Four Light Neutrinos in Singular Seesaw Mechanism with Abelian Flavor Symmetry Postprint

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Abstract

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Full Text

Preamble

Four Light Neutrinos in Singular Seesaw Mechanism with Abelian Flavor Symmetry

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Abstract

The four light neutrino scenario, which explains the atmospheric, solar, and LSND neutrino experiments, is studied in the framework of the seesaw mechanism. By taking both the Dirac and Majorana mass matrices of neutrinos to be singular, the four-neutrino mass spectrum consisting of two almost degenerate pairs separated by a mass gap of ~ 1 eV is naturally generated. Moreover, the right-handed neutrino Majorana mass can be at the 10^{14} GeV scale, unlike in the usual singular seesaw mechanism. Abelian flavor symmetry is used to produce the required neutrino mass pattern. A specific example of the flavor charge assignment is provided to show that maximal mixings between $\nu_\mu - \nu_\tau$ and $\nu_e - \nu_s$ are respectively attributed to the atmospheric and solar neutrino anomalies, while small mixing between the two pairs accounts for the LSND results. The implications for other fermion masses are also discussed.

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I. Introduction

The recent Super-Kamiokande data on the zenith-angle-dependent deficit of atmospheric muon neutrinos have provided compelling evidence for neutrino masses and mixing [?]. This might represent the first discovery of physics beyond the Standard Model (SM) and has drawn considerable theoretical attention. The mass pattern of neutrinos provides valuable information for exploring the physics related to the flavor puzzle in the SM. Several mechanisms have been suggested to accommodate massive neutrinos. One of the most popular scenarios is the seesaw mechanism, which naturally explains the smallness of neutrino masses by introducing heavy right-handed neutrinos [?]. Another example is the supersymmetric extension of the SM with R-parity violation, where trilinear lepton number violating interactions induce small neutrino masses at the loop level [?]. Since the complete flavor problem in the SM remains unresolved, the detailed neutrino mass pattern in these models remains uncertain.

Observations of atmospheric and solar neutrinos provide information about neutrino mass-squared differences and mixing angles under the assumption of neutrino oscillations. According to Super-Kamiokande data, about 35% of muon-type atmospheric neutrinos change flavor into non-electron neutrinos, implying $\Delta m^2 \sim 10^{-3}$ eV^2 and $\sin^2 2\theta_{\mu x} \simeq 1$ [?]. The solar neutrino deficit problem [?] can be explained by either matter-enhanced oscillations (MSW effects) [?] or vacuum oscillations [?]. The MSW solution allows two parameter spaces: $\Delta m^2 \sim 10^{-5}$ eV^2 with $\sin^2 2\theta_{ey} \simeq 10^{-2}$ or $\Delta m^2 \sim 10^{-4}$ eV^2 with $\sin^2 2\theta_{ey} \simeq 0.8$. The vacuum oscillation solution requires $\Delta m^2 \sim 10^{-10}$ eV^2 with $\sin^2 2\theta_{ey} \simeq 1$.

If only solar and atmospheric neutrino data are considered, they can be understood within frameworks containing three light neutrinos [?, ?]. However, when we further consider the LSND experiment [?], something previously unexpected must be introduced into the theory. When ascribing the detection of flavor-changing events to neutrino oscillations, the experiment indicates that $\Delta m^2 \sim 1$ eV^2 and $\sin^2(2\theta_{e\mu}) \sim 10^{-2}$. To accommodate all these experiments, three light neutrinos are insufficient; at least one additional light neutrino ν_s is needed [?]. This neutrino must be sterile, as it does not participate in Z^0 decay.

Although the KARMEN group has recently reported that a large portion of the favored parameter region of LSND is excluded [?], full confirmation of the LSND results still awaits future experiments. For example, a particular value of $\Delta m^2 \sim 6$ eV^2 compatible with LSND results does not contradict KARMEN data, since this Δm^2 is examined most sensitively at LSND while least sensitively at KARMEN. Moreover, neutrinos with mass scales of eV play an important role in understanding the dark matter problem. In astrophysics, Cold + Hot Dark Matter cosmological models (CHDM) agree best with data on cosmic microwave background anisotropies and the large-scale distribution of galaxies and clusters in the nearby universe [?]. With about 70% cold dark matter and 10% baryonic matter, a few-eV neutrino mass is required to account for the remaining 20% hot dark matter.

In this paper, we adopt the four light neutrino scenario. Phenomenological studies [?] have shown that the following mass patterns are favored: in terms of mass eigenstates, the four neutrinos are grouped into two pairs separated by a gap of ~ 1 eV, with the two neutrinos in each pair being almost degenerate compared to the gap. The atmospheric neutrino anomaly can be explained by oscillations within one pair, and the solar neutrino deficit by oscillations within the other. The LSND data is explained by oscillations between neutrinos from different pairs. In terms of weak eigenstates, approximately speaking, ν_τ can pair with either ν_μ or ν_e.

Constructing this spectrum of four very light neutrinos is a theoretically challenging problem [?]. One of the most appealing explanations for the smallness of neutrino masses is the seesaw mechanism, which introduces three very heavy right-handed neutrinos $N_\alpha(\alpha = e, \mu, \tau)$ with Majorana masses M. The generic presence of such N_α fields in many extensions of the SM, such as SO(10) GUTs and E_6 string theories, adds further appeal to this mechanism. However, since the ordinary seesaw mechanism predicts three light neutrinos and three very heavy neutrinos, there is no room for light sterile neutrinos. While the so-called singular seesaw mechanism, where the Majorana mass matrix of the N_α fields is singular, has been suggested to address this problem, a drawback emerges: the characteristic mass scale of lepton number violation becomes too low, sacrificing one of the key merits of the seesaw mechanism itself.

In this letter, we propose a four-neutrino scenario within the framework of the seesaw mechanism that maintains M at GUT scales, thereby preserving all the original attractions of the seesaw mechanism, while naturally producing a mass spectrum with two almost degenerate pairs separated by a mass gap. Additionally, we discuss the physical origin of our scenario from the perspective of Abelian flavor symmetry through a specific example that explains all three neutrino anomalies.

II. A Model for Four Light Neutrinos

The natural generation of neutrino masses much lighter than the electroweak scale makes the seesaw mechanism popular. By introducing three right-handed neutrinos $N_\alpha(\alpha = e, \mu, \tau)$ with Majorana mass M, the ordinary seesaw mechanism involves two mass scales: the heavy neutrino mass M and the seesaw-suppressed neutrino mass m^2/M, where m is the Dirac mass. Assigning m at the electroweak scale, the SM-singlet N_α fields can have masses in the phenomenologically interesting range such that $M \sim 10^{16}$ GeV, corresponding to a light neutrino mass $m_\nu \sim 1$ eV.

To accommodate three neutrino experiments, one of the right-handed neutrinos should be light. This requires the singular seesaw mechanism: the right-handed neutrino Majorana mass matrix is singular [**?**]. In Ref. [**?**], this mechanism was used to produce the four light neutrino mass pattern. However, their approach suffers from a drawback: the mass of the heavy right-handed neutrinos is at the keV scale, which is further explained by introducing a double seesaw mechanism. This occurs because in Ref. [**?**], three mass scales M, m, and m^2/M are introduced. When m is charged with explaining the LSND results and hot dark matter while m^2/M addresses the solar neutrino problem, M becomes much smaller than the GUT scale.

We observe that if the Dirac mass matrix is also singular, the right-handed neutrino mass can be restored to the GUT scale even within the singular seesaw mechanism. For illustrative purposes, we consider the mass matrix of the following simple form:

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & m_{22} & m_{23} \\ 0 & 0 & 0 & m_{22} & 0 & m_{32} \\ 0 & 0 & 0 & m_{23} & m_{32} & 0 \\ 0 & m_{22} & m_{23} & 0 & M_{22} & M_{23} \\ m_{22} & 0 & m_{32} & M_{22} & 0 & M_{23} \\ m_{23} & m_{32} & 0 & M_{23} & M_{23} & M_{33} \end{pmatrix}$$

which is of rank four. In the mass spectrum, there are two heavy neutrinos of masses $\sim M$, two light neutrinos of masses $\sim m^2/M$, and two massless neutrinos. In other words, the three scales in this case are M, m^2/M, and 0. The four light neutrinos are naturally divided into two pairs with a mass hierarchy of $\sim m^2/M$, and each pair consists of two degenerate mass eigenstates. Thus, the seesaw mechanism, under the assumption that both the Dirac and Majorana mass matrices are singular, can naturally produce the seemingly bizarre but required mass pattern to explain the three neutrino anomalies. Moreover, M can be around 10^{13} GeV if m is taken to be at the electroweak scale and m^2/M at the 1 eV scale, maintaining a key merit of the seesaw mechanism: it can be easily implemented in numerous theories for physics beyond the SM.

Now let us explore the physical origin of the singular seesaw mechanism that could lead to the specific neutrino mass matrix texture in (1). It is natural to expect that some symmetry induces such a neutrino mass pattern. This symmetry should also provide the large mixing needed for the atmospheric neutrino anomaly, which is no longer automatic in (1).[1] Furthermore, a soft breaking of the symmetry is necessary to generate small masses for the two massless neutrinos and to lift the degeneracy within each pair.

It is known that Abelian flavor symmetry breaking by small parameters could explain the hierarchy in fermion masses [**?**]. We will apply this principle to discuss neutrino masses. In the following discussion, supersymmetry is implied. The flavor symmetry is spontaneously broken by a vacuum expectation value (VEV) of an electroweak singlet field X. As long as the flavor charges balance under the Abelian flavor symmetry, the following interactions are allowed:

$$L_\alpha H N_\beta \left(\frac{X}{\Lambda}\right)^{m_{\alpha\beta}}, \quad M N_\alpha N_\beta \left(\frac{X}{\Lambda}\right)^{n_{\alpha\beta}}$$

where L_α ($\alpha = e, \mu, \tau$), H, and N_α denote the lepton doublets, one Higgs field, and the right-handed neutrino fields, respectively. Λ is the flavor symmetry breaking scale, and the condition $m_{\alpha\beta}, n_{\alpha\beta} \geq 0$ is required for the holomorphy of the superpotential. The order parameter for this new symmetry is defined by $\lambda \equiv \langle X \rangle / \Lambda$.

We intend the neutrino mass pattern in (1) to be achieved through selection rules for the interactions in (2) via proper assignment of flavor charges to L_α and N_α. An additional requirement is that the atmospheric neutrino anomaly arises from $\nu_\mu - \nu_\tau$ oscillation. Compared to analogous analyses for three light neutrino scenarios that do not account for the LSND result [**?**], the choice of flavor charges here is more constrained. One subtle point is that one of the right-handed neutrino masses must be made vanishingly small.

As a specific example, we consider the following assignment of Abelian flavor charges:

$$\begin{aligned}
&L_e(2t-a), \quad L_\mu(a), \quad L_\tau(-a), \\
&E_e^c(a+6), \quad E_\mu^c(a+6), \quad E_\tau^c(a+6), \\
&N_e(2r+a), \quad N_\mu(-a), \quad N_\tau(a+2), \quad X(-1)
\end{aligned}$$

where the integers a, t, and r are constrained as $t < a < r$. The E_α^c fields are the anti-particle fields of the SU(2) singlet charged leptons. To obtain the physical mixing angles of neutrinos, we must simultaneously consider the mass matrix for the charged lepton sector. The gauge boson and Higgs fields possess vanishing flavor charges. Note that all flavor charges for the second and third generations are expressed by a single parameter a.

[1] Compared to the neutrino mass matrix which yields a degenerate neutrino pair with maximum mixing in the original singular seesaw mechanism [**?**], the matrix in (1) gives vanishing mass to two neutrinos. In our case, their mass eigenstates can always be rotated to the weak eigenstates.

The flavor charge assignment in (4) and (5) produces the Dirac and Majorana mass matrices of neutrinos as:

$$M_D = m \begin{pmatrix} Y_{11}\lambda^{r+t} & 0 & Y_{13}\lambda^{t+1} \\ Y_{21}\lambda^{r+a} & 1 & Y_{23}\lambda^{a+1} \\ Y_{31}\lambda^{r-1} & 0 & \text{(missing entry)} \end{pmatrix}$$

$$M_M = \lambda M \begin{pmatrix} \zeta_1\lambda^{2r+a-1} & \zeta_2\lambda^{r-1} & \zeta_3\lambda^{r+a} \\ \zeta_2\lambda^{r-1} & \zeta_3\lambda^{r+a} & \zeta_4\lambda^{a+1} \\ \text{(symmetric)} & \text{(entries)} & \text{(implied)} \end{pmatrix}$$

and the charged lepton mass matrix as:

$$M_l = m\lambda^2 \begin{pmatrix} \eta_{13}\lambda^{t+1} & 0 \\ \eta_{22}\lambda & \eta_{23}\lambda^{a+1} \end{pmatrix}$$

where the Y' s, ζ' s, and η' s are order-one coefficients. To leading order, only the tau lepton acquires mass $\lambda^2 m$ while the muon and electron remain massless. The mass matrix of four light neutrinos is obtained as follows, to leading order:

$$M_\nu \sim \frac{m^2}{M} \begin{pmatrix} \lambda^{2r+2t+1} & \lambda^{2r+t+a+1} \\ \lambda^{2r+t+a+1} & \lambda^{2r+2a+1} \end{pmatrix}$$

Here ϵ denotes the ratio of the weak scale to the GUT scale, $\epsilon \sim m/M$. The neutrino mass spectrum yields $m_{\nu 1} = m_{\nu 2} = 0$, $m_{\nu 3} = m_{\nu 4} = \sin\theta_{34} = 1$.

To attribute the LSND data to oscillation between the two groups, we require $\Delta m^2 \sim 1$ eV^2, which can be satisfied with masses:

$$m_{\nu 3} \sim m_{\nu 4} \sim \frac{m^2}{M} \sim 1 \text{ eV}, \quad M \sim 10^{13} \text{ GeV}.$$

One merit of our charge assignment is that we can obtain a larger mass scale for M according to the small value of λ. If $\lambda \sim 10^{-1}$, which is typical for the Cabbibo angle, the tau lepton mass is properly obtained: $m_\tau \sim 1$ GeV.

The full charged lepton mass matrix in (7) is solved by standard methods. The eigenvalues of M_l are 0, $\lambda^3 m$, and $\lambda^2 m$. In our mechanism with the flavor charge assignment in (4) and (5), the muon acquires mass with the appropriate magnitude (~ 100 MeV), but the electron remains massless even under Abelian flavor symmetry breaking.

The matrix M_l is diagonalized by:

$$R_l^\dagger M_l R_l = \text{Diag}(m_e, m_\mu, m_\tau).$$

Since R_L diagonalizes the hermitian mass-squared matrix $M_l M_l^\dagger$, we have:

$$R_L \sim \begin{pmatrix} 1 & \eta_{23}\lambda^{a+1} & 0 \\ -\eta_{23}\lambda^{a+1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In the neutrino sector, the mass matrix of four light neutrinos can be obtained by the method described in Ref. [?]. The matrix M_M is diagonalized to give three eigenvalues $\lambda^{2r+a}M$, M, and M by a rotation matrix R_M:

$$R_M \sim \begin{pmatrix} 1 & \zeta_2\lambda^{r-1} & \zeta_2\lambda^{r-1}/\sqrt{2} \\ -\zeta_2\lambda^{r-1} & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}.$$

We finally obtain the symmetric mass matrix of four light neutrinos:

$$M_\nu \sim \frac{m^2}{M} \begin{pmatrix} \lambda^{r+t+1}/\epsilon & \lambda^{r+a+1}/\epsilon & \lambda^{r+t+1}/\epsilon & \lambda^{r+a+1}/\epsilon \\ \lambda^{r+a+1}/\epsilon & \lambda^{r}/\epsilon & \lambda^{r+a+1}/\epsilon & \lambda^{2r+a+1}/\epsilon^2 \\ \lambda^{r+t+1}/\epsilon & \lambda^{r+a+1}/\epsilon & \lambda^{r+t+1}/\epsilon & \lambda^{r+a+1}/\epsilon \\ \lambda^{r+a+1}/\epsilon & \lambda^{2r+a+1}/\epsilon^2 & \lambda^{r+a+1}/\epsilon & \lambda^{r}/\epsilon \end{pmatrix}$$

where the charged lepton mixing effects are incorporated so that the charged lepton fields have been rotated to mass eigenstates. In this matrix, each element denotes the order-of-magnitude estimate. The mixing angle for the LSND results comes from the λ^r/ϵ term:

$$\sin\theta_{\mathrm{LSND}} \sim \lambda^r/\epsilon \sim 10^{-2}.$$

Since this mixing angle is small, the masses of ν_3 and ν_4 approximately come from the (22), (23), (32), and (33) components of M_ν, implying the mass difference $\Delta m_{34} \sim \lambda^a$. The Super-Kamiokande data are explained as $\Delta m_{34}^2 \sim 10^{-3}$ eV^2, which requires $a = 3$, where we have used $\lambda \sim 10^{-1}$. Secondly, the masses of ν_1 and ν_2 approximately come from the (11), (14), and (44) components of M_ν. Since $t < a$ in our charge assignment, we naturally have maximal mixing between ν_1 and ν_2 with the mass-squared difference:

$$\sin\theta_{12} \simeq 1, \quad \Delta m_{12}^2 \sim 10^{-10}\ \mathrm{eV}^2,$$

which corresponds to the vacuum oscillation solution for the solar neutrino problem.

Let us summarize the characteristic features of our mechanism:

1. The four-neutrino mass spectrum consisting of two almost degenerate pairs separated by a mass gap ~ 1 eV is naturally generated.

2. The mass scale of the right-handed neutrinos in our case is high enough even in the singular seesaw mechanism. The understanding of neutrino mass and mixing can be put into the same category as that of charged leptons and quarks.
3. The large mixing for the atmospheric neutrino problem is not automatic. It is achieved by introducing Abelian family symmetry. The $\nu_\mu - \nu_\tau$ large mixing is feasible in this case.
4. The lightest neutrino masses are not generated by the seesaw mechanism. The large mixing of the vacuum oscillation solution for the solar neutrino problem can be accommodated.

III. Discussion and Summary

In this four light neutrino scenario, the flavor charge assignment or the texture of the neutrino mass matrix may imply that the first-generation fermions are exceptional regarding their masses. For the second and third generations, neutrinos are treated essentially the same as in three light neutrino scenarios using the ordinary seesaw mechanism [**?**]. Thus, we expect that the charged lepton and quark masses of these two generations can be naturally understood within the framework of Abelian flavor symmetry.

For the first generation, although it is possible to produce appropriate masses for the charged fermions by introducing exotic flavor quantum numbers, the Yukawa couplings with flavor symmetry might not be the source of these masses. Below, we point out a possible origin for the masses of first-generation fermions that was mentioned in Ref. [**?**]. In a supersymmetric model with R-parity violation and baryon number conservation, the following interactions are allowed by gauge symmetry and flavor symmetry:

$$QLD^c \left(\frac{X}{\Lambda}\right)^m, \quad LLE^c \left(\frac{X}{\Lambda}\right)^n$$

with positive definite integers m and n, where we have suppressed generation indices. Here Q and D^c are the SU(2) quark doublet and down-type anti-quark singlet superfields, respectively. If the sneutrino fields acquire non-vanishing VEVs, these interactions generate masses for charged leptons and down-type quarks. When the sneutrino VEVs are around $(10^{-3} - 10)$ GeV, the correct magnitudes for electron and down quark masses can be obtained.[2] One interesting feature is that the up quark remains massless, thus eliminating the strong CP problem [**?**]. A similar idea was recently carried out in detail in Ref. [**?**].

In summary, within the framework of the singular seesaw mechanism, we have studied the four light neutrino scenario that explains atmospheric, solar, and LSND neutrino experiments. By taking the Dirac neutrino mass matrix to be

[2] A GeV-scale sneutrino VEV can generally result in a MeV tau neutrino mass at tree level. However, this conclusion is model-dependent. For example, the model described in Ref. [**?**] has vanishing tau neutrino mass at tree level.

singular as well, the right-handed neutrino Majorana mass can be at the 10^{14} GeV scale. Abelian flavor symmetry is used to produce the required neutrino mass pattern.

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Note: Figure translations are in progress. See original paper for figures.