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Full Text

Preamble

Reanalysis of the Four-Quark Operators Relevant to Λ_b Lifetime from QCD Sum Rule

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Abstract

The Λ_b matrix element of the four-quark operator relevant to its lifetime is reanalyzed using QCD sum rules. The new ingredients introduced are that (1) more vacuum condensates are considered; (2) a different quark-hadron duality is adopted; and (3) the possible deviation from the vacuum saturation assumption for the four-quark condensates is considered. With $1 = 4$, the lifetime-related hadronic parameter r is calculated to be (3.6 ± 0.9) , and the lifetime ratio $\tau(\Lambda_b)/\tau(B_0) = (0.83 \pm 0.04)$, which is consistent with the experimental data.

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I. INTRODUCTION

Heavy baryon lifetimes provide a testing ground for the standard model, especially for QCD in some aspects, because they can be calculated systematically by heavy quark expansion [1]. Theoretically, if we do not assume the failure of the local duality assumption, the heavy hadron lifetime differences appear, at most, at the order of $1/mQ^2$ [2]. The recent experimental result on the lifetime ratio of the Λ_b baryon and B meson is [3]

$$\tau(\Lambda_b)/\tau(B_0) = 0.79 \pm 0.06,$$

which has shown some deviation from the theoretical expectation. This is one of the issues concerned with tests of the heavy quark expansions and has drawn a lot of theoretical attention [4–8], and may imply the potential importance of the $O(1/m^3)$ effect for the above heavy baryon and heavy meson lifetime difference [4,9,10].

It is at the order $1/m^3$ that four-quark operators appear, whose contributions to decay widths are enhanced by a phase-space factor of $(16\pi^2)$ with respect to the leading terms in the operator product expansion. Consequently, one should include them in the prediction for non-leptonic decay rates [11]. To the order of $1/m^3$, the lifetime ratio was calculated as follows [4]:

$$\tau(\Lambda_b)/\tau(B_0) = 1 + \pi(\Lambda_b) [p_1 B_1(\text{mb}) + p_2 B_2(\text{mb}) + p_3 1(\text{mb}) + p_4 2(\text{mb}) + [p_5 + p_6 B(\text{mb})]r(\text{mb})],$$

where the term proportional to π arises from the contributions of four-quark operators. The numerical values of the coefficients c_G and π_i ($i = 1-6$) have been calculated in Ref. [4]. μ^2 and $\mu_G^2(B)$ are the averages of b-quark kinetic and chromomagnetic energy, respectively. To the order of $1/m^2$, the lifetime ratio is 0.98. At the scale mb, the values of π_i are $\pi_1 = 0.195$, $\pi_2 = 0.004$, $\pi_3 = 0.021$, $\pi_4 = 0.012$, $\pi_6 = 0.173$.

$B_1, B_2, 1, 2, r$ and B are the parameterization of the hadronic matrix elements of the following four-quark operators:

$$\begin{aligned} B_1 f B_2 &= (1/2m\Lambda_b) \Lambda_b | \bar{b} \gamma_\mu (1-\gamma_5) q \bar{q} \gamma_\mu (1-\gamma_5) b | \Lambda_b \\ B_2 f B_2 &= (1/2m\Lambda_b) \Lambda_b | \bar{b} \gamma_\mu (1-\gamma_5) t a q \bar{q} \gamma_\mu (1-\gamma_5) t a b | \Lambda_b \\ 1 f B_2 &= (1/2m\Lambda_b) \Lambda_b | \bar{b} \gamma_\mu (1-\gamma_5) q \bar{q} \gamma_\mu (1+\gamma_5) b | \Lambda_b \\ 2 f B_2 &= (1/2m\Lambda_b) \Lambda_b | \bar{b} \gamma_\mu (1-\gamma_5) t a q \bar{q} \gamma_\mu (1+\gamma_5) t a b | \Lambda_b \\ r f \Lambda_2 &= (1/2m\Lambda_b) \Lambda_b | O | \Lambda_b \\ B f \Lambda_2 &= (1/2m\Lambda_b) \Lambda_b | \bar{O} | \Lambda_b \end{aligned}$$

where $(f_B/200 \text{ MeV})^2$ arises from the contributions of four-quark operators. In Eq. (2), the energy scale for the parameters is at mb. In Eq. (3), generally the renormalization scale is arbitrary, and the parameters depend on it. It can be taken naturally at the low hadronic scale to apply the heavy quark expansion.

These parameters should be calculated by some nonperturbative QCD method. The QCD sum rule [12], which is regarded as a nonperturbative method rooted in QCD itself, has been used successfully to calculate the properties of various hadrons. In Refs. [5] and [6], the mesonic parameters B_i and i have been calculated by QCD sum rules within the framework of heavy quark effective theory (HQET) [13]. The baryonic parameters r and B have been calculated in Ref. [7]. As a result, to the order of $1/m_b^3$, the theoretical calculation still cannot explain the experimental result (1).

II. THEORETICAL CONSIDERATION

In this paper, we try to match the experimental data by considering more subtleties in the theoretical analysis. As analyzed in Ref. [14], the lifetime ratio (1) depends crucially on the value of the baryonic parameter r . Therefore, we shall carry out a reanalysis for the baryonic parameter r using QCD sum rules. We note that in the analysis of Ref. [7], more condensates could be included. They are the gluon condensate and six-quark condensate. The gluon condensate may be important because its dimension is four. In spite of being dimension nine, the six-quark condensate may also have significant contribution, because the diagram is not suppressed by any loop factor. Experience from heavy baryon masses tells us that the pure tree diagram is important [15-18]. Therefore, in our calculation, these two kinds of condensates will be considered, in addition to those considered in Ref. [7].

The conclusion that $B = 1$ does not change. It follows from the valence quark approximation which is used both in Ref. [7] and in our QCD sum rule analysis.

To calculate r , the following three-point Green's function is constructed:

$$\Pi(\omega, \omega) = i^2 \int dx dy e^{ik \cdot x - ik \cdot y} \langle 0 | T \{ \bar{j}_v(x) \tilde{O}(0) j_{v^\dagger}(y) \} | 0 \rangle$$

where $\omega = v \cdot k$ and $\omega = v \cdot k$. The Λ_Q baryonic current j_v was given in Refs. [15-18]:

$$j_v = abc \, q_a^T C \gamma_5 (a + b \, v) \tau \, q_b \, h_{v,c}$$

where a and b are certain constants which will be discussed later, h_v is the heavy quark field in HQET with velocity v , C is the charge conjugate matrix, and τ is the flavor matrix.

In Eq. (5), \tilde{O} denotes the four-quark operator:

$$\tilde{O} = h_v \gamma_\mu h_v \bar{q} \gamma_\mu q$$

Note that $\langle \Lambda_b | O | \Lambda_b \rangle = \langle \Lambda_b | \tilde{O} | \Lambda_b \rangle$ in the valence quark approximation [7], where $O = h_v \gamma_\mu q \bar{q} \gamma_\mu h_v$.

In terms of the hadronic expression, the parameter r appears in the ground state contribution of $\Pi(\omega, \omega)$:

$$\Pi(\omega, \omega) = (f \Lambda^2 r) / (\Lambda - \omega)(\Lambda - \omega) + \text{higher states}$$

where the “higher states” denotes the contribution of resonances and continuum. The quantity $f\Lambda$ has appeared in Eq. (4), $\Lambda = m\Lambda_Q - m_Q$, and the quantity $f\Lambda$ is defined as

$$0|\bar{j}v|\Lambda_Q = f\Lambda u$$

with u being the unit spinor in HQET. The QCD sum rule calculations for $f\Lambda$ were given in Refs. [15-18]. On the other hand, this Green’s function $\Pi(\omega, \omega)$ can be calculated in terms of quark and gluon language with vacuum condensates. This establishes the sum rule. For the resonance part of Eq. (10), we adopt the assumption of quark-hadron duality.

The calculation of $\Pi(\omega, \omega)$ in HQET is straightforward. The fixed-point gauge [19] is used. The tadpole diagrams in which the light quark lines from the four-quark vertex are contracted have been subtracted. The following values of the condensates are used:

$$\begin{aligned} \bar{q}q &= -(0.23 \text{ GeV})^3, \\ g^2G^2 &= 0.8 \text{ GeV}^4, \\ \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q &= -(0.075 \pm 0.015) \text{ GeV}^4. \end{aligned}$$

Note we have adopted the new value for the gluon condensate from the recent analysis of the heavy quarkonium spectrum [20]. However, our final results are not very sensitive to the gluon condensate since its contribution is only about 6% of the whole sum rule as we shall see later.

Except for the quark-gluon mixed condensate, our calculation would be consistent with that of Ref. [7] if the gluon and six-quark condensates were omitted. While the calculation of $\Pi(\omega, \omega)$ is large, however, the hadron ground state property should be obtained at small (ω, ω) . These contradictory requirements are achieved by introducing double Borel transformation for ω and ω . It is defined as:

$$B\tilde{\tau}B\tilde{\tau}\Pi(\omega, \omega) = \lim_{n,m \rightarrow \infty, \omega, \omega \rightarrow \infty} (\omega_{n+1}/n!) (-d/d\omega)^n (\omega_{m+1}/m!) (-d/d\omega)^m \Pi(\omega, \omega)$$

with $\omega/n = \tilde{\tau}$ and $\omega/m = \tilde{\tau}$ fixed. There are two Borel parameters $\tilde{\tau}$ and $\tilde{\tau}$. They appear symmetrically, so $\tilde{\tau} = \tilde{\tau} = 2T$ is taken in the following analysis. The reason for the factor 2 is similar to that explained in Ref. [21].

III. DUALITY ASSUMPTION

Generally, duality is used to simulate the higher states by the whole quark and gluon contribution above some threshold energy ω_c . The whole contribution of the three-point correlator $\Pi(\omega, \omega)$ can be expressed by the dispersion relation:

$$\Pi(\omega, \omega) = \int_0^\infty d\omega' \frac{\text{Im}\Pi(\omega', \omega')}{(\omega - \omega')(\omega' - \omega)}.$$

With the redefinition of the integral variables $\omega' = \omega + \omega'$ and $\omega = \omega - \omega'$, the integration becomes $\int_0^\infty d\omega' = \int_0^\infty d\omega' + \int_0^\infty d\omega' = 2 \int_0^\infty d\omega' + \dots$.

It is in [13] that the quark-hadron duality is assumed [13]:

$$\text{higher states} = \int_{\omega_c}^{\infty} d\omega \rho(\omega) \frac{[\text{Im}\Pi(s)]}{(s - \omega)(s - \omega^*)}.$$

This kind of assumption was suggested for calculating the Isgur-Wise function in Ref. [21] and was argued for in Ref. [22]. As pointed out in [22,13], in calculating three-point functions the duality is valid after integrating the spectral density over the “off-diagonal” variable $s = 1/2(s_1 + s_2)$. The sum rule after the integration with the variable s is obtained as:

$$f_{\Lambda}^2 r e^{-\Lambda/T} = \int_{\omega_c}^{\infty} d\omega e^{-\omega/T} \left[\frac{(a^2 + b^2)}{(840\pi^6)} \int_0^1 ds_1 \int_0^1 ds_2 \left[\frac{1}{(s_1 - \omega)(s_2 - \omega)} \right] + \frac{(a^2 + b^2)}{(2048\pi^6)} \int_0^1 ds_1 \int_0^1 ds_2 \left[\frac{1}{(s_1 - \omega)(s_2 - \omega^*)} \right] + \frac{(17(a^2 + b^2))}{(48\pi^4)} \int_0^1 ds_1 \int_0^1 ds_2 \left[\frac{1}{(s_1 - \omega)(s_2 - \omega)} \right] + \frac{(a^2 + b^2)}{(32\pi^4)} \int_0^1 ds_1 \int_0^1 ds_2 \left[\frac{1}{(s_1 - \omega)(s_2 - \omega^*)} \right] \right],$$

where a, b are the scale parameters used to indicate the possible deviation from the factorization assumption for the four- and six-quark condensates. $a, b = 1$ corresponds to the vacuum saturation approximation. $a = (3-8)$ is often introduced in order to include the nonfactorizable contribution and to fit the data [23]. There is no discussion of b in the literature, so we use $b = 1$.

As argued in [17], the choice $a = b = 1$ tends to yield the optimal interpolating current for the Λ_b baryon in HQET. We shall adopt $a = b = 1$ in our numerical analysis.

The parameters f_{Λ} , Λ , etc. were obtained by the HQET sum rule analysis of the two-point correlator [15-18]:

$$(a + b)2f_{\Lambda}^2 e^{-\Lambda/T} = \int_{\omega_c}^{\infty} d\omega e^{-\omega/T} \left[\frac{(a^2 + b^2)}{(20\pi^4)} \int_0^1 ds_1 \int_0^1 ds_2 \left[\frac{1}{(s_1 - \omega)(s_2 - \omega)} \right] + \dots \right].$$

We have not included α_s corrections in Eq. (19), because they are also neglected in the sum rule for r (18). The values of the parameters are $f_{\Lambda}^2 = (2.9 \pm 0.4) \times 10^{-2} \text{ GeV}^3$, $\Lambda = (0.9 \pm 0.2) \text{ GeV}$, and the Borel parameter T is in the window $(0.2-0.4) \text{ GeV}$ with the threshold ω_c taken to be $(1.2 \pm 0.1) \text{ GeV}$.

In order to minimize the dependence on the parameters, we divide Eq. (18) by Eq. (19) to extract r . The variation of the matrix element with ω_c for two values of $a = 1, 4$ is given in Fig. 1 [Figure 1: see original paper]. The value of ω_c is $(1.2 \pm 0.35) \text{ GeV}$, which is almost the same as that in the two-point correlator sum rule. We obtain for $a = 4$: $r = (3.6 \pm 0.9)$. The sum rule window is $T = (0.15-0.35) \text{ GeV}$. By taking $f_B = 200 \text{ MeV}$, we get $r = (1.6 \pm 0.4) \times 10^{-2} \text{ GeV}^3$. If we use $a = 1$, we get $r = (1.3 \pm 0.3)$ and $r = (5.5 \pm 1.2) \times 10^{-3} \text{ GeV}^3$.

In our numerical calculation, the contribution of the perturbative term is smaller than those of the condensates due to small T in the duality window and the high dimensionality of the spectral function ($\delta = 8$ in eq. (18)). There are several known examples where the perturbative part has small contribution in the sum rule [16,17,24], without affecting the reliability of the results. Nevertheless, a hierarchical structure among various condensate terms exists in those cases and in the present case. In the present case, because of the cancellation between the contributions of the quark condensate and the mixed condensate, as well as the smallness of the contributions of the gluon and the six-quark condensates, the

influence of the four-quark condensate is significant and is enhanced by assuming deviation from vacuum saturation.

In Ref. [7] the authors used the following duality assumption:

$$\text{higher states} = \int_{\omega_c}^{\infty} d\omega \rho(\omega) \frac{[\text{Im}\Pi(\omega)]}{(\omega - \omega_c)(\omega - \omega_c^*)}.$$

In their calculation, the matrix element $r = (0.4-1.2) \times 10^{-3} \text{ GeV}^3$ in the working region of the Borel parameter when ω_c varies from 1.1 GeV to 1.3 GeV. In other words, their analysis is very sensitive to the continuum threshold, which might imply that the above duality assumption is not good. In contrast, the dependence on ω_c is not so strong in our approach.

The $1/\text{mb}$ corrections to the above results can be analyzed in principle. While having little influence on our calculation, they formally belong to the $O(1/\text{mb}^4)$ effects to the Λ_b lifetime. It should be noted that the value of r we have obtained above is at some hadronic scale, rather than the scale mb , because we have been working in the framework of HQET, in which the natural scale is $\mu_{\text{had}} \sim \text{mb}$. The renormalization group evolution of the relevant operators was calculated in Ref. [25,4]. Information on parameters B and r at scale mb is necessary to obtain the lifetime ratio of Eq. (2). By choosing $\alpha_s(\mu_{\text{had}}) = 0.5$ (corresponding to $\mu_{\text{had}} = 0.67 \text{ GeV}$), Ref. [4] gives:

$$\begin{aligned} r(\text{mb}) &= [1.54 + 0.18 B(\mu_{\text{had}})] r(\mu_{\text{had}}), \\ B(\text{mb}) &= 1.54 + 0.18 B(\mu_{\text{had}}). \end{aligned}$$

That is, $r(\text{mb}) = (2.3 \pm 0.6)$ for $\mu = 1$ or $r(\text{mb}) = (6.2 \pm 1.6)$ for $\mu = 4$, using $B(\mu_{\text{had}}) = 0.58$ from Eqs. (21) and (23).

The Λ_b and B_0 lifetime ratio given in Eq. (2) is expressed specifically as:

$$\begin{aligned} \tau(\Lambda_b)/\tau(B_0) &= 0.98 - 0.17 \mu_1(\text{mb}) + 0.20 \mu_2(\text{mb}) - (0.013 + 0.022 B(\text{mb}))r(\text{mb}) \\ &= (0.83 \pm 0.04) \text{ for } \mu = 4, \end{aligned}$$

or

$$\tau(\Lambda_b)/\tau(B_0) = (0.93 \pm 0.02) \text{ for } \mu = 1,$$

where the values $\mu_1(\text{mb}) = 0.08$ and $\mu_2(\text{mb}) = 0.01$ have been taken from the QCD sum rules [5]. From Eq. (30), we see that with the vacuum saturation ($\mu = 1$), although r is enhanced by about six times compared to that in Ref. [7], it is still not large enough to account for the data in Eq. (1). The lifetime ratio between Λ_b and B mesons can be explained if we also take into account the nonfactorizable contribution of the four-quark condensate, as can be seen from Eq. (29).

IV. SUMMARY AND DISCUSSION

In summary, we have reanalyzed the QCD sum rule for the Λ_b matrix element of the four-quark operator relevant to the lifetime of Λ_b . Compared to the previous analysis [7], the new ingredients we have introduced are that (i) more

condensates are considered; (ii) a different quark-hadron duality is adopted; and (iii) the possible deviation from the vacuum saturation assumption for the four-quark condensates is considered. Of these ingredients, the last two points have more important influence on numerical results than the first point, as the effect due to more condensates is small (10%). The second point is more essential than the third point. Note that due to the second point, r is about six times enhanced compared to the result of Ref. [7]. And due to the third point, r is about three times enhanced by comparing the results of Eqs. (23) and (21). With a fixed duality assumption for the sum rule, our result shows that the baryonic parameter r is significantly dependent on the nonfactorizable effect of the four-quark condensate, with $r(\text{mb}) = (2.3 \pm 0.6)$ for $1 = 1$ and $r(\text{mb}) = (6.2 \pm 1.6)$ for $1 = 4$. With the latter value, the Λ_b baryon and B_0 meson lifetime ratio has been calculated to be $\tau(\Lambda_b)/\tau(B_0) = (0.83 \pm 0.04)$, which is close to the experimental data $\tau(\Lambda_b)/\tau(B_0) = (0.79 \pm 0.06)$. From these results we draw the conclusion that under the local duality assumption it is possible to resolve the issue of the Λ_b baryon and B_0 meson lifetime ratio in the framework of heavy quark expansion.

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Figure captions

Fig. 1. Sum rule for $\Lambda b|\tilde{O}|\Lambda b / f\Lambda^2$, where \tilde{O} has been given in Eq. (8). From top to bottom, these curves correspond to $\omega c = 1.3, 1.2, 1.1$ GeV respectively. The sum rule window is $T = (0.15-0.35)$ GeV.

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