# Splitting of the — spectrum in a renormalized light-cone QCD-inspired model postprint

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## Abstract

We show that the splitting between the light pseudo-scalar and vector meson states is due to the strong short-range attraction in the 1S0 sector which makes the pion and the kaon light particles. We use a light-cone QCD-inspired model of the mass squared operator with harmonic confinement and a Dirac-delta interaction. We apply a renormalization method to define the model, in which the pseudo-scalar ground state mass fixes the renormalized strength of the Dirac-delta interaction.

## Full Text

## Preamble

**Splitting of the - Spectrum in a Renormalized Light-Cone QCD-Inspired Model**

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**Abstract.** We demonstrate that the splitting between light pseudoscalar and vector meson states arises from strong short-range attraction in the $^1S_0$ sector, which renders the pion and kaon light particles. We employ a light-cone QCD-inspired model for the mass-squared operator with harmonic confinement and

a Dirac-delta interaction, applying a renormalization procedure that fixes the renormalized delta strength using the pseudoscalar ground-state mass.

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## I. Introduction

The effective light-cone QCD theory [**?**, **?**] attempts to describe the Fock-state components of the light-front meson wavefunction for bound constituent quarks, constructed recursively from the lowest Fock component. This lowest Fock component is an eigenfunction of an effective mass-squared operator parameterized in terms of an interaction containing both a Coulomb-like potential and a Dirac-delta term. Both interaction terms originate from effective one-gluon exchange, with the delta term representing the hyperfine interaction. Despite having only the canonical number of free parameters plus one additional parameter—the renormalized strength of the Dirac-delta interaction—this effective theory successfully describes the ground-state masses of pseudoscalar mesons and, in particular, the pion structure [**?**]. However, the model lacks confinement, preventing the study of excited states above the breakup threshold. The next logical step is to introduce a confining interaction, which raises questions about how the model' s elegant renormalization features are modified and what implications this has for the pseudoscalar meson spectrum.

In this work, we extend the light-cone QCD-inspired theory to include confinement and perform renormalization using the light pseudoscalar masses as input to fix the renormalized strength of the Dirac-delta interaction active in the $^1S_0$ channel. We apply this extended model to study the splitting of excited pseudoscalar states from excited $^3S_0$ vector meson states as a function of the ground-state pseudoscalar mass. The $\pi$-$\rho$ mass splitting, arising from the attractive delta interaction, serves as the source of the scalar mass. We show that despite the model' s simplicity, there is reasonable qualitative agreement with experimental values [**?**].

The nontrivial mass splitting between the $\pi$ and $\rho$ meson spectra is a consequence of the small pion mass on the hadronic scale, which fixes the renormalized strength of the Dirac-delta interaction. We emphasize that the renormalized delta strength, fitted to the pion mass, encodes complex short-range physics that makes the pion a strongly bound system. In this initial study, we replace the Coulomb-like and confining interactions with a harmonic oscillator potential, enabling an analytic formulation as we will demonstrate. The confining interaction parameters in the mass-squared operator are fitted to the $^3S_1$ meson ground-state mass and the slope of the excited-state trajectory with principal quantum number [**?**]. We expect that using the detailed Coulomb-like potential and a specific confinement form will have only a small overall effect and will not alter our conclusions regarding the splitting of the pion and kaon spectra relative to their corresponding vector meson spectra.

We should add that the observation of an almost linear relationship between the mass-squared of excited states and $n$, as noted in Ref. [?], connects naturally with our model. The trajectory slope in the $(n, M^2)$ plane defines the harmonic oscillator strength of the mass-squared operator. Here we reveal some of the physics underlying Ref. [?] and demonstrate the relation between the $\pi$ and $\rho$ spectra through the pion mass scale, which defines the model' s renormalization condition.

This paper is organized as follows. In Section II, we review the light-cone QCD-inspired theory for the mass-squared operator in a constituent quark-antiquark system and extend it to include confinement. We demonstrate renormalization using subtracted equations for the model' s transition matrix. In Section III, we present a pedestrian approach to this theory using a harmonic oscillator interaction in the mass-squared operator equation and show results within this model. In Section IV, we offer several remarks on the qualitative aspects of the model and summarize our work.

## II. Extended Light-Cone QCD-Inspired Theory

We extend the renormalized effective QCD theory from Ref. [?] to include confinement. The bare mass operator equation for the lowest light-front Fock component of a bound constituent quark-antiquark system with masses $m_1$ and $m_2$ in the spin-0 channel is given by [?, ?]:

$$M^2\psi(x,\vec{k}_\perp) = \left[\frac{\vec{k}_\perp^2 + m_1^2}{x} + \frac{\vec{k}_\perp^2 + m_2^2}{1-x}\right]\psi(x,\vec{k}_\perp) + \int dx' d\vec{k}'_\perp \left[\frac{\alpha}{Q^2} + \lambda\delta(x-x')\delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp) + W_{\mathrm{conf}}(Q^2)\right]\psi(x',$$

where $M$ is the bound-state mass, $\psi$ is the projection of the light-front wavefunction onto the quark-antiquark Fock state, $W_{\mathrm{conf}}(Q^2)$ represents the confining interaction, $Q$ is the space part of the four-momentum transfer, $\alpha$ is the Coulomb-like potential strength, and $\lambda$ is the bare coupling constant of the Dirac-delta interaction.

For convenience, we transform to the instant-form representation [?, ?]. The Jacobian of the transformation from $(x, \vec{k}_\perp)$ to $\vec{k}$ is:

$$x(k_z) = \frac{E_1 + k_z}{E_1 + E_2}$$

with the dimensionless function and measure:

$$dx d\vec{k}_\perp = m_r A(k) d\vec{k}, \quad A(k) = \frac{E_1 + E_2}{\sqrt{E_1 E_2}}$$

where the reduced mass is $m_r = m_1 m_2/(m_1 + m_2)$. The individual energies are $E_i = \sqrt{m_i^2 + \vec{k}^2}$ for $i = 1, 2$.

The mass operator equation in instant-form momentum variables becomes:

$$M^2\phi(\vec{k}) = [E_1 + E_2]^2\phi(\vec{k}) - \int d\vec{k}' \frac{A(k)A(k')}{[E_1 + E_2]^2 - [E_1' + E_2']^2 + i\epsilon} W_{\text{conf}}(Q^2)\phi(\vec{k}')$$

where $m_s = m_1 + m_2$, the phase-space factor is included in $1/A(k)A(k')$, and $\psi(x, \vec{k}_\perp) = A(k)\phi(\vec{k})$. The momentum transfer is chosen in rotationally invariant form as $Q^2 = (\vec{k} - \vec{k}')^2$.

Bound-state masses come from diagonalizing the mass-squared operator, but the operator is mathematically ill-defined. These masses also correspond to poles of the theory' s Green' s function. Given the need for regularization and renormalization due to the singular interaction, we employ the renormalization method for singular interactions developed in the context of nonrelativistic scattering equations [**?**] to derive the T-matrix and obtain the Green' s function poles.

### A. Mass Operator and Green' s Function

The operator form of the mass-squared equation is:

$$M^2 = M_0 + V + V_\delta + V_{\text{conf}}$$

where the free mass operator $M_0 = E_1 + E_2$ is the sum of quark energies, $V$ is the Coulomb-like potential, $V_\delta$ is the short-range singular interaction, and $V_{\text{conf}}$ provides quark confinement. The matrix elements follow from the instant-form equation.

Due to the confining interaction, the complete basis states for the renormalized theory are eigenstates of the mass-squared operator without the singular interaction:

$$[M_0 + V + V_{\text{conf}}]|\phi_n\rangle = M_n^2|\phi_n\rangle$$

where $M_n^2$ is the eigenvalue and $n$ denotes the quantum numbers.

Our renormalization procedure uses Green' s function methods. The free Green' s function is:

$$G_0(M^2) = \frac{1}{M^2 - (M_0 + V + V_{\text{conf}}) + i\epsilon}$$

The poles of $G_0$ give the eigenvalues of the confining Hamiltonian. The complete Green' s function includes the delta potential and satisfies:

$$G(M^2) = G_0(M^2) + G_0(M^2)V_\delta G(M^2)$$

The poles of $G(M^2)$ on the real axis are the eigenvalues of the full mass-squared operator.

### B. Renormalized T-Matrix

Analogous to scattering theory, we introduce the transition matrix from which the full Green' s function derives:

$$G(M^2) = G_0(M^2) + G_0(M^2)T_R(M^2)G_0(M^2)$$

The matrix elements of the renormalized transition matrix $T_R(M^2)$ satisfy the subtracted Lippmann-Schwinger equation [**?**, **?**]:

$$T_R(M^2) = T_R(\mu^2) + T_R(\mu^2)[G_0(M^2) - G_0(\mu^2)]T_R(M^2)$$

The subtraction point $\mu^2$ is arbitrary and can be moved without affecting physical results, as $T_R(\mu^2)$ changes according to the Callan-Symanzik equation.

To solve this, we need $T_R(\mu^2)$. Given the separable structure of the singular interaction, we write:

$$T_R(\mu^2) = \lambda_R(\mu^2)|\eta\rangle\langle\eta|$$

where $\lambda_R(\mu^2)$ is the renormalized delta strength, fixed by the pseudoscalar $^1S_0$ meson ground state.

The solution of the subtracted equation with this ansatz is:

$$T_R(M^2) = \frac{|\eta\rangle t_R(M^2)\langle\eta|}{1 - \langle\eta|G_0(M^2) - G_0(\mu^2)|\eta\rangle t_R(M^2)}$$

where the reduced matrix element is:

$$t_R(M^2) = \lambda_R(\mu^2) + \lambda_R(\mu^2)\langle\eta|G_0(M^2) - G_0(\mu^2)|\eta\rangle t_R(M^2)$$

Divergences in the sum are exactly canceled by differences between free Green' s functions. The masses are given by zeros of $t_R(M^2)$, which defines the zero angular momentum bound states.

The ground-state pion or other pseudoscalar mesons provide input to define the reduced matrix element. Choosing the subtraction point at the pseudoscalar ground-state mass (e.g., the pion mass) makes the renormalized strength zero, yielding a zero at mass $\mu$. This physical input can be varied continuously to study $^1S_0$ and $^3S_0$ spectrum splitting.

## III. Splitting of the Spectrum in a Pedestrian Approach

For analytic treatment while preserving physical content, we use equal-mass constituent quarks and simplify the equation to:

$$[\mathcal{H}_{\text{h.o.}} + g\delta(\vec{r})]\,\phi(\vec{r}) = M^2\phi(\vec{r})$$

where $g$ is the bare delta strength and the harmonic oscillator mass-squared operator is:

$$\mathcal{H}_{\text{h.o.}} = 4(\vec{k}^2 + m_q^2) + \omega^2 r^2 + a$$

in units $\hbar = c = 1$.

The eigenvalue equation becomes:

$$\left[4(\vec{k}^2 + m_q^2) + \omega^2 r^2 + a\right]\Psi_n(\vec{r}) = M_n^2\Psi_n(\vec{r})$$

where $n$ is the principal quantum number for s-wave excited states and $\Psi_n(\vec{r})$ are harmonic oscillator eigenstates.

The nearly linear $M^2$ vs. $n$ relationship observed in Ref. [**?**] connects naturally to our model. The trajectory slope in the $(n, M^2)$ plane defines $\omega$ for each system, while the ground-state mass determines $a$. The constituent quark mass $m_q$ comes from the vector meson ground-state mass. In this approach, we treat up-down and strange quark masses as degenerate, fitting each case separately. The eigenvalues are:

$$M_n^2 = M_{\text{v.g.s.}}^2 + 2\omega n$$

where $M_{\text{v.g.s.}}$ is the $^3S_1$ meson ground-state mass.

The reduced T-matrix becomes:

$$t_R(M^2) = \frac{g}{1 - g\sum_n \frac{|\Psi_n(0)|^2}{M^2 - M_n^2}}$$

The s-wave eigenfunction at the origin is:

$$\Psi_n(0) = \left(\frac{\alpha}{\pi}\right)^{3/4}\frac{1}{\sqrt{(2n+1)!!}}$$

where $\alpha^{-1}$ is the oscillator length.

The final reduced T-matrix for our pedestrian model is:

$$t_R(M^2) = \frac{(2\pi\alpha)^{3/2}}{\sum_n \frac{1}{(2n+1)!!}\left[\frac{1}{M^2 - M^2_{\text{v.g.s.}} - 2\omega n} - \frac{1}{\mu^2 - M^2_{\text{v.g.s.}} - 2\omega n}\right]}$$

Zeros of this equation give the eigenvalues of the mass-squared operator.

Model parameters are $\omega$, the ${}^3S_1$ meson ground-state mass, and the renormalized delta strength. We have the same number of parameters as the original model [**?**]: the canonical 7 ($\alpha$ and 6 constituent quark masses) plus one–the renormalized delta strength. Below we present numerical results for $\pi$-$\rho$ and $K$-$K^*$ spectrum splitting due to the delta interaction in the pseudoscalar ${}^1S_0$ channel.

[Figure 1: see original paper] shows our results for $\pi$-$\rho$ mass splitting for the first six levels as a function of ground-state pseudoscalar mass $\mu$, interpolating from pion to rho meson spectra. The parameter $\omega = 1.39$ GeV$^2$ comes from Ref. [**?**]. While the ground state shows large splitting, this diminishes for excited states, though the model consistently gives ${}^1S_0$ masses below corresponding ${}^3S_1$ masses. The $\rho(1700)$ is suggested to be a D-wave meson [**?**] and does not fit well in our ${}^3S_1$ resonance picture.

Although the kaon trajectory is more uncertain [**?**], we address ${}^1S_0$ and ${}^3S_1$ excited-state splitting to verify model consistency. Figure 2 shows results for kaon states $K$ and $K^*$, confirming the pattern observed in Figure 1: large ground-state mass differences translate to much smaller differences for excited states, with ${}^1S_0$ masses consistently below corresponding ${}^3S_1$ masses. The kaon oscillator parameter, $\omega = 1.19$ GeV$^2$, is fitted to the mass-squared difference between $K^*(1410)$ and $K^*(892)$.

## IV. Remarks on the Qualitative Aspects of the Model and Summary

The pseudoscalar mass spectrum represents a true challenge to theoretical modeling beyond physics described by confinement alone [**?**]. The $\pi$-$\rho$ mass difference and the huge splitting between the $\pi$ and its first resonant state $\pi(1300)$ must arise from potential parameters fitted with a hyperfine (short-range) interaction specifically tuned to the pion mass. Successful description of splitting among $\pi$, $K$, $K^*$ pairs depends crucially on the short-range structure of the S-wave hyperfine potential, as concluded in the seminal work by Godfrey and Isgur [**?**]. The work of Anisovich, Anisovich, and Sarantsev [**?**] showing meson trajectories in the $(n, M^2)$ and $(J, M^2)$ planes brought phenomenological order to the meson spectrum, though the underlying physics remained unclear. We believe the present work reveals some of that physics.

Our model' s main ingredients are: dynamics described by a mass-squared operator and the Dirac-delta interaction that brings the small pion mass into the model. We note that attractive short-range interactions have been extensively studied in meson structure and spectrum. The Godfrey-Isgur model was used to construct light-cone wavefunctions for pion, rho-meson, and nucleon [?], with a high-momentum tail above 1 GeV/c attributed to short-range attraction. Impact on pion and rho structure was tested in electroweak observables with some success [?], though existing data were not sensitive enough for definitive conclusions about hard constituent quarks [?].

Another covariant quark model approach using the instantaneous Bethe-Salpeter equation [?] also emphasized short-range attraction' s importance for large mass splittings in the pseudoscalar ($J = 0$) sector, attributed to the two-body point-like part of ' t Hooft' s instanton-induced $U_A(1)$ symmetry breaking interaction [?].

The pion light-cone wavefunction has been probed experimentally via diffractive dissociation of 500 GeV/c $\pi^-$ into dijets, providing the first direct measurement of the pion valence-quark momentum distribution [?]. Results give a stringent test of the asymptotic wavefunction [?], matching the high-momentum tail of the pion wavefunction [?]. This experiment supports our model' s singular interaction.

From these examples, the necessity of strong short-range attraction to build the pion is clear, both in light pseudoscalar meson spectrum modeling and recent pion valence wavefunction measurements. The Dirac-delta interaction parameterizes the complex short-range physics of QCD, brought into the model via the small pion mass. The splitting of pion and kaon spectra from their ${}^3S_1$ counterparts is a direct consequence of the delta interaction and its renormalization, the structure of the mass-squared operator with confinement, and all implied relativistic effects.

In summary, we extended the renormalized light-cone QCD-inspired effective theory to include confinement in the mass-squared operator with a Dirac-delta interaction. Using a pedestrian harmonic oscillator approach, we derived an algebraic equation whose solutions give ${}^1S_0$ state masses as a function of the pseudoscalar meson ground-state mass. The model naturally accommodates large ground-state splitting between light ${}^1S_0$ and ${}^3S_1$ mesons and its suppression in excited states, in qualitative agreement with data. Simultaneously, it provides a sound physical basis for understanding $q\bar{q}$ state systematics in the $(n, M^2)$ and $(J, M^2)$ planes. Thus, extending the light-cone QCD-inspired model to include confinement while preserving its simplicity and renormalizability yields a reasonable picture of the light meson spectrum.

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[Figure 1: see original paper] Mass of the excited $q\bar{q}$ state ($M$) as a function of the pseudoscalar meson ground-state mass ($\mu$) for $I = 1$. Numerical results for $M$ come from zeros of Eq. (25). Experimental data are from Ref. [**?**]. Data on the left and right of the figure correspond to $^1S_0$ and $^3S_1$ mesons, respectively.

[Figure 2: see original paper] Mass of the excited $q\bar{q}$ state ($M$) as a function of the pseudoscalar meson ground-state mass ($\mu$) for $I = 1/2$ in the strange sector. Numerical results for $M$ come from zeros of Eq. (25). Experimental data are from Ref. [**?**]. Data on the left and right of the figure correspond to $^1S_0$ and $^3S_1$ mesons, respectively.

*Note: Figure translations are in progress. See original paper for figures.*

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