark should be made on the slepton SUSY breaking parameters. Lepton flavor violation, as in the process $\mu \rightarrow e\gamma$, can be induced by the SUSY breaking parameters through loops. Experimental data on such processes put severe constraints on these parameters. The symmetries assumed in this model, however, do not constrain these parameters strongly enough. Similar to what has been done for the quark case [?], further assumptions are needed: (i) the slepton mass-squared matrices corresponding to definite chiralities of leptons are proportional to the unit matrix; (ii) the slepton mass-squared matrix corresponding to mixing associated with different chiralities is proportional to the lepton mass matrix.

However, these conditions cast doubt on the electron and neutrino mass generation mechanism through loops proposed in Ref. [?]. In that mechanism, we hoped to violate condition (i) to give the electron mass through loops. But the violation is required to be small and can be measured by some dimensionless quantities δ , defined as the square of the ratios of the flavor off-diagonal masses to the average slepton mass. From experimental constraints [?], $\delta \lesssim (m_{\tilde{l}}/\text{GeV})^{-2}$ when the averaged slepton mass $m_{\tilde{l}}$ is compatible with the photino mass $m_{\tilde{\gamma}}$. According to Ref. [?], the electron mass should be $m_e \simeq \frac{1}{4\pi} \delta \tilde{m} v_d$, where \tilde{m} is the soft mass parameter associated with different chirality mixing, which might give a realistic numerical result for the electron mass if \tilde{m} is appropriately chosen.

3. Neutrino Masses

Loop-induced Majorana neutrino masses are inevitable in this model. The best approach to studying this problem is to work in the mass eigenstate basis. The mechanism for loop-level neutrino masses was discussed in Ref. [?]. From Eq. (5), we see that only the electron-type neutrino can acquire mass through loops, as shown in Figure 2 [Figure 2: see original paper]. The induced mass is

$$m_{\nu_e} \simeq \frac{(\lambda_\mu)^2 \tilde{m}_0 m_\mu^2}{16\pi^2 m_{\tilde{t}}^2}.$$

In the above equation, we have reexpressed the parameter \tilde{m} as \tilde{m}_0 , which is a more natural choice in the lepton case. For reasonable values of \tilde{m}_0 and $m_{\tilde{t}}$, with $\tilde{m}_0 \simeq m_{\tilde{t}} \simeq 100$ GeV, we predict the Majorana ν_e mass to be around 0.1 eV. Note that only the muon and its superpartner contribute to the ν_e mass in the loop. At this stage, the model predicts vanishing masses for ν_μ and ν_τ , and there is no mixing between ν_e and them.

It is necessary to discuss the tree-level induced neutrino mass. Usually, in the case of large sneutrino vev, one expects the neutrino to acquire a large mass through the see-saw mechanism due to mixing with the Zino, unless an unnatural fine-tuning is made. However, this model avoids such tree-level neutrino masses, as shown explicitly below. The Lagrangian for the neutralino masses is given by

$$\mathcal{L}_{\text{neutralino}} = i(\nu_3, \tilde{\phi}_u^0, \tilde{\phi}_d^0, \tilde{Z}, \tilde{X}) \begin{pmatrix} 0 & \mu & 0 & av_u & \lambda'v_u \\ \mu & 0 & 0 & av_d & -\lambda'v_d \\ 0 & 0 & 0 & 0 & \lambda'v_d \\ av_u & av_d & 0 & M_{\tilde{Z}} & 0 \\ \lambda'v_u & -\lambda'v_d & \lambda'v_d & 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_3 \\ \tilde{\phi}_u^0 \\ \tilde{\phi}_d^0 \\ \tilde{Z} \\ \tilde{X} \end{pmatrix} + \text{h.c.},$$

where $a = (g^2 + g'^2)^{1/2}$ with g and g' the $SU(2)$ and $U(1)$ gauge coupling constants, and $M_{\tilde{Z}}$ is the Zino mass from the soft breaking term. It is easy to see that the above mass matrix is of rank 4 (instead of 5), with the eigenstate ν'_3 expressed in Eq. (2) corresponding to the zero eigenvalue. However, this does not mean that there is no fine-tuning problem in this model. The problem has actually been transferred to the smallness of the coupling constants of terms like $H_u H_d$, $H_u L_i$, and $X H_u L_i$, which we have not written in Eq. (1). While the smallness of these couplings needs further explanation, less fine-tuning is required in this model because of supersymmetry.

Summary

In summary, the supersymmetric model for fermion mass hierarchy proposed in Refs. [?] faces restrictive limitations from phenomenological considerations. The analysis requires that lepton universality violation be near its experimental bound. The lepton-number-violating decay modes $\tau \rightarrow 2e\mu$ and $\tau \rightarrow 3\mu$ may be observable in future experiments. The electron neutrino Majorana mass is predicted to be about 0.1 eV.

Finally, a remark should be made on the electron mass. This model gives an interesting hierarchical pattern for leptons. However, the idea of radiative

generation for the electron mass [?] is seriously problematic. Such an approach does not simplify the understanding of the lepton mass hierarchy; instead, it merely transfers the problem from the Yukawa sector to other sectors of the model. Therefore, it is fair to say that the idea of radiative mass generation is not appealing within this model's framework. It would be preferable if the electron mass originated from some new physics not yet included in the Lagrangian.

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References

- [1] D. Du and C. Liu, *Mod. Phys. Lett.* A8, 2271 (1993).
- [2] D. Du and C. Liu, *Mod. Phys. Lett.* A10, 1837 (1995).
- [3] C. Liu, *Int. J. Mod. Phys.* A11, 4307 (1996).
- [4] For a review, see L. Hall, *Mod. Phys. Lett.* A5, 467 (1990).
- [5] J. Ellis et al., *Phys. Lett.* B150, 142 (1985).
- [6] C. Aulakh and R. Mohapatra, *Phys. Lett.* B119, 136 (1982).
- [7] S. Dimopoulos and L. Hall, *Phys. Lett.* B207, 210 (1988); V. Barger, G. Giudice and T. Han, *Phys. Rev.* D40, 2987 (1989); M. Hirsch, H.V. Klapdor-Kleingrothaus, S.G. Kovalenko, *Phys. Rev. Lett.* 75, 17 (1995); M. Fabbrichesi and F. Vissani, IC/96/220, SISSA 140/96/EP, hep-ph/9611237; J.E. Kim, P. Ko and D.G. Lee, SNUTP 96-073; For a review, see G. Bhattacharyya, talk at SUSY 96 (Maryland, 1996).
- [8] G. Ross and J. Valle, *Phys. Lett.* B151, 375 (1985).
- [9] H. Fritzsch, *Phys. Lett.* B73, 317 (1978); H. Terazawa, INS-Report-298 (Univ. of Tokyo, 1977); H. Harari, H. Haut and J. Weyers, *Phys. Lett.* 78B, 459 (1978); Y. Chikashige, et al., *Phys. Lett.* 94B, 499 (1980); C. Jarlskog, in: *Proc. Int. Symp. on Production and Decays of Heavy Flavors* (Heidelberg, Germany, 1986); Y. Nambu, in: *Proc. XI. Warsaw Symp. on High energy physics* (Kazimierz, Poland, 1988); P. Kaus and S. Meshkov, *Mod. Phys. Lett.* A3, 1251 (1988); Y. Koide, *Z. Phys.* C45, 39 (1989); H. Fritzsch and J. Plankl, *Phys. Lett.* B237, 451 (1990).
- [10] W.J. Marciano, in TAU 94, p. 587, Eds. L. Rolandi (*Nucl. Phys. B Proc. Suppl.* 40, 1995); A. Pich, FTUV/96-87, IFIC/96-96, hep-ph/9612308, in TAU 96 (Colorado, 1996); K. Hagiwara, KEK-TH-461 (Dec, 1995).
- [11] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *Nucl. Phys.* B447, 321 (1996); J. Hagelin, S. Kelley and T. Tanaka, *Nucl. Phys.* B415, 293 (1994).
- [12] K. Babu and R. Mohapatra, *Phys. Rev. Lett.* 64, 1705 (1990); R. Barbieri, M. Guzzo, A. Masiero and D. Tommasini, *Phys. Lett* B252, 251 (1990); K. Enqvist, A. Masiero and A. Riotto, *Nucl. Phys.* B373, 95 (1992); R.M. Godbole, P. Roy and X. Tata, *Nucl. Phys.* B401, 67 (1993).

Figure Captions

Fig. 1 Feynman diagrams for the decays $\tau \rightarrow 2e\mu$ and $\tau \rightarrow 3\mu$.

Fig. 2 The mechanism for electron-neutrino mass generation.

[Figure 1: see original paper]

[Figure 2: see original paper]

Note: Figure translations are in progress. See original paper for figures.

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