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Full Text

Preamble

Heavy Baryon Masses in Large N_c HQET
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Abstract

We argue that in the large N_c HQET, the masses of the s-wave low-spin heavy baryons are approximately equal to the heavy quark mass plus the proton mass. To subleading order, the heavy baryon mass $1/N_c$ expansion not only has the same form but also has the same coefficients as that of the light baryon. Based on this, we perform numerical analysis.

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Heavy baryons provide a testing ground for the Standard Model. Those containing a single heavy quark, like Λ_c , Λ_b , $\Sigma()$ and $\Sigma()$, can be studied within

the heavy quark effective theory (HQET) [1]. For complete calculations, some additional nonperturbative methods must be used. In this Letter, we discuss the simple incorporation of the large N_c method [2] in HQET.

HQET is an effective field theory of QCD in the heavy quark limit [1]. In a systematic manner, it provides a description for heavy hadrons. Under the heavy quark limit, there is no heavy quark pair production. The large mass of the heavy quark, which interacts with the light quark system with typical energy Λ_{QCD} , plays no role except for contributing to the total energy of the hadron. With the velocity super-selection rule, the heavy quark mass m_Q , defined perturbatively as the pole mass, can be removed by field redefinition. The heavy quark field h_v is defined by $P+Q(x) = \exp(-im_Q v \cdot x) h_v(x)$, where $P+ = \frac{1}{2}(1+v)$. To leading order in $1/m_Q$, the effective Lagrangian for the heavy quark is $\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v$.

Besides the heavy quark symmetry [1], we note explicitly from Eq. (2) that the heavy quark becomes effectively massless (modulo m_Q). The heavy hadron mass M is expanded as $M = m_Q + \Lambda$, where Λ is the heavy hadron mass in HQET, which is independent of heavy quark flavors. The quantity Λ cannot be determined further from HQET. It is at this stage that we apply the large N_c method.

As one of the most important and interesting methods in nonperturbative QCD, the large N_c limit [2] is often applied despite the realistic value $N_c = 3$. Nonperturbative properties of mesons can be observed from the analysis of planar diagrams, and baryons from the Hartree-Fock picture. Recently, there has been renewed interest in large N_c applications to baryons due to the work of Ref. [3], which shows that there is a contracted $SU(2f)$ light quark spin-flavor symmetry in the baryon sector, obtained by combining large N_c counting rules and the chiral Lagrangian. This symmetry can be directly derived in the Hartree-Fock picture [4] or by other methods [5]. Similar results were also obtained previously [6]. Further applications of this spin-flavor symmetry to heavy baryons have been made by Jenkins [3] in discussing baryon-pion couplings and baryon hyperfine splittings. Interesting relations among the baryonic Isgur-Wise functions were obtained in Refs. [7] and [8]. Masses of heavy baryons with any finite number of heavy quarks were studied by $1/N_c$ expansion of QCD in Ref. [9].

Inspired by these approaches, we consider HQET at the large N_c limit. Physically, the heavy quark limit and the large N_c limit are non-commutative. Different orders of the limits correspond to different pictures. In large N_c HQET, there is nothing new in the meson case, so we focus on heavy baryons.

We argue that the mass of s-wave low-spin heavy baryons in HQET, Λ , equals the proton mass in the large N_c limit. Let us continue thinking about the Hartree-Fock picture not in full QCD but in HQET. The heavy baryons contain $(N_c - 1)$ light quarks and one “massless” heavy quark. The mass or energy of the baryon is determined by the summation of the energies of individual quarks. The kinetic energy of the heavy quark is typically Λ_{QCD} , like that of the light

quarks. The interaction energy between the heavy quark and any of the light quarks is typically Λ_{QCD}/N_c . Therefore, the interaction energy between the heavy quark and the whole light quark system scales as Λ_{QCD} . However, the total interaction energy of the light quark system itself scales as $N_c\Lambda_{\text{QCD}}$.

In the limit $N_c \rightarrow \infty$, the light quarks drown out the heavy quark. The energy of the heavy baryon is determined by its light quark system. This light quark system also dominates the proton in the large N_c limit. Therefore, we come to the conclusion that in the large N_c limit, the masses of s-wave low-spin heavy baryons defined in HQET equal the proton mass.

From the same logic as in the previous paragraph, we can easily deduce the results for the baryon-pion coupling constants. These constants are also determined by the light quark system, so they are the same for light baryons and heavy baryons. Additionally, the heavy baryon also has the light quark spin-flavor symmetry. These results were obtained by Jenkins in Ref. [3].

Of course, all results are subject to $1/N_c$ corrections, which deserve more detailed consideration. The corrections violate the light quark spin-flavor symmetry. Let us first discuss the spin symmetry violation in Λ . The baryon mass can be written as $\Lambda = N_c\Lambda_{\text{QCD}} + c_1 J^2/N_c$, where J is the angular momentum of the light quark system. The mass parameter c_1 is yet undetermined but is of order Λ_{QCD} . The factor N_c must appear to maintain the correct N_c scaling for Λ . In the extreme case where all quark spins align in the same direction, $J^2 = N_c^2$. Only by dividing by a factor N_c does the term J^2 in Eq. (4) have the correct N_c scaling. Note this term is $1/N_c^2$ suppressed compared to $N_c\Lambda_{\text{QCD}}$.

On the other hand, the light baryon mass m has the same form of $1/N_c$ expansion: $m = N_c\Lambda_{\text{QCD}} + \tilde{c}_1 J^2/N_c$, where J is the baryon spin. Further, we argue in the following that $c_1 = \tilde{c}_1$.

Consider again the extreme case where in the mass $1/N_c$ expansion, the sub-leading term becomes leading, with $J^2 = N_c(N_c + 2)$. Because of light quark dominance, we have $m = \Lambda$ in the limit $N_c \rightarrow \infty$. This immediately results in the conclusion given by Eq. (6).

Another lowest-order $1/N_c$ effect lies in light quark flavor symmetry breaking. For now, we ignore spin symmetry violation. After including baryons with strangeness number -1 , the masses for heavy and light baryons can be expanded as $\Lambda = N_c\Lambda_{\text{QCD}} + c_2(-S)$ and $m = N_c\Lambda_{\text{QCD}} + \tilde{c}_2(-S)$, respectively, where S is the baryon strangeness number which can be 0 or -1 . Again, we argue that $c_2 = \tilde{c}_2$.

In Eq. (7), spin symmetry is not violated. The strange quark spin decouples from the strong interaction. The only contribution of the strange quark mass to baryon masses is the strange quark mass itself. Therefore, c_2 and \tilde{c}_2 are nothing but the strange quark mass defined in the large N_c limit. To order $1/N_c$, terms like I^2 and $I \cdot J$ should be included in the expansion (7). However, in the realistic case, $I = J$, and these terms can be effectively absorbed into the

Jl^2 term in Eq. (4).

For a complete analysis of heavy baryon masses, $1/m_Q$ corrections must be considered. To order $1/m_Q$, the heavy baryon mass M is expanded as $M = m_Q + \Lambda - 1/(2m_Q) + 2 \langle \mathbf{S}_Q \cdot \mathbf{J}_l \rangle / m_Q$, where \mathbf{S}_Q is the heavy quark spin and $1 = \langle \mathbf{H}(v) | \bar{h} v (iD)^2 h v | \mathbf{H}(v) \rangle$, $2 \langle \mathbf{S}_Q \cdot \mathbf{J}_l \rangle = -Z_Q \langle \mathbf{H}(v) | \bar{h} v \mathbf{g} \cdot \mathbf{G} h v | \mathbf{H}(v) \rangle$, with Z_Q being the renormalization factor. In the leading order of $1/N_c$, 1 scales as unity and is independent of the light quark structure, while 2 vanishes. These can be seen directly from the definition (10) with light quark spin-flavor symmetry and from the fact that 2 is zero for Λ_Q baryons. Therefore, we arrive at the following $1/N_c$ expansions for 1 and 2 :

$$\begin{aligned} 1 &= c' + c'' \langle \mathbf{S}_Q \cdot \mathbf{J}_l \rangle / N_c + c''' S / N_c \\ 2 &= c' \langle \mathbf{S}_Q \cdot \mathbf{J}_l \rangle / N_c + c'' S / N_c \end{aligned}$$

We perform numerical analysis for non-strange baryons in the following. The heavy baryon mass is given by Eq. (9). For Λ and m , the $1/N_c$ expansions are given by Eqs. (4) and (5) with $c_1 = \tilde{c}_1$, and for 1 and 2 by Eq. (11) with $S = 0$. To be consistent, the accuracy of the analysis is maintained to the order of Λ_{QCD}/m_Q . This means the term c' in Eq. (11) is also neglected. Formally, the uncertainty will be due to $1/m_Q^2$ and $1/N_c^3$ corrections, which are about 10 MeV.

With the measured masses of the proton, neutron, and Δ , we obtain $N_c \Lambda_{\text{QCD}} = 866$ MeV and $c_1 = 293$ MeV. This gives $\Lambda_{\Lambda_Q} = 866$ MeV and $\Lambda_{\Sigma^{(*)}} = 1060$ MeV. Although there is no data for c'' , the following quantity can be predicted with theoretical accuracy of 10 MeV:

$$3(M_{\Sigma c} + 2M_{\Sigma^* c}) = M_{\Lambda c} + \Lambda_{\Sigma} - \Lambda_{\Lambda c} = 2479 \text{ MeV}$$

Similarly, the corresponding quantity for bottom quark is predicted as:

$$M_{\Sigma b} + 2M_{\Sigma^* b} = 5835 \pm 50 \text{ MeV}$$

This shows that the recently proposed $\Sigma^{(*)}c$ masses in a new interpretation [10] of the heavy baryon spectrum deviate by 100 MeV from our result. It also implies that $M_{\Sigma c} = 2492$ MeV by taking $M_{\Sigma c} = 2453$ MeV. Our numerical analysis is actually the same as that in Ref. [9].

Comparing with Ref. [9], what are the different points of this paper? We began with HQET, which gives a clear physical picture for heavy baryons, and emphasized that the heavy baryon mass in HQET, Λ , is of order proton mass. Then we showed that the next-to-leading order $1/N_c$ expansions of Λ and the light baryon mass not only have the same form but also have the same coefficients. These points cannot be taken for granted in large N_c HQET. They justify some of the numerical analysis of Ref. [9].

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