

Some Remarks on the Baryon-Meson Couplings in the $1/N_c$ Expansion (Postprint)

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Abstract

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Full Text

Preamble

SNUTP 96-121 Some Remarks on the Baryon-Meson Couplings in the $1/N_c$ Expansion Chun Liu¹ Center For Theoretical Physics, Seoul National University Seoul, 151-742, Korea

Abstract

The original results for the baryon-pion couplings in the large N_c QCD can be understood in a simpler way in the Hartree-Fock picture. The large N_c relation and its $1/N_c$ correction between the heavy baryon-meson coupling and the light baryon-meson coupling are emphasized.

Application to the baryon-meson interactions is straightforward. The implications of recent experimental result for the strong coupling constants of the heavy baryon chiral lagrangian are discussed. Baryons provide us testing ground to Quantum Chromodynamics (QCD). Experiments have collected rich data for light baryons, and are accumulating more and more data for heavy baryons. For theoretical calculations, the task is to apply model-independent methods of non-perturbative QCD. Tools suitable for this task include lattice QCD, $1/N_c$

expansion, chiral Lagrangian, Heavy Quark Effective Theory (HQET), QCD sum rules, and so on. In this paper, we discuss some aspects of the $1/N_c$ expansion of baryon-meson strong coupling constants.

In the large N_c limit [1], the meson properties can be obtained by analyzing planar diagrams, and baryon properties by considering the Hartree-Fock picture. Hadrons can be understood qualitatively quite well. Recently, there has been renewed interest in the $1/N_c$ expansion due to the work of Dashen, Manohar, and Jenkins [2]. By combining large N_c counting rules with the chiral Lagrangian, they pointed out that there exists a contracted $SU(2N_f)$ light quark spin-flavor symmetry in the baryon sector in the large N_c limit. This symmetry can also be derived in the Hartree-Fock picture [3,4]. (A similar result was obtained previously [5].) The observation of light quark spin-flavor symmetry has led to many quantitative applications of the $1/N_c$ expansion to both light and heavy baryons [2,6-10].

Within the framework of large N_c HQET, we discussed the heavy baryon masses [9], and emphasized in the Hartree-Fock picture that the heavy baryon mass equals the heavy quark mass plus the proton mass in the large N_c limit. Actually this point was first pointed out in Ref. [11] by Chow and Wise. In the Hartree-Fock picture, by observing the dominance of light quarks in the large N_c limit, we further deduced that the heavy baryon-pion coupling constant equals the light baryon-pion coupling constant, which is a result of Jenkins in Ref. [2]. This paper will develop this deduction. Our discussion is not restricted to heavy baryons; it applies to both light and heavy baryons. Moreover, it is not limited to baryon interactions with pions only; the interactions can involve mesons as well. In fact, the discussion considers baryon interactions with any light meson. However, it emphasizes the relation between heavy baryon-meson coupling and light baryon-meson coupling.

The baryons of most general interest are in the ground state, meaning the quarks have no orbital angular momentum excitations in the constituent picture. For simplicity, only two flavors of light quarks are considered. In terms of their quantum numbers, spin J and isospin I , they form towers of states with $(I, J) = (1/2, 1/2), (3/2, 3/2), \dots, (N_c/2, N_c/2)$ for light baryons and $(I, J) = (0, 0), (1, 1), \dots, (N_c/2, N_c/2)$ for heavy baryons. In the large N_c limit, because of the light quark spin-flavor symmetry, baryons belonging to the same tower of (I, J) are degenerate. Each tower is an irreducible representation of this symmetry. As in Ref. [9], we have been working in the Hartree-Fock picture for baryons. Many interesting results can be obtained in a simple way. In the following, we just consider the ground state baryons. However, all the arguments can be generalized to the excited baryon cases.

The first result is that in the large N_c limit, all the coupling constants of the baryon interactions with a fixed light meson are equal to each other. When we talk about the coupling constant, the explicit factors of N_c and the Clebsch-Gordan coefficients have been factored out. Generally, every vertex of the baryon-meson interactions in a given tower of (I, J) can be expressed as a com-

bination of the fields of the initial baryon, the final baryon, and the light meson, multiplied by some coupling coefficient. Due to the symmetry group of the interaction, the coupling coefficient can be parameterized into the product of a certain constant and Clebsch-Gordan coefficients. This constant is then called the coupling constant (or coupling for short). Note that both the initial and final baryon states in the interaction belong to the same tower of (I, J) .

Consider the light baryon case. The quark spin-flavor symmetry tells us that all the light baryon-meson couplings are equal. And because the interaction is determined by the light quarks inside the baryons, the heavy baryon-meson couplings are also equal to each other due to this symmetry. In the $N_c \rightarrow \infty$ limit, heavy baryons are dominated by the light quark systems that also dominate light baryons. In this case, the heavy baryon-meson coupling constant equals the light baryon-meson coupling constant. Therefore all the ground state baryon and light meson interaction couplings are the same in the large N_c limit. Of course, for different light mesons, the couplings should be different.

Note that it is the observation of light quark dominance in large N_c baryons that establishes the large N_c equality relation between heavy baryon-meson coupling and light baryon-meson coupling in the Hartree-Fock picture. Light quark spin independence [3], or light quark spin-flavor symmetry, only gives the coupling equality relation within a given tower of (I, J) . The “heavy-light” equality relation, however, is subject to $1/N_c$ corrections.

The second result concerns the $1/N_c$ corrections of the above conclusion. $1/N_c$ corrections violate the spin-flavor symmetry. And we note the baryon spectrum has the relation $I = J$. Therefore the coupling of the baryon-meson interaction has the following $1/N_c$ expansion:

$$g = g_0 \left[1 + c_1 \frac{J_1^2 + J_2^2}{N_c} + c_2 \frac{L^2}{N_c} + c_3 \frac{J_1 \cdot J_2}{N_c} + \mathcal{O} \left(\frac{1}{N_c^2} \right) \right]$$

where g is the coupling constant in the large N_c limit, which is the same for both heavy and light baryons. c ($i = 1, 2, 3$) are unknown coefficients. J_1 and J_2 denote the baryon spins of the initial and final states, respectively, and L stands for the total angular momentum of the light meson, which counts the orbital angular momentum and the spin of the meson in the rest frame of the initial baryon. When the $1/N_c$ corrections are considered, the coupling constant is a function of J_1^2 and J_2^2 , however, it can be reexpressed in terms of J_1^2 and L^2 . The factor N_c should appear so as to keep the N_c scaling for g . In the extreme case where all the quark spins align in the same direction in the baryon, J_1^2 or J_2^2 in the above equation have the correct N_c scaling.

However, L^2 is always a fixed quantity which does not depend on N_c . So generally this term is suppressed by N_c . Because strong interaction is CP invariant, the terms proportional to J_1^2 and J_2^2 have the same coefficient. For different

towers of (I, J) , that means for the light and heavy baryons, the coefficients c are not necessarily the same.

From Eq. (1) and the above discussion, we see the following points: (a) Within a given tower of (I, J) , the equal relation of the coupling constants receives corrections actually only at the order of $1/Nc^2$. This is because within the tower of (I, J) , the term c is a constant. The couplings can be redefined by absorbing this constant, so that there is an explicit equal relation among the redefined couplings, which gets corrections at order $1/Nc^2$. Furthermore, the redefinition can be made to absorb all the terms of $(L^2)^n$ ($n = 1, 2, 3, \dots$), which are constants within the tower of (I, J) . In the mixing terms, the factor L^2/Nc can be absorbed into the coefficients. Therefore, the coupling constant can be expanded only by the powers of J^2/Nc^2 and J^2/Nc^2 within a given tower of (I, J) .

- (b) Between the different towers of (I, J) , the equal relation receives correction at the order of $1/Nc$. This is simply because the value of the coefficient c in the light baryon case is generally different from that in the heavy baryon case. So their difference gives an order of $1/Nc$ correction to the coupling constant equal relation. Furthermore, in the extreme situation where $J^2 \ll Nc/4$, in Eq. (1) the subleading terms which are proportional to $(J^2)^m (J^2)^n$ (m and n are non-negative integers) have the leading behavior. Because of the light quark dominance, the large Nc equal relation of the couplings now gives that the summation of the coefficients of these subleading terms for the light baryons equals that for the heavy baryons. Under the specific assumption that the large Nc limit and the heavy quark limit are commutative, we may also have the equal relation of the coefficient c for the light baryons and the heavy baryons.

The first result of $1/Nc$ corrections (a) can be obtained directly from Ref. [4] in the Hartree-Fock picture. However, because we also have the large Nc coupling equal relation between different towers of (I, J) , it is necessary to consider its $1/Nc$ correction. The discussion on the second result (b) has made the origin of the $1/Nc$ correction to the “heavy-light” coupling equal relation clear.

For the light meson being a pion, the main conclusions for the baryon-meson interactions we have obtained are the same as those originally obtained in Ref. [2]. However, working in the Hartree-Fock picture, the way of understanding is simpler, and is not subject to the soft pion limit. All the results can be immediately generalized to baryon interactions with other light mesons, like the baryon-meson interactions.

Similar to the baryon-pion interaction vertex described in Ref. [2], the baryon-vertex is $\bar{B}_2 G_\rho^{ai} B_1 \rho_{ai}$, where a denotes the isospin of the meson and i labels the spin component of the meson in the baryon rest frame. The coupling coefficient can be written as:

$$\langle I_2 I_{2z}, J_2 J_{2z} | I_1 I_{1z}, J_1 J_{1z} \rangle = N_c g_\rho(J_1, J_2) \sqrt{2J_1 + 1} \sqrt{2J_2 + 1} \begin{pmatrix} I_1 & 1 & I_2 \\ I_{1z} & a & I_{2z} \end{pmatrix} \begin{pmatrix} J_1 & 1 & J_2 \\ J_{1z} & i & J_{2z} \end{pmatrix}$$

where $g_\rho(J_1, J_2)$ is the coupling constant which is of order 1 because the N_c dependence has been factored out. The Clebsch-Gordan coefficient is determined from angular momentum and isospin conservation. The coupling constant $g_\rho(J_1, J_2)$, which is the main topic of this Letter, is given by Eq. (1).

In this way, the NN and $N\Delta$ interaction couplings are related to each other, where N and Δ denote nucleon and Δ -baryon. It is interesting to note that the p-wave NN and $N\Delta$ interaction vertices [2] have the same form as that of the s-wave NN and $N\Delta$ interactions, therefore the ratios of the couplings, in which the Clebsch-Gordan coefficients are canceled, have the relation:

$$\frac{g_{N\Delta\rho}}{g_{NN\rho}} = \frac{g_{N\Delta\pi}}{g_{NN\pi}}$$

which is valid up to $1/N_c^2$. Actually this relation has been suggested by phenomenological models [12]. Correspondingly, we also have the same relation for the heavy baryon case:

$$\frac{g_{\Lambda_Q \Sigma_Q \rho}}{g_{\Lambda_Q \Sigma_Q \pi}} = \frac{g_{NN\rho}}{g_{NN\pi}}$$

However, this relation will receive correction at the order of $1/N_c$.

Finally let us make a comment on the recent experimental result for the strong couplings in heavy baryon chiral Lagrangians [13, 14]. There are two coupling constants, g_1 and g_2 , where the notation of Ref. [13] is adopted. They can be determined from the transitions $\Sigma_c^* \rightarrow \Sigma_c \pi$ and $\Sigma_c \rightarrow \Lambda_c \pi$, respectively. In the large N_c limit, the equal coupling relation gives:

$$g_1 = g_2 = \frac{g_N}{\sqrt{2}}$$

where $g_N \approx 1.25$ is the light baryon-pion coupling constant. From the CLEO data [15], it was obtained that [16]:

$$\frac{g_2}{g_1} \approx 0.57$$

This is surprising, because the relation deviates from the large N_c expectation remarkably. But such a deviation is not unexpected at order $1/N_c$. This means that the coefficient c_1 of Eq. (1) for the heavy baryon case is different from

that for the light baryon case. In other words, the large N_c limit and the heavy quark limit are not commutative. Another large N_c relation does not receive corrections until order $1/N_c^2$. Therefore, on the other hand, the ratio $g_1/g_N \approx 0.14$ is expected to be quite accurate. It will be checked in experiments in the near future.

In summary, the original results for the baryon-pion couplings in large N_c QCD can be understood in a simpler way in the Hartree-Fock picture. The large N_c relation and its $1/N_c$ correction between the heavy baryon-meson coupling and the light baryon-meson coupling are stressed. While the simplification may be inspiring, we have made a simple application to the baryon-meson interactions. We have also discussed implications of the recent experimental result for the strong coupling constants of the heavy baryon chiral Lagrangians.

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