Isospin and Z1/3 Dependence of the Nuclear Charge Radii (Postprint)

Authors: S.Q. Zhang, J. Meng, S.-G. Zhou, J.Y. Zeng

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Abstract

Based on the systematic investigation of the data available for A 40, a Z1/3 dependence for the nuclear charge radii is shown to be superior to the generally accepted A1/3 law. A delicate scattering of data around Rc/Z1/3 is inferred as owing to the isospin effect and a linear dependence of Rc/Z1/3 on N/Z (or (N−Z)/2) is found. This inference is well supported by the microscopic Relativistic Continuum Hartree-Bogoliubov (RCHB) calculation conducted for the proton magic Ca, Ni, Zr, Sn and Pb isotopes including the exotic nuclei close to the neutron drip line. With the linear isospin dependence provided by the data and RCHB theory, a new isospin-dependent Z1/3 formula for the nuclear charge radii is proposed.

Full Text

Preamble

Isospin and $Z^{1/3}$ Dependence of the Nuclear Charge Radii

S.Q. Zhang[1], J. Meng[1,2,3*], S.-G. Zhou[1,2,3], and J.Y. Zeng[2,3,4]

[1]Department of Technical Physics, Peking University, Beijing 100871
[2]Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080
[3]Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000
[4]Department of Physics, Peking University, Beijing 100871

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Abstract

Based on a systematic investigation of available data for $A \geq 40$, a $Z^{1/3}$ dependence for nuclear charge radii is shown to be superior to the generally accepted

$A^{1/3}$ law. A delicate scattering of data around $R_c/Z^{1/3}$ is inferred to arise from isospin effects, and a linear dependence of $R_c/Z^{1/3}$ on N/Z (or $(N-Z)/2$) is found. This inference is well supported by microscopic Relativistic Continuum Hartree-Bogoliubov (RCHB) calculations conducted for the proton-magic Ca, Ni, Zr, Sn, and Pb isotopes, including exotic nuclei close to the neutron drip line. With the linear isospin dependence provided by both data and RCHB theory, a new isospin-dependent $Z^{1/3}$ formula for nuclear charge radii is proposed.

Introduction

Nuclear radius is one of the most fundamental bulk properties of an atomic nucleus [**?**, **?**]. Among all size quantities describing nuclei, nuclear charge radii have been investigated experimentally by various techniques and methods [**?**], including muonic atom spectroscopy [**?**], isotope shifts from optical and K X-ray spectroscopy [**?**], and high-energy elastic electron scattering [**?**], among others. Recently, more nuclei far from the β-stability line have become experimentally accessible thanks to the development of radioactive ion beam facilities [**?**, **?**]. Nuclear sizes connected with exotic phenomena such as skins and halos have become a hot topic, with important implications not only for nuclear physics but also for astrophysics and atomic physics. Accurate studies of nuclear charge radii are crucial for understanding not only the proton distribution inside the nucleus but also halo and skin structures. In particular, a simple and reliable formula for nuclear charge radii would be very useful for extracting the decoupling of protons and neutrons in exotic nuclei and for providing information about the effective nucleon-nucleon interaction widely used in nuclear models. In this Letter, we examine the available experimental charge radii data for $A \geq 40$ and study their global behavior, proposing a new $Z^{1/3}$ formula with isospin effects instead of the widely accepted $A^{1/3}$ law.

Based on considerations of nuclear saturation properties, nuclear charge radii R_c are usually described by the $A^{1/3}$ law [**?**, **?**]:

$$R_c = r_A A^{1/3},$$

where A is the mass number and $R_c = \sqrt{5/3}\langle r^2\rangle^{1/2}$, with $\langle r^2\rangle^{1/2}$ being the root-mean-square (rms) charge radius. For very light nuclei, because of their small A and large fluctuations in charge distribution due to shell effects with short periods, the charge distribution radius as a bulk property has little meaning. A detailed analysis of charge radius data for $A \geq 40$ shows that r_A is by no means constant, but systematically decreases with A; specifically, $r_A \approx 1.31$ fm for light nuclei ($A \sim 40$) and $r_A \approx 1.20$ fm for very heavy nuclei (see upper left panel in Fig. 1 [Figure 1: see original paper]). This fact implies that some physics is missing in the $A^{1/3}$ law.

Definite evidence for violation of the $A^{1/3}$ law is also found in measurements of isotope shifts in mean square charge radii [**?**, **?**]. In particular, $\delta\langle r^2\rangle_{A+2,A}$ values (associated with the addition of two neutrons) are often considerably smaller than expected from the $A^{1/3}$ law ($\delta\langle r^2\rangle_{A+2,A} = \frac{4}{3A}\langle r^2\rangle_A$). A typical example is that the observed charge radii of calcium isotopes $^{40-50}$Ca remain almost the same (except for very small changes induced by deformation or shell effects), despite significant changes in the mass number A. In contrast, there is also evidence that observed $\delta\langle r^2\rangle_{A+2,A}$ values (associated with the addition of two protons) are often greater than expected from the $A^{1/3}$ law (e.g., $\delta\langle r^2\rangle$ for ^{46}Ti-^{44}Ca, ^{50}Ti-^{48}Ca, etc.).

Along the β-stability line, the ratio Z/A gradually decreases with A; for light nuclei $Z/A \approx 1/2$, while for the heaviest β-stable nucleus $^{238}_{92}$U, $(Z/A)^{1/3} \approx 0.7285$, thus $(1/2)^{1/3}/(Z/A)^{1/3} \approx 1.09$, which is very close to the r_A ratio of 1.30/1.20 shown in the upper left panel of Fig. 1. A naive point of view is that the charge radius of a nucleus may be more directly related to its charge number Z rather than its mass number A. Therefore, compared to the $A^{1/3}$ law, a $Z^{1/3}$ dependence for nuclear charge radii may be more reasonable:

$$R_c = r_Z Z^{1/3},$$

as noted in Ref. [?]. An analysis of the limited charge radii data then available showed that r_Z remains almost constant, i.e., $r_Z = 1.65(2)$ fm for $A \geq 40$. The $Z^{1/3}$ dependence of nuclear charge radii was also used to modify the Coulomb energy term in the semi-empirical nuclear mass formula [?], improving agreement between calculated and experimental results. Moreover, the $A^{-1/3}$ law for nuclear giant (monopole, dipole, and quadrupole) resonance energy ($\propto 1/R$) could also be improved if the $A^{-1/3}$ dependence is replaced by a $Z^{-1/3}$ dependence [?].

In the past two decades, vast new experimental information on the electromagnetic structure of nuclear ground states has become available [?], with improved accuracy. In particular, muon factories at Los Alamos (LAMPF) and Villigen (PSI, formerly SIN) began operation in 1974. Almost all stable nuclei have been measured by the muonic X-ray transition technique, and corresponding charge radii have been accurately deduced (experimental relative error about 10^{-3}). Moreover, modern techniques for optical isotope shift measurements have made it possible to reach even short-lived (down to 1 s) unstable isotopes [?]. Therefore, it is worthwhile to reexamine fundamental nuclear properties and investigate whether this vast body of improved experimental results follows the $Z^{1/3}$ dependence.

The measured $\langle r^2\rangle^{1/2}$ values for 536 nuclei with $A \geq 40$ compiled in Refs. [?] are analyzed in Fig. 1 using both $A^{1/3}$ and $Z^{1/3}$ dependence. The dependence of charge radii on quadrupole deformation β has been taken into account for rare-earth deformed nuclei:

$$r_A = r_A^d\left(1 + \frac{5}{8\pi}\beta^2\right),$$

$$r_Z = r_Z^d\left(1 + \frac{5}{8\pi}\beta^2\right),$$

while for spherical nuclei ($\beta = 0$): $r_A = r_A^d$, $r_Z = r_Z^d$, with β values taken from Refs. [?, ?].

In the upper left and right panels of Fig. 1, charge radii for the 159 most stable nuclei with $A \geq 40$ along the β-stability line are analyzed using $A^{1/3}$ and $Z^{1/3}$ dependence, respectively. In the middle left and right panels, the same analysis is performed for the measured $\langle r^2 \rangle^{1/2}$ of 536 nuclei with $A \geq 40$. Two significant features are observed: (A) The agreement between data and calculations using the $Z^{1/3}$ dependence is much better than that using the $A^{1/3}$ law. While r_A^d shows a global regular decrease with A, r_Z^d remains nearly constant ($r_Z^d = 1.631(11)$ fm). The relative rms deviations σ for the $Z^{1/3}$ dependence ($\sigma = 7.57 \times 10^{-3}$ for stable nuclei and 1.00×10^{-2} for 536 nuclei) are much smaller than those for the $A^{1/3}$ law ($\sigma = 1.90 \times 10^{-2}$ for stable nuclei and 1.63×10^{-2} for 536 nuclei). (B) Although the rms deviation for the $Z^{1/3}$ dependence is significantly reduced, an isospin-induced scattering of data can be observed in the middle panels of Fig. 1 compared to the top panels. In fact, r_Z^d generally increases with N for most isotopic chains, such as Cd, Sn, Xe, and Nd isotopes, except for a few lighter isotopic chains like Sr and some cases where a small anomalous decrease of r_Z^d with N occurs due to shell closure at $N = 50$. Therefore, it seems necessary to investigate an isospin-dependent correction for the scattering of r_Z^d. While isospin effects have been considered based on the $A^{1/3}$ law in Refs. [?, ?], the fact that the $Z^{1/3}$ dependence describes nuclear charge radii much better than the $A^{1/3}$ law suggests that the $Z^{1/3}$ dependence provides a more reasonable starting point for describing the isospin dependence of nuclear charge radii.

To confirm that the isospin-dependent $Z^{1/3}$ formula to be developed for nuclear charge radii is also valid for nuclei far from the β-stability line, charge radii data for exotic nuclei are needed. However, as such data are not available, we require that our new isospin-dependent $Z^{1/3}$ formula should be consistent with a reliable microscopic nuclear model.

The fully self-consistent and microscopic relativistic continuum Hartree-Bogoliubov (RCHB) theory, which extends the relativistic mean field (RMF) [?] with Bogoliubov transformation in coordinate representation [?], is a good candidate for this purpose. The RCHB theory can satisfactorily describe ground-state properties for nuclei both near and far from the β-stability line. A remarkable success of RCHB theory is the self-consistent reproduction of the halo in ^{11}Li [?] and prediction of the exotic phenomenon of giant halo [?]. In combination with the Glauber model, RCHB theory successfully reproduces interaction cross sections in Na isotopes [?] and charge-changing cross sections of C, N, O, F isotopes (ranging from the β-stability line to the neutron drip line) on a ^{12}C target at 930 MeV/u [?, ?]. These successes encourage us to apply RCHB theory to describe charge radii of nuclei both close to and far from the β-stability line, checking its validity against available data and providing information for nuclei far from stability.

The detailed formalism and numerical techniques of RCHB theory can be found in Ref. [?] and references therein. In the present calculations, we follow the procedures in Refs. [?, ?, ?] and solve the RCHB equations in a box with size $R = 20$ fm and step size 0.1 fm. The parameter set NL-SH [?] is used, which aims to describe both stable and exotic nuclei. A density-dependent δ-force in the pairing channel with $\rho_0 = 0.152\ \text{fm}^{-3}$ is employed, with its strength V_0 fixed by the Gogny force as in Ref. [?]. The contribution from continuum states is restricted within a cutoff energy $E_{\text{cut}} \sim 120$ MeV.

As typical examples, we studied even-even Ca, Ni, Zr, Sn, and Pb isotopes ranging from the β-stability line to the neutron drip line. The two-neutron separation energy S_{2n} is one of the essential quantities for testing a nuclear model. In Fig. 2 [Figure 2: see original paper], the calculated S_{2n} values (open symbols) for even-even Ca, Ni, Zr, Sn, and Pb isotopes from RCHB theory are compared with available data (solid symbols) [?], showing satisfactory agreement. In particular, the deviation between calculated binding energies and available data is within 1%. In the present calculation, neutron drip-line nuclei are predicted at ^{72}Ca, ^{98}Ni, ^{140}Zr, and ^{176}Sn. In the S_{2n} versus N curve for each isotopic chain, some kinks appear due to neutron shell or subshell closure. For example, closed shells at $N = 20, 28$ and the subshell at $N = 40$ correspond to kinks in the S_{2n} versus N curve for Ca isotopes at ^{40}Ca, ^{48}Ca, and ^{60}Ca, respectively. While the kink at $N = 20$ for ^{40}Ca may also be due to the Wigner term for $N = Z = 20$, there are no kinks at ^{70}Ca and ^{176}Sn, indicating the disappearance of magic numbers $N = 50$ and 126 for these nuclei in RCHB.

The rms charge radii $\langle r^2\rangle^{1/2}$ obtained from RCHB theory (open symbols) and available data (solid symbols) for even-even Ca, Ni, Zr, Sn, and Pb isotopes are shown in Fig. 3 [Figure 3: see original paper]. The RCHB calculations reproduce the data very well (within 1.5%). For a given isotopic chain, an approximate linear N dependence of the calculated rms charge radii $\langle r^2\rangle^{1/2}$ is clearly seen in Fig. 3, showing that the variation of $\langle r^2\rangle^{1/2}$ for a given isotopic chain deviates from both the simple $Z^{1/3}$ dependence and the simple $A^{1/3}$ law (denoted by dashed lines in Fig. 3). Therefore, a strong isospin dependence of nuclear charge radii is necessary for nuclei with extreme N/Z ratios.

In Fig. 4 [Figure 4: see original paper], the experimental and RCHB-predicted $r_Z^d = R_c/Z^{1/3}$ values for proton-magic isotopes are presented as a function of $\eta = N/Z$. The coefficient r_Z^d clearly increases linearly with η (except for some deviations due to deformation or shell effects), and the slopes are nearly the same for these isotopic chains. The linear η (or isospin $T_Z = (N - Z)/2$) dependence of r_Z^d for an isotopic chain may be understood as the effect of first-order perturbation corrections to the nuclear wave function from an isospin T_Z-dependent interaction [?]. Based on analysis of data in the middle and upper panels of Fig. 1 and RCHB predictions in Figs. 3 and 4, we propose the following isospin-dependent $Z^{1/3}$ formula for nuclear charge radii:

$$R_c = aZ^{1/3}[1 + b(\eta - \eta^*)], \quad \eta = N/Z,$$

where η^* is N/Z for nuclei along the β-stability line (directly extracted from the nuclear mass formula [**?**]), $a = r_Z^{d*}(1 + \frac{5}{8\pi}\beta^2)$, $r_Z^{d*} = 1.631(11)$ fm obtained from the upper right panel of Fig. 1, and $b = 0.062(9)$ obtained from least-squares fitting.

Analysis of available data using Eq. (4) with these values of r_Z^{d*} and b is displayed in the lower right panel of Fig. 1. The data are better reproduced by Eq. (4) than by Eq. (2) (the rms deviation is reduced by about 40%). The same analysis performed for the $A^{1/3}$ dependence was less successful (see lower left panel of Fig. 1). In Refs. [**?**, **?**], a similar equation for $A^{1/3}$ dependence was used to describe the isospin dependence of charge radii, achieving better agreement by fitting both r_A and b simultaneously. However, a simple explanation for r_A (saturation property) and b (isospin effect) in Eq. (4) is then missing. It is expected that the modified $Z^{1/3}$ formula (Eq. (4)) will become more useful as more data are obtained for nuclei far from the β-stability line.

Summary

We have systematically investigated nuclear charge radii with $A \geq 40$. It is clearly seen that the $Z^{1/3}$ dependence is superior to the $A^{1/3}$ law. A delicate scattering of data around $R_c/Z^{1/3} = 1.631$ is inferred to arise from isospin effects, and a linear dependence of $R_c/Z^{1/3}$ on N/Z (or $(N-Z)/2$) is found. This inference is well supported by microscopic RCHB calculations for proton-magic Ca, Ni, Zr, Sn, and Pb isotopes, including exotic nuclei close to the neutron drip line, which reproduce available S_{2n} and nuclear charge radii data well. With the linear dependence of the coefficient r_Z^d on N/Z (or $(N-Z)/2$) extracted from both data and RCHB theory, a new isospin-dependent $Z^{1/3}$ formula for nuclear charge radii is proposed, which improves the description of available data for nuclei near the β-stability line and should be very useful for new data obtained for nuclei far from the β-stability line.

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Figure Captions

FIG. 1. The nuclear charge radius data for r_A in $A^{1/3}$ and r_Z in $Z^{1/3}$ law with and without isospin dependence. See text for details.

FIG. 2. Two-neutron separation energies S_{2n} of even Ca, Ni, Zr, Sn, Pb isotopes as a function of N, including data (solid symbols) from Ref. [**?**] and RCHB calculation with a δ-force (open symbols).

FIG. 3. The rms charge radii versus neutron number N for even-even Ca, Ni, Zr, Sn, Pb isotopes. RCHB calculation with δ-force is represented by open symbols, while corresponding data are denoted by solid symbols. Dashed lines represent predictions by the $A^{1/3}$ law with $r_A^d = 1.228$ fm.

FIG. 4. The experimental (solid symbols) and RCHB-predicted (dashed lines) coefficient $r_Z^d = R_c/Z^{1/3}$ for nuclear charge radii as a function of isospin quantity $\eta = N/Z$ in even-even Ca, Ni, Zr, Sn, and Pb isotopes. An asymptotic behavior is drawn as a solid line.

Note: Figure translations are in progress. See original paper for figures.