Study of Proton Magic Even-Even Isotopes and Giant Halos of Ca Isotopes with Relativistic Continuum Hartree-Bogoliubov Theory Postprint

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Abstract

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Full Text

Preamble

Study of Proton Magic Even-Even Isotopes and Giant Halos of Ca Isotopes with Relativistic Continuum Hartree-Bogoliubov Theory

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We study the proton magic O, Ca, Ni, Zr, Sn, and Pb isotope chains from the proton drip line to the neutron drip line with the relativistic continuum Hartree-Bogoliubov (RCHB) theory. Particularly, we investigate in detail the properties of even-even Ca isotopes due to the appearance of giant halos in neutron-rich Ca nuclei near the neutron drip line. The RCHB theory reproduces the experimental binding energies E_b and two-neutron separation energies S_{2n} very well. The predicted neutron drip-line nuclei are ^{28}O, ^{72}Ca, ^{98}Ni, ^{136}Zr, ^{176}Sn, and ^{266}Pb, respectively. Halo and giant halo properties predicted in Ca isotopes with $A > 60$ are investigated in detail through analysis of two-neutron separation energies, nucleon density distributions, single-particle energy levels, and the occupation probabilities of energy levels including continuum states. The spin-orbit splitting and the diffuseness of nuclear potential in these Ca isotopes are also studied. Furthermore, we examine the neighboring lighter isotopes in the drip-line Ca region and find some possibility of giant halo nuclei in the Ne-Na-Mg drip-line nuclei.

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I. INTRODUCTION

The study of exotic nuclei, so-called due to their large N/Z ratios (isospin) and interesting properties such as halo and skin, has attracted worldwide attention [**?**, **?**]. With recent developments in accelerator technology and detection techniques, it has become possible to produce these exotic nuclei and study their detailed properties using radioactive ion beam (RIB) facilities. Since Tanihata et al. discovered the first case of halo in an exotic nucleus ^{11}Li with RIB in 1985 [**?**], more and more exotic nuclei have been investigated with various modern experimental methods to better understand this attractive phenomenon.

For nuclei far from the β-stability valley with small nucleon separation energy, the valence nucleons in exotic nuclei extend over quite a wide space to form low-density nuclear matter. It is expected that the "halo" in exotic nuclei provides an interesting case to study the nuclear environment relevant to astrophysics in the laboratory. Very neutron-rich nuclei, particularly those near the neutron drip lines and near closed shells, play an important role in nuclear astrophysics. Their properties such as binding energies, neutron separation energies, deformation parameters, etc., strongly affect the way neutron-rich stable isotopes are formed in nature by the so-called r-process.

Furthermore, exotic nuclei are expected to exhibit other interesting phenomena such as the disappearance of traditional shell gaps and the occurrence of new shell gaps, which result in new magic numbers. Reference [?] has reported that the new magic number $N = 16$ appears in the light neutron drip-line region. Ozawa et al. have suggested a mechanism for the formation of new magic numbers, which is intimately related to neutron halo formation and is common in nuclei near the neutron drip line.

It is very helpful to use a self-consistent microscopic model to study the properties of exotic nuclei. During the past two decades, the relativistic mean field (RMF) theory has received wide attention because it is very successful in describing many nuclear phenomena for nuclei both far from stability and stable. Compared with non-relativistic mean field theory, RMF can reproduce the correct nuclear incompressibility coefficient and saturation properties (Coester line) in nuclear matter and naturally gives the spin-orbit coupling potential. Reviews of RMF theory are given in Refs. [?, ?, ?]. The starting point of RMF theory is a Lagrangian that describes nucleons as Dirac spinors moving in a mean field composed of interactions between nucleons (protons and neutrons) and mesons (σ, ω, ρ), plus the Coulomb field. From this viewpoint, RMF theory is more microscopic in the sense of describing the nuclear system at the meson level.

Usually for exotic nuclei, their Fermi surfaces are very close to the continuum threshold. In these cases, the valence nucleons could be easily scattered to continuum states due to pairing correlation. Thus, theories that can properly handle both pairing and continuum states are needed to describe the properties of exotic nuclei. For the pairing interaction, the simple BCS method and general Bogoliubov quasi-particle transformation are two candidates. The former is very useful and successful for stable nuclei; however, when extended to exotic nuclei, the occupation probability becomes finite for those continuum states and involves unphysical fermion gas. In this case, we should use the Bogoliubov transformation to handle pairing correlation in exotic nuclei instead of the simple BCS method. Based on RMF theory, the relativistic Hartree-Bogoliubov (RHB) equation can be derived by quantizing the meson fields as well as the nucleon fields in the Lagrangian density [?]. Furthermore, in order to describe self-consistently both continuum and bound states and the coupling between them, the RHB theory must be solved in coordinate space, i.e., the newly developed Relativistic Continuum Hartree-Bogoliubov (RCHB) theory [?, ?].

The RCHB theory has been extremely successful in describing the ground-state properties of nuclei both near and far from the β-stability line. A remarkable success of RCHB theory is the new interpretation of the halo in ^{11}Li [?] and the prediction of the exotic phenomenon of giant halos in Zr ($A > 122$) isotopes [?]. Giant halos are very interesting phenomena in exotic nuclei because the halos are formed by more than two neutrons scattered as Cooper pairs to the continuum region. However, the exotic Zr ($Z = 40$) isotopes with $N > 82$ are rather heavy for observation by present RIB facilities. With current RIB techniques, light drip-line nuclei are more likely to be accessed through transfer reactions or other reaction mechanisms. It is thus extremely valuable for us to investigate giant halo phenomena in lighter nuclei like Ca ($Z = 20$) isotopes [?].

The calcium isotope chains have received much attention due to rich experimental results on binding energy, density distribution, single-particle energy, radius, etc. Though those data are now limited near the stability line, it is useful to calculate these quantities to test microscopic theory against future experiments. Also, shell effects exist in this chain due to the short shell period as the traditional magic numbers are $N = 20, 28, 50$ and the sub-magic number $N = 40$. The properties of Ca isotopes have been investigated with various methods based on mean field theory, e.g., the non-relativistic Hartree-Fock (HF) and Hartree-Fock-Bogoliubov (HFB) method [?], the Skyrme Hartree-Fock (SHF) method [?], relativistic density-dependent Hartree-Fock method [?], and self-consistent Hartree-Fock calculation plus random-phase approximation [?]. We apply here the RCHB theory to investigate the ground-state properties of the Ca isotope chain, especially probing the halo properties in exotic nuclei near the neutron drip line.

In a previous letter [?], we briefly reported halos discovered in Ca isotopes near the drip-line region with RCHB theory. Here we give details of the investigation of the mechanism for the appearance of giant halos. Besides the detailed discussion on Ca isotopes, the ground-state properties of other proton magic isotope chains (O, Ni, Zr, Sn, and Pb) are discussed in this article. The paper is organized as follows: In Sec. II, a brief outline of RCHB theory is given. In Sec. III, we provide numerical results for these proton magic even-even nuclei and various properties for Ca isotopes, such as two-neutron separation energies S_{2n}, radii, density distributions, single-particle energy levels, contributions from continua, spin-orbit splitting, and potential diffuseness. In Sec. IV, the prospects for theoretical progress and experimental expectations of giant halo nuclei are reviewed. Finally, Sec. V summarizes our main results.

II. RELATIVISTIC CONTINUUM HARTREE-BOGOLIUBOV THEORY

The RCHB theory, which is the extension of RMF theory with Bogoliubov transformation in coordinate representation, was suggested in Ref. [?], and its detailed formalism and numerical solution for a particular case can be found in Ref. [?] and references therein. The basic ansatz of RMF theory is a Lagrangian density whereby nucleons are described as Dirac particles that interact via the exchange of various mesons and the photon. The mesons are the scalar sigma (σ), vector omega (ω), and isovector vector rho ($\vec{\rho}$). The rho ($\vec{\rho}$) meson provides the necessary isospin asymmetry. The scalar sigma meson moves in a self-interacting field having cubic and quadratic terms with strengths g_2 and g_3, respectively. The Lagrangian then consists of the free baryon and meson parts and the interaction part with minimal coupling, together with the nucleon mass M, and m_σ, g_σ, m_ω, g_ω, m_ρ, g_ρ the masses and coupling constants of the respective mesons:

$$L = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - U(\sigma) - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma^\mu\omega_\mu\psi - \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\vec{\rho}_\mu - g_\rho\bar{\psi}$$

The field tensors for the vector mesons are given as:

$$\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu,$$

$$\vec{R}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu - g_\rho(\vec{\rho}_\mu \times \vec{\rho}_\nu),$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu.$$

For a realistic description of nuclear properties, nonlinear self-coupling for the scalar mesons turns out to be crucial [**?**]:

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4.$$

The classical variation principle gives the following equations of motion:

$$[\vec{\alpha}\cdot\vec{p} + V_V(\vec{r}) + \beta(M + V_S(\vec{r}))]\psi_i = \epsilon_i\psi_i$$

for the nucleon spinors and

$$(-\Delta + m_\sigma^2)\sigma(\vec{r}) = g_\sigma\rho_s(\vec{r}) - g_2\sigma^2(\vec{r}) - g_3\sigma^3(\vec{r}),$$

$$(-\Delta + m_\omega^2)\omega^\mu(\vec{r}) = g_\omega j^\mu(\vec{r}),$$

$$(-\Delta + m_\rho^2)\vec{\rho}^\mu(\vec{r}) = g_\rho\vec{j}^\mu(\vec{r}),$$

$$-\Delta A^\mu(\vec{r}) = e j_{\mathrm{em}}^\mu(\vec{r}),$$

for the mesons, where

$$V_V(\vec{r}) = g_\omega\omega^0(\vec{r}) + g_\rho\vec{\tau}\vec{\rho}^0(\vec{r}) + e\frac{1-\tau_3}{2}A^0(\vec{r}),$$

$$V_S(\vec{r}) = g_\sigma \sigma(\vec{r}),$$

are the vector and scalar potentials respectively, and the source terms for the mesons are

$$j^\mu(\vec{r}) = \sum_{i=1}^{A} \bar{\psi}_i \gamma^\mu \psi_i,$$

$$\vec{j}^\mu(\vec{r}) = \sum_{i=1}^{A} \bar{\psi}_i \gamma^\mu \vec{\tau} \psi_i,$$

$$j^\mu_{\mathrm{em}}(\vec{r}) = \sum_{i=1}^{A} \bar{\psi}_i \gamma^\mu \frac{1-\tau_3}{2} \psi_i,$$

$$\rho_s(\vec{r}) = \sum_{i=1}^{A} \bar{\psi}_i \psi_i,$$

where the summations are over the valence nucleons only.

It should be noted that, as usual, the present approach neglects the contribution of negative energy states, i.e., the no-sea approximation, which means that the vacuum is not polarized. The coupled equations are nonlinear quantum field equations, and their exact solutions are very complicated. Thus, the mean-field approximation is generally used: the meson field operators are replaced by their expectation values, and the nucleons move independently in the classical meson fields. The coupled equations are self-consistently solved by iteration.

For spherical nuclei, i.e., systems with rotational symmetry, the potential for the nucleon and the sources of meson fields depend only on the radial coordinate r, characterized by the angular momentum quantum numbers l, j, m, and the isospin $t = \pm\frac{1}{2}$ for neutron and proton respectively, plus other quantum numbers i. The Dirac spinor has the form:

$$\psi(\vec{r}) = \begin{pmatrix} G^i_{lj}(r) Y_{ljm}(\theta, \phi) \\ i F^i_{lj}(r)(\vec{\sigma} \cdot \hat{r}) Y_{ljm}(\theta, \phi) \end{pmatrix} \chi_t(t),$$

where $Y_{ljm}(\theta, \phi)$ are the spinor spherical harmonics and $G^i_{lj}(r)$ and $F^i_{lj}(r)$ are the radial wave functions for upper and lower components. They are normalized according to

$$\int dr(|G^i_{lj}(r)|^2 + |F^i_{lj}(r)|^2) = 1.$$

The radial equation of the spinor can be reduced as:

$$\epsilon_i G^i_{lj}(r) = \left(-\frac{d}{dr} + \frac{\kappa}{r}\right) F^i_{lj}(r) + (M + V_S(r) + V_V(r)) G^i_{lj}(r),$$

$$\epsilon_i F^i_{lj}(r) = \left(\frac{d}{dr} + \frac{\kappa}{r}\right) G^i_{lj}(r) - (M + V_S(r) - V_V(r)) F^i_{lj}(r),$$

where

$$\kappa = \begin{cases} -(j+1/2) & \text{for } j = l + 1/2, \\ +(j+1/2) & \text{for } j = l - 1/2. \end{cases}$$

The meson field equations become simple radial Laplace equations of the form:

$$\left(\frac{d^2}{dr^2} - m_\phi^2\right) \phi(r) = s_\phi(r),$$

where m_ϕ are the meson masses for $\phi = \sigma, \omega, \rho$ and for photon ($m_\phi = 0$). The source terms are:

$$s_\phi(r) = \begin{cases} -g_\sigma \rho_s(r) - g_2 \sigma^2(r) - g_3 \sigma^3(r) & \text{for the } \sigma \text{ field}, \\ g_\omega \rho_v(r) & \text{for the } \omega \text{ field}, \\ g_\rho \rho_3(r) & \text{for the } \rho \text{ field}, \\ e\rho_c(r) & \text{for the Coulomb field}, \end{cases}$$

with

$$4\pi r^2 \rho_s(r) = \sum_{i=1}^{A} (|G_i(r)|^2 - |F_i(r)|^2),$$

$$4\pi r^2 \rho_v(r) = \sum_{i=1}^{A} (|G_i(r)|^2 + |F_i(r)|^2),$$

$$4\pi r^2 \rho_3(r) = \sum_{p=1}^{Z} (|G_p(r)|^2 + |F_p(r)|^2) - \sum_{n=1}^{N} (|G_n(r)|^2 + |F_n(r)|^2),$$

$$4\pi r^2 \rho_c(r) = \sum_{p=1}^{Z} (|G_p(r)|^2 + |F_p(r)|^2).$$

The Laplace equation can in principle be solved by the Green' s function:

$$\phi(r) = \int r'^2 dr' G_\phi(r,r') s_\phi(r'),$$

where for massive fields

$$G_\phi(r,r') = \frac{1}{rr'}\left(e^{-m_\phi|r-r'|} - e^{-m_\phi|r+r'|}\right),$$

and for the Coulomb field

$$G_\phi(r,r') = \begin{cases} 1/r & \text{for } r > r', \\ 1/r' & \text{for } r < r'. \end{cases}$$

Equations (10) and (11) could be solved self-consistently in the usual RMF approximation. For RMF, however, as the classical meson fields are used, the equations of motion for nucleons derived from Eq. (1) do not contain pairing interaction. In order to include pairing interaction, one must quantize the meson fields, which leads to a Hamiltonian with two-body interaction. Following the standard procedure of Bogoliubov transformation, a Dirac Hartree-Bogoliubov equation can be derived, allowing a unified description of mean field and pairing correlation in nuclei. For details, see Ref. [?] and references therein. The RHB equations are as follows:

$$\begin{pmatrix} h-\lambda & \Delta \\ -\Delta^* & -h^*+\lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix},$$

where

$$h(\vec{r},\vec{r}') = [\vec{\alpha}\cdot\vec{p} + V_V(\vec{r}) + \beta(M + V_S(\vec{r}))]\delta(\vec{r},\vec{r}')$$

is the Dirac Hamiltonian and the Fock term has been neglected as is usually done in RMF.

The pairing potential is:

$$\Delta_{kk'}(\vec{r},\vec{r}') = \frac{1}{2}\sum_{\tilde{k}\tilde{k}'} V_{kk',\tilde{k}\tilde{k}'}(\vec{r},\vec{r}';\vec{r}_1,\vec{r}_1')\kappa_{\tilde{k}\tilde{k}'}(\vec{r}_1,\vec{r}_1'),$$

obtained from one-meson exchange interaction $V_{kk',\tilde{k}\tilde{k}'}(\vec{r},\vec{r}';\vec{r}_1,\vec{r}_1')$ in the pp-channel and the pairing tensor $\kappa = V^* U^T$.

The nuclear density is:

$$\rho(\vec{r},\vec{r}') = V^*(\vec{r})\psi_{ilj}V(\vec{r}').$$

As in Ref. [?], $V_{kk',\tilde{k}\tilde{k}'}$ used for the pairing potential in Eq. (19) is either the density-dependent two-body force of zero range with interaction strength V_0 and nuclear matter density ρ_0:

$$V(\vec{r}_1, \vec{r}_2) = V_0 \delta(\vec{r}_1 - \vec{r}_2) \left[1 - \sigma_1 \cdot \sigma_2\right],$$

or Gogny-type finite-range force with parameters μ_i, W_i, B_i, H_i and M_i $(i = 1, 2)$ as the finite-range part of the Gogny force [?]:

$$V(\vec{r}_1, \vec{r}_2) = \sum_{i=1,2} e^{-((\vec{r}_1 - \vec{r}_2)/\mu_i)^2} \left(W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau\right).$$

A Lagrange multiplier λ is introduced to fix the particle number for neutron and proton as $N = \mathrm{Tr}\rho_n$ and $Z = \mathrm{Tr}\rho_p$.

In order to describe both continuum and bound states self-consistently, the RHB theory must be solved in coordinate representation, i.e., the Relativistic Continuum Hartree-Bogoliubov (RCHB) theory [?]. It is then applicable to both exotic and normal nuclei. In Eq. (17), the spectrum of the system is unbound from above and below the Fermi surface, and the eigenstates occur in pairs of opposite energies. When spherical symmetry is imposed on the solution of the RCHB equations, the wave function can be conveniently written as

$$U(\vec{r}) = \begin{pmatrix} G_{lj}^U(r) Y_{ljm}(\theta, \phi) \\ i F_{lj}^U(r) (\vec{\sigma} \cdot \hat{r}) Y_{ljm}(\theta, \phi) \end{pmatrix} \chi_t(t),$$

$$\psi_i^V = \begin{pmatrix} G_{lj}^V(r) Y_{ljm}(\theta, \phi) \\ i F_{lj}^V(r) (\vec{\sigma} \cdot \hat{r}) Y_{ljm}(\theta, \phi) \end{pmatrix} \chi_t(t).$$

The above equation depends only on radial coordinates and can be expressed as the following integro-differential equation:

$$\frac{dG^U(r)}{dr} - \frac{\kappa}{r} G^U(r) - (E + \lambda - V_V(r) + V_S(r)) F^U(r) + \int r' dr' \Delta(r, r') F^V(r') = 0,$$

$$\frac{dF^U(r)}{dr} + \frac{\kappa}{r} F^U(r) + (E + \lambda - V_V(r) - V_S(r)) G^U(r) + \int r' dr' \Delta(r, r') G^V(r') = 0,$$

$$\frac{dG^V(r)}{dr} - \frac{\kappa}{r} G^V(r) - (E - \lambda + V_V(r) + V_S(r)) F^V(r) + \int r' dr' \Delta(r, r') F^U(r') = 0,$$

$$\frac{dF^V(r)}{dr}+\frac{\kappa}{r}F^V(r)+(E-\lambda+V_V(r)-V_S(r))G^V(r)+\int r'dr'\Delta(r,r')G^U(r')=0,$$

where the nucleon mass is included in the scalar potential $V_S(r)$. Equation (25), in the case of δ-force given in Eq. (22), reduces to normal coupled differential equations and can be solved with the shooting method using Runge-Kutta algorithms. For the case of Gogny force, the coupled integro-differential equations are discretized in space and solved by finite element methods (see Ref. [?]). Instead of solving Eqs. (10) and (11) self-consistently for the RMF case, we now have to solve Eqs. (25) and (11) self-consistently for the RCHB case.

As the calculation for Gogny force is very time-consuming, we use it for one nucleus and fix the interaction strength of the δ-force given in Eq. (22).

III. RESULTS AND DISCUSSIONS

A. Binding Energies and Two-Neutron Separation Energies

The numerical techniques of RCHB theory can be found in Ref. [?] and references therein. In the present calculations, we follow the procedures in Ref. [?] and solve the RCHB equations in a box with size $R = 20$ fm and step size 0.1 fm. The parameter set NL-SH [?] is used, which aims to describe both stable and exotic nuclei. The use of the TM1 parameter set provides similar results, and we show here only those of the NL-SH parameter [?]. The density-dependent δ-force in the pairing channel with $\rho_0 = 0.152$ fm^{-3} is used, and its strength $V_0 = 650$ MeV·fm^{-3} is fixed by the Gogny force as in Ref. [?]. The contribution from continua is restricted within a cutoff energy $E_{\text{cut}} \sim 120$ MeV.

In this work, the RCHB calculation is restricted to spherical shape, which is a good approximation for most proton magic nuclei with $Z = 8, 20, 28, 50, 82$. The RCHB code was also applied to Zr isotopes, considering the sub-magic number $Z = 40$ and the fact that most investigations with non-relativistic codes and relativistic RMF+BCS codes show that nuclei above ^{122}Zr are spherical [?].

We calculated ground-state properties of all even-even O, Ca, Ni, Zr, Sn, and Pb isotopes ranging from the proton drip line to the neutron drip line with the RCHB code. We present the binding energy E_b calculated from the RCHB method for these isotope chains and corresponding available data [?] in the last two columns of Tables I-IV. We do not include tables for Sn and Pb isotopes here to save space. The difference (ΔE_b) between experimental and calculated binding energies is less than 3.5 MeV for most nuclei, which is less than 1% of the experimental values. Large differences ($\Delta E_b \sim 10$ MeV) for some Zr isotopes are found, mainly due to deformation.

The two-neutron separation energy S_{2n}, defined as $S_{2n} = E_b(Z, A+2) - E_b(Z, A)$, is quite a sensitive quantity for testing microscopic theory. The two-neutron separation energy becomes negative when the nucleus becomes unstable against two-neutron emission. Hence, the drip-line nucleus for a given isotope chain is the one before the nucleus with negative S_{2n}. In Fig. 1 [Figure 1: see original paper], both theoretical and available experimental S_{2n} are presented as a function of neutron number N for the O, Ca, Ni, Zr, Sn, and Pb isotope chains. Good agreement between experiment and calculation is clearly seen.

FIG. 1. The two-neutron separation energies S_{2n} for proton magic isotopes are plotted as a function of neutron number. The panels from left to right and top to bottom are for O, Ca, Ni, Zr, Sn, and Pb, respectively. Experimental values are denoted by solid symbols and calculated ones by open symbols.

TABLE I. Ground-state properties of even O isotopes calculated with RCHB theory using the NL-SH parameter. Listed are neutron (r_n), proton (r_p), matter (r_m), and charge (r_c) root mean square radii, as well as binding energies E_b. Corresponding experimental values r_c and E_b are included when available.

TABLE II. Same as Table I, but for even Ca isotopes.

TABLE III. Same as Table I, but for even Ni isotopes.

TABLE IV. Same as Table I, but for even Zr isotopes.

From Fig. 1 and Tables I-IV, the proton magic even-mass nuclei at the neutron drip line are predicted as ^{28}O, ^{72}Ca, ^{98}Ni, ^{136}Zr, ^{174}Sn, and ^{266}Pb, respectively. Of these neutron drip-line nuclei, we have experimental information only for the O isotopes. Experimental efforts were made to investigate ^{26}O [**?**, **?**] and ^{28}O [**?**] to determine whether they are bound. These nuclei were found to be unstable, and the heaviest O isotope was concluded to be ^{24}O. Hence, the present calculation is not successful in reproducing the O neutron drip line.

Numerous theoretical studies of binding energies of neutron-rich O nuclei have been made [**?**], but these calculations failed to reproduce that ^{24}O is the drip-line nucleus, as is the case for RCHB with NL-SH presented here. It is known that in relativistic mean field theory, one parameter set is used to describe all nuclei in the nuclear chart, which is quite a challenge. In fact, it is well known that mean field theory is difficult for light nuclei. Therefore we have NL1, NL-SH, NL3, and TM1 for heavy systems and NL2 and TM2 for light systems. The heavy oxygen isotopes lie just at the borderline between light and heavy groups. Therefore, it is not surprising that NL-SH, NL1, NL2, NL3, TM1, TM2, and their non-relativistic counterparts mostly do not correctly predict the neutron drip line, which is already known experimentally to be at ^{24}O.

One might suspect that the drip-line nucleus for Ca is predicted to be ^{70}Ca instead of ^{72}Ca. Most calculations predict Ca isotopes with $N > 50$ would be unbound, as additional neutrons should fill the continuum region above the $N = 50$ shell. For example, in Refs. [**?**, **?**], the drip-line nucleus is predicted as ^{70}Ca using HFB and Skyrme HF methods, respectively.

The reason for the unexpected bound nucleus ^{72}Ca is caused by the halo effect, which also results in the disappearance of the normal $N = 50$ magic number in the RCHB calculation. Looking along the S_{2n} vs. N curve in Fig. 1, strong kinks can be clearly seen at $N = 8$ for O, $N = 20, 28, 40$ for Ca, $N = 28, 50$ for Ni, $N = 50, 82$ for Zr, and $N = 126$ for Pb isotope chains. All these numbers correspond to neutron magic numbers, which arise from large gaps in single-particle energy levels. However, no kink appears at the $N = 50$ magic number for Ca isotopes in Fig. 1.

In stable nuclei, the $N = 50$ magic number is given by the large energy gap between the $1g_{9/2}$ and $3s, 2d$ levels. However, in the vicinity of the drip-line region of Ca isotopes, the single-particle energy of $3s_{1/2}$ decreases due to the halo effect and its zero centrifugal potential barrier, causing the large gap between $1g_{9/2}$ and $3s_{1/2}$ to disappear. Therefore, the nucleus ^{70}Ca is no longer a double-magic nucleus, and furthermore ^{72}Ca is also bound. This disappearance of the $N = 50$ magic number at the neutron drip line is due to the halo property of the neutron density with spherical shape being maintained, different from the disappearance of the $N = 20$ magic number due to deformation (the unbound ^{26,28}O mentioned above). Most recently, a new magic number $N = 16$ has been discovered in the neutron drip-line light-nuclei region, which is also considered to be due to the halo effect [**?**]. It can be expected that near the drip line, more disappearances of traditional magic numbers and regenerations of new magic numbers would be found with the same mechanism.

Another remarkable phenomenon in Fig. 1 is that the S_{2n} values for exotic Ca isotopes near the neutron drip line are very close to zero in the large mass region, i.e., $S_{2n} \approx 2.06, 1.24, 0.76, 0.43, 0.21, 0.04$ MeV for $A = 62, 64, 66, 68, 70, 72$ isotopes, respectively. If one regards ^{60}Ca as a core, then the several valence neutrons occupying levels above the $N = 40$ sub-shell for these $A > 60$ exotic nuclei are all weakly bound and can be easily scattered to continuum levels due to pairing interaction, especially for $^{66-72}$Ca. This case is very similar to S_{2n} in Zr isotopes with $N > 82$ (see Fig. 1) [**?**]. Note, however, that for Sn isotopes in the vicinity of the drip line [**?**], S_{2n} decreases rather fast with mass number, and for Ni isotopes [**?**], S_{2n} are quite large (~ 2 MeV) and undergo a sudden drop to negative values at the drip-line nucleus. Such behavior of S_{2n} in Ca isotopes with $N > 40$ gives us a hint that giant halos exist in these nuclei, just as happens in Zr isotopes with $N > 82$ [**?**].

B. Nuclear Root Mean Square Radii

We discuss here the nuclear radii, which are important basic physical quantities for describing atomic nuclei along with nuclear binding energies. In mean field theory, the root mean square (rms) proton, neutron, and matter radii (r_p, r_n, r_m) can be directly deduced from their density distributions ρ:

$$r_{p(n)} = \langle r^2_{p(n)} \rangle^{1/2} = \left(\frac{\int \rho_{p(n)} r^2 d\tau}{\int \rho_{p(n)} d\tau} \right)^{1/2},$$

$$r_m = \langle r_m^2 \rangle^{1/2} = \left(\frac{\int(\rho_p + \rho_n) r^2 d\tau}{\int(\rho_p + \rho_n) d\tau} \right)^{1/2}.$$

The theoretical rms charge radii r_c are connected with the proton radii r_p as:

$$r_c^2 = r_p^2 + 0.64\ \text{fm}^2,$$

when the small isospin dependence is ignored for exotic nuclei [**?**].

FIG. 2 [Figure 2: see original paper]. The root mean square charge radii for proton magic even-mass nuclei are plotted as a function of neutron number. The dotted curves correspond to radii calculated with the $A^{1/3}$ formula.

FIG. 3 [Figure 3: see original paper]. The root mean square neutron radii for proton magic even-mass nuclei are plotted as a function of neutron number. The solid curve corresponds to radii calculated with the $N^{1/3}$ formula.

FIG. 4 [Figure 4: see original paper]. The root mean square radii for proton, neutron, charge, and matter distributions are plotted as a function of neutron number for Ca isotopes. Available experimental values are denoted by solid symbols.

It is interesting to investigate the neutron number dependence of proton radii (or charge radii) and neutron radii for these proton-magic isotope chains. In Fig. 2, we show the rms charge radii r_c obtained from RCHB theory (open symbols) and available data (solid symbols) for even-even Ca, Ni, Zr, Sn, and Pb isotopes. Values for O isotopes are not plotted here due to systematic deviations of mean field theory for light isotopes as discussed above. As can be seen, the RCHB calculations reproduce the data very well (within 1.5%). For a given isotopic chain, an approximate linear N dependence of the calculated rms charge radii r_c is clearly observed. However, the variation of r_c for a given isotopic chain deviates from the generally accepted simple $A^{1/3}$ law (denoted by dashed lines in Fig. 2), showing that strong isospin dependence of nuclear charge radii is necessary for nuclei with extreme N/Z ratios. Recently, we have investigated these charge radii along with a large amount of experimental data and extracted a formula with $Z^{1/3}$ dependence and isospin dependence to better describe these charge radii across the nuclear isotope chart [**?**].

We also plot in Fig. 3 the neutron radii r_n from RCHB calculations for even-even nuclei of the O, Ca, Ni, Zr, Sn, and Pb isotope chains. The predicted r_n curve using the simple empirical equation $r_n = r_0 \cdot N^{1/3}$ with $r_0 = 1.139$ fm normalized to ^{208}Pb is also shown. Except for O and Ca isotopes, it is very interesting to see that this simple formula for r_n agrees with the calculated neutron radii except for two anomalies. One appears in the Ca chain above $N = 40$, and the other in the Zr chain above $N = 82$. The increase in exotic Ni and Sn nuclei is not as rapid as that in Ca and Zr chains. The regions of abnormal increase in neutron radii are the same as those for S_{2n}. Both behaviors are connected with the formation of giant halos. As calcium is a light element, giant halos in exotic Ca isotopes would be much more easily accessed experimentally than in the Zr chain.

In Table II we present the calculated neutron, proton, matter, and charge radii for all even Ca isotopes, along with several available charge radii data. These radii versus mass number A are plotted in Fig. 4. For nuclear charge radii r_c, the well-known parabolic behavior along $^{40-48}$Ca is not reproduced in our calculation, mainly due to the improper spherical approximation for the actually deformed ^{42,44,46}Ca nuclei. It can be clearly seen that radii r_p, r_n, r_m, and r_c all increase with mass number A. The matter radii r_m as well as r_n increase much faster than r_p and r_c. Besides the normal increase of r_n and r_m with A, a slight up-bend tendency occurs in the neutron drip-line region from $A = 62$ up to $A = 72$.

C. Density Distribution and Halos

We shall concentrate from here on the giant halo properties for Ca isotopes, mainly due to easier access by present experimental techniques. For exotic nuclei, physics connected with the low-density region in the tails of neutron and proton distributions has attracted much attention in nuclear physics as well as in other fields such as astrophysics. It is therefore of great importance to examine the matter distribution and see how densities change with proton-to-neutron ratio in these nuclei. As the density here is obtained from a fully microscopic and parameter-free model well supported by experimental binding energies, we now proceed to examine the density distributions of the entire Ca isotope chain and study the relation between halo development and shell effects within the model.

FIG. 5 [Figure 5: see original paper]. (a) Neutron, (b) proton, and (c) matter density distributions for Ca isotopes. The arrows represent the density variation tendency with neutron number N.

The density distributions for some Ca isotopes are given for neutron, proton, and matter densities in Figs. 5a-c, respectively. In these figures, the arrows represent the density variation tendency with mass number A. From Fig. 5c, the matter densities extend outward with A at the nuclear surface ($3 < r < 6$ fm), while the variation is small in the center ($r < 3$ fm). For the proton density in Fig. 5b, the density for $r < 3$ fm decreases dramatically, and at the surface ($r \sim 4$ fm) extends outward monotonously with A, although the proton number is fixed at 20. This is in accordance with the increase of proton radii r_p and charge radii r_c shown in Fig. 4. The increase of radius is due to the nuclear saturation property as the total mass number A of the nuclei increases. The neutron densities, represented in Fig. 5a, extend greatly outward with A on the surface and neutron skin is developing. In the center, most neutron densities also increase with A, though the increase is not as dramatic as the decrease of proton density. As a result, the matter densities vary (mainly decrease) slightly in the interior region.

Halo phenomena are always related to wide extension of nuclear density distributions in space. To clearly exhibit the large spatial extension, the density distributions are also shown on a logarithmic scale in the insets of Figs. 5a, 5b, and 5c for neutron, proton, and matter density distributions. From the inset of Fig. 5a, the proton density decreases once more with A in the exterior region $r > 8$ fm, while heavier Ca isotopes extend outward in the region $4 < r < 6$ fm and the proton density decreases dramatically with A in the center $r < 3$ fm. Thus there exist two switching regions for the proton density: one lies at the nuclear surface ($r \sim 4$ fm) due to nuclear saturation property, and the other appears in the region beyond $6 - 8$ fm, which is related to neutron deficiency.

From the neutron density shown in the inset of Fig. 5a, it is clearly seen that neutron density distributions for Ca isotopes with $N > 40$ extend much more widely than those for lighter ones. The tail becomes larger and larger as the neutron number N increases from 42 to 52, which is quite different from the cases in Ni and Sn isotopes [**?**, **?**]. In those two isotope chains, the tails of neutron densities reach saturation beyond specific nuclei, i.e., $A = 90$ for Ni and $A = 160$ for Sn. However, no saturation point is seen for Ca isotopes as shown in Fig. 5a.

For the tails of matter density distributions in the inset of Fig. 5c, they are mainly determined by proton densities for proton-rich nuclei and by neutron densities for neutron-rich nuclei. The nucleus with the smallest tail for matter distribution is shown to be ^{44}Ca, which is nearest to the β-stability line, not the most neutron-deficient nucleus ^{34}Ca. For neutron-rich nuclei with N from 42 to 52, the tail behaviors of r_m are consistent with those of r_n, providing strong evidence that giant neutron halos exist in exotic Ca nuclei ($A > 60$).

FIG. 6 [Figure 6: see original paper]. Radii r corresponding to given proton and neutron densities $\rho_p(r) = \rho_n(r) = 10^{-2}, 10^{-4}$, and 10^{-6} fm^{-3}, respectively, as a function of mass number in the Ca isotope chain.

To provide a simple quantitative idea of density distributions and halos, Fig. 6 shows the radii at which proton and neutron densities $\rho_p(r)$ and $\rho_n(r)$ equal $10^{-2}, 10^{-4}$, and 10^{-6} fm^{-3}, respectively, as a function of neutron number for the Ca isotope chain. As the central density of nuclear matter is about $\rho_0 \sim 0.16$ fm^{-3} and the proton and neutron densities are about half of ρ_0, the radius for $\rho_{p(n)} = 10^{-2}$ fm^{-3} corresponds to densities decreasing to 10% of central density at the nuclear surface. The radius for $\rho_{p(n)} = 10^{-6}$ fm^{-3} corresponds to the tails of density distribution. From this figure, for ^{42}Ca we see that proton and neutron densities have similar values at the same r, i.e., the neutron density distribution is very similar to that of proton for β-stable nuclei. For neutron-rich nuclei, if $\rho_n = \rho_p$, the neutron radius r is much larger. For neutron-deficient nuclei, the opposite is seen. For $\rho_n = 10^{-6}$ fm^{-3}, an increasing tendency for the r curve can be clearly seen, particularly showing a strong kink at $N = 62$.

This kink is also an important signal for giant neutron halos, along with S_{2n} and neutron radii r_n.

D. Single-Particle Levels in Canonical Basis and Contribution of Continuum

The above results can be understood more clearly by considering the microscopic structure of the underlying wave functions and the single-particle energies in the canonical basis [?]. In Fig. 7 [Figure 7: see original paper], neutron single-particle levels in the canonical basis are shown for even Ca isotopes from mass number $A = 34$ to $A = 72$. Shell closure ($N = 20$ and 28) and sub-shell closure ($N = 40$) are clearly seen as large gaps between levels. A dotted line in the figure represents the neutron chemical potential λ_n, which jumps three times at magic or submagic neutron numbers on its way to almost zero at ^{72}Ca. These jumps correspond to shell (or sub-shell) closure, just as seen in the S_{2n} case. λ_n comes close to zero for nuclei near the neutron drip line, i.e., $\lambda_n \approx -0.92$ MeV for ^{62}Ca, -0.64 MeV for ^{64}Ca, -0.44 MeV for ^{66}Ca, -0.28 MeV for ^{68}Ca, -0.16 MeV for ^{70}Ca, and -0.07 MeV for ^{72}Ca. Meanwhile, no jump at $N = 50$ in the λ_n curve agrees with the disappearance of this traditional magic number mentioned above in the S_{2n} case. From Fig. 7, the large gap between the $1g_{9/2}$ orbit and the $s-d$ shell has disappeared in the neutron drip-line region due to the lowering of $3s_{1/2}$ and $2d_{5/2}$ orbits. For example, this gap for ^{72}Ca is only 1.02 MeV.

FIG. 7. Single-particle neutron energies in the canonical basis versus mass number A for Ca isotopes. Fermi surfaces are shown by the dot-dashed line.

As the neutron chemical potential λ_n approaches zero, Ca isotopes with $A > 60$ are all weakly bound nuclei. This means that additional neutrons will occupy weakly bound states very close to the continuum region. These neutrons supply very small binding energies and result in nearly vanishing two-neutron separation energies S_{2n}. Furthermore, pairing interaction scatters neutron pairs from such weakly bound states to continua as the Fermi level is close to zero. The single-particle occupation of continua then affects nuclear properties. For Ca isotopes with $A > 60$, the added neutrons occupy weakly bound states and continua: $1g_{9/2}, 3s_{1/2}, 2d_{5/2}, 2d_{3/2}$, etc. Such orbits play important roles as discussed below.

FIG. 8 [Figure 8: see original paper]. Occupation probabilities in the canonical basis for various even Ca isotopes as a function of single-particle energy. The chemical potential is indicated with a vertical line. The number N_h of neutrons in the halo is also shown.

In Fig. 8, we present occupation probabilities v^2 of neutron levels near the Fermi surface (i.e., $-20 \leq E \leq 10$ MeV) in the canonical basis for several neutron-rich even Ca isotopes ($^{58-72}$Ca). The neutron chemical potential λ_n is indicated by a vertical line. For nuclei with mass number $A < 60$, the chemical potential is quite large (e.g., -13.1 MeV for ^{40}Ca, -6.74 MeV for ^{48}Ca, and

-3.69 MeV for ^{58}Ca), and occupation probabilities for continua are nearly zero. As the neutron number goes beyond the sub-shell $N = 40$, the Fermi surface approaches zero and occupation of the continuum becomes more important. Summing the occupation probabilities v^2 for states with $E > 0$, one obtains the contribution of continua n_h [?]. They are approximately 2.2, 0.6, 1.1, 1.7, 1.9, and 2.7 for ^{62}Ca, ^{64}Ca, ^{66}Ca, ^{68}Ca, ^{70}Ca, and ^{72}Ca, respectively. In Ref. [?], from two to roughly six neutrons in total scattering in the continuum were predicted in Zr isotopes with $N > 82$, while the number of neutrons scattering to the continuum in Ca isotopes is reduced to about $1 \sim 3$.

FIG. 9 [Figure 9: see original paper]. Single-particle levels in the canonical basis for neutrons in ^{66}Ca. The neutron potential $V(r) + S(r)$ is denoted by the solid line and the Fermi surface is shown by the dashed line. Occupied probabilities for single-particle levels are shown by their line length. The root mean square radius r_{nlj} (in fm) for each level is listed in parentheses behind the corresponding level.

As a typical example, we investigate in detail the single-particle levels and contributions from continua in the exotic nucleus ^{66}Ca. Fig. 9 shows neutron single-particle levels in the canonical basis for ^{66}Ca. The length of each level is proportional to its occupied probability v^2. The neutron Fermi surface ($\lambda_n \approx -0.435$ MeV) is represented by a dashed line, and the nuclear mean field potential $V(r) + S(r)$ is denoted by the solid curve. We define the root mean square radius r_{nlj} for a single-particle level denoted by nlj as:

$$r_{nlj} = \left(\frac{\int \rho_{nlj} r^2 d\tau}{\int \rho_{nlj} d\tau} \right)^{1/2},$$

where ρ_{nlj} presents the probability density of each level with corresponding wave function ψ_{nlj}. This root mean square radius r_{nlj} for each level is shown in parentheses behind the corresponding level label in units of fm. Also in Table V, we list the single-particle energies ϵ_{nlj}, rms radii r_{nlj}, and occupied probabilities v^2_{nlj} for all neutron levels with energies $\epsilon < 10$ MeV in ^{66}Ca calculated from RCHB theory.

Here, the state $1g_{9/2}$ is weakly bound with energy -0.48 MeV, while the $3s_{1/2}, 2d_{5/2}, 2d_{3/2}$, and $1g_{7/2}$ levels are in the continuum with energies $0.64, 1.41, 2.85$, and 5.67 MeV, respectively. We note that due to the absence of a centrifugal barrier, the orbit $3s_{1/2}$ lies below states $2d_{3/2}$ and $2d_{5/2}$. The occupied probabilities v^2 of these states are: 0.514 for $1g_{9/2}$, 0.089 for $3s_{1/2}$, 0.071 for $2d_{5/2}$, 0.027 for $2d_{3/2}$, and 0.017 for $1g_{7/2}$. We obtain about 0.85 neutrons in these continuum states, which is about 4/5 of the total continuum contribution $n_h = 1.1$ for ^{66}Ca (see Fig. 8). The $3s_{1/2}$ state has an rms radius of 7.24 fm, compared with rms radii of neighboring states (~ 5 fm) and the total neutron rms radius (4.314 fm), due to its zero centrifugal barrier.

Therefore, nucleons occupying the $3s_{1/2}$ state contribute considerably to the nuclear rms radius.

The relative contributions ρ_{nlj}/ρ_n of different single-particle orbits to the full neutron density as a function of radial distance r are shown in Fig. 10 [Figure 10: see original paper] for the typical nucleus ^{66}Ca. For comparison, we also present the total neutron density ρ_n with the shaded area in arbitrary units. In the interior of nuclei ($r < 4$ fm), the contribution to neutron distribution mainly comes from low-lying energy levels such as $1s_{1/2}, 1p_{3/2}, 1p_{1/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}$. In the nuclear surface ($r \sim 5$ fm) and the beginning part of the tail ($6 < r < 10$ fm), the weakly bound $f-p$ shell states and the very weakly bound $1g_{9/2}$ orbit play dominant roles for neutron distribution. Their contributions gradually decrease with radius, while contributions from the continuum (e.g., $3s_{1/2}, 2d_{5/2}$) become more important. For $r > 15$ fm, the $3s_{1/2}$ state is dominant (about 60 percent), and other continuum states $2d_{5/2}, 2d_{3/2}, 4s_{1/2}$ and the very weakly bound $1g_{9/2}$ state also play their roles. It is well demonstrated that contributions from continua are crucial to the tail, which is closely connected with halo phenomena.

TABLE V. Properties of neutron single-particle levels in ^{66}Ca, including single-particle energies, occupation probabilities, and rms radii.

TABLE VI. Properties of the $3s_{1/2}$ level for $^{60-72}$Ca: energy ϵ (MeV), rms radius r (fm), and occupation probability.

In Table VI, the energies, rms radii, and occupied probabilities of the neutron single-particle level $3s_{1/2}$ for even isotopes $^{60-72}$Ca are presented. Its energy decreases with neutron number N, i.e., it lies in the continuum for $^{60-68}$Ca and becomes slightly bound for ^{70,72}Ca. As both the rms radii and v^2 for the $3s_{1/2}$ state increase monotonously with mass number A, it drives the tail of neutron density distribution to become larger and larger as shown above. Of course, with more neutrons added to ^{66}Ca up to the heaviest bound nucleus predicted as ^{72}Ca, contributions to the tail from other continuum states $2d_{5/2}, 4s_{1/2}$, and $2d_{3/2}$ will also become more important due to their larger occupation.

FIG. 10 [Figure 10: see original paper]. Relative contributions of different orbits to the total neutron density as a function of radius. The shaded area indicates the total neutron density in arbitrary units.

E. Vector and Scalar Potentials and Their Isospin Evolution

One of the essential differences between non-relativistic single-particle equations and the relativistic Dirac equation in nuclear physics is that the relativistic equation contains from the beginning two potentials V and S, which have different behaviors under Lorentz transformation [**?**]. We now investigate the vector and scalar potentials in Ca isotope chains in detail. Fig. 11 [Figure 11: see original paper] presents the vector V and scalar S potentials for protons (left panels) and neutrons (right panels) in Ca chains as a function of radial distance r.

We see from the figure that both vector and scalar potentials have a Woods-Saxon shape, similar to nuclear density distribution. They nearly vanish outside the nucleus and are more or less constant in the nuclear interior, namely $S \approx -420$ MeV as an attractive potential and $V \approx 350$ MeV as a repulsive potential.

FIG. 11. Vector and scalar potentials $(V(r), S(r))$ for Ca isotopes. The left panel shows those for protons, while the right panel shows those for neutrons.

We can clearly see that the scalar potential $S(r)$ for protons is the same as that for neutrons in all cases. It is isospin-independent as the scalar potential comes from the σ meson field, which takes the same form for either protons or neutrons. On the other hand, the vector potential $V(r)$ is isospin-dependent: the potential for protons is somewhat different from that for neutrons. The difference comes from the ρ meson and Coulomb interaction. For a given nucleus, the vector potential for protons is slightly larger than that for neutrons in the nuclear interior, and it does not vanish outside the nucleus due to the long-range Coulomb interaction.

F. Spin-Orbit Splitting in Exotic Nuclei

An advantage of relativistic mean field theory is that spin-orbit coupling arises naturally. The spin-orbit splitting has been discussed through various themes with relativistic or non-relativistic microscopic theory [**?**, **?**, **?**]. We now examine the spin-orbit splitting for all even Ca isotopes in the relativistic continuum Hartree-Bogoliubov theory.

The spin-orbit splitting energy E_{ls} for two partners $(nlj = l-1/2, nlj = l+1/2)$ is defined as:

$$E_{ls} = \frac{E_{nlj=l-1/2} - E_{nlj=l+1/2}}{2l+1}.$$

FIG. 12 [Figure 12: see original paper]. Spin-orbit splitting energies E_{ls} in Ca isotopes as a function of mass number A for proton (lower panel) and neutron (upper panel) spin-orbit partners $(1d_{3/2}, 1d_{5/2})$, $(1g_{7/2}, 1g_{9/2})$, $(1p_{1/2}, 1p_{3/2})$, $(1f_{5/2}, 1f_{7/2})$, and $(2p_{1/2}, 2p_{3/2})$.

In Fig. 12, the spin-orbit splitting energies E_{ls} in Ca isotopes are shown as a function of mass number A for proton and neutron spin-orbit partners. For lighter Ca isotopes, some doublets sit in the continuum region with large positive energies in the canonical basis, which would result in some uncertainty in spin-orbit splitting. Therefore we limit level energies E to below 10 MeV. In Fig. 12, the splitting decreases monotonically from the proton-rich side to the neutron-rich side for most doublets except for $(1p_{1/2}, 1p_{3/2})$ and $(2p_{1/2}, 2p_{3/2})$. The splitting of the p doublets between two closures (i.e., ^{40}Ca, ^{48}Ca) fluctuates quite a lot. Above $N = 28$, the splitting increases a little, then declines with neutron number. For doublets $(1p_{1/2}, 1p_{3/2})$ and $(1d_{3/2}, 1d_{5/2})$, the spin-orbit splitting for neutrons and protons is very close. For doublets $(1f_{5/2}, 1f_{7/2})$

and $(2p_{1/2}, 2p_{3/2})$, there are quite large differences. Moreover, the splitting for neutrons is usually smaller than that for protons.

FIG. 13 [Figure 13: see original paper]. Derivative of the potential difference, $d(V(r) - S(r))/dr$, for protons (left) and neutrons (right) in Ca isotopes as a function of radius.

It would be very helpful to examine the origin of spin-orbit splitting in the Dirac equation. For a Dirac nucleon moving in scalar and vector potentials, its equation of motion can be decoupled and reduced to either the upper or lower component. If reduced to the lower component, it relates to another interesting topic—pseudo-spin symmetry [**?**]. Here we focus on spin-orbit splitting, which relates to the upper component. The Dirac equation can be reduced for the upper component as [**?**]:

$$\frac{d^2G^i_{lj}(r)}{dr^2}-\frac{\kappa(1+\kappa)}{r^2}G^i_{lj}(r)=-\frac{1}{E+2M-V_V(r)+V_S(r)}\frac{d(2M-V_V(r)+V_S(r))}{dr}\frac{dG^i_{lj}(r)}{dr}-(E+2M-V_V(r)+V_S$$

where

$$\kappa = \begin{cases} -l-1, & j = l+1/2, \\ +l, & j = l-1/2. \end{cases}$$

The spin-orbit splitting is provided by the corresponding spin-orbit term [**?**]:

$$V_{ls} = \frac{1}{E+2M-V(r)+S(r)}\frac{d(2M-V(r)+S(r))}{dr}.$$

It can be seen clearly that spin-orbit splitting is energy-dependent and depends also on the derivative of the potential $2M - V(r) + S(r)$ as well as the particle distribution. Therefore, the so-called spin-orbit potential in RCHB theory is defined as [**?**]:

$$V_{ls} = \frac{d(2M-V(r)+S(r))}{dr}.$$

The derivative of the potential difference, $d(V(r) - S(r))/dr$, for protons and neutrons in Ca isotopes is given in Fig. 13. The potential difference $V(r)-S(r)$ for both protons and neutrons is almost the same, as $V(r) - S(r)$ is a large quantity ($\sim$ 700 MeV), and the difference in spin-orbit potential for protons and neutrons could be neglected. Therefore, proton and neutron V_{ls} are almost the same in the present model. This is why the spin-orbit splitting for neutron and proton doublets $(1p_{1/2}, 1p_{3/2})$ and $(1d_{3/2}, 1d_{5/2})$ are very close, as shown in Fig. 12.

FIG. 14 [Figure 14: see original paper]. Summed mean field potential, $V(r)+S(r)$, for protons and neutrons in even Ca isotopes. Arrows in the figure show the direction of increasing nuclear potential diffuseness.

FIG. 15 [Figure 15: see original paper]. Mean field anti-nucleon potential, $V(r)-S(r)$, for protons and neutrons in even Ca isotopes. Arrows in the figure show the direction of increasing nuclear potential diffuseness.

From ^{34}Ca to ^{48}Ca, the amplitude of V_{ls} increases monotonically, and from ^{48}Ca to ^{72}Ca, the amplitude decreases monotonically due to surface diffuseness. Meanwhile, the maximum point of the potential V_{ls} has an outward tendency. Thus, the systematic decrease of spin-orbit splitting is partially related to the decrease of V_{ls}. Furthermore, the systematic decrease also comes from the diffuseness of the nuclear potential. In Figs. 14 and 15, we see that the potentials $V(r)-S(r)$ and $V(r)+S(r)$ extend outward into the nuclear surface, making the diffuseness increase with neutron number.

FIG. 16 [Figure 16: see original paper]. Spin-orbit splitting E_{ls} as a function of specific quantum numbers in ^{42,52,62,72}Ca. The left panel shows splitting for p orbits with quantum number n; the right panel shows splitting for $n=1$ orbits with quantum number l.

So far we have investigated how proton and neutron spin-orbit splitting changes with mass number A, i.e., the isospin dependence. The decreasing tendency is mainly caused by the derivative of the anti-nucleon potential and the diffuseness of the mean field. For a given nucleus, the potential is the same for different spin-orbit doublets, so it is necessary to investigate how the splitting changes with different quantum numbers, as in Fig. 16. To avoid confusion from showing too many doublets, we present splitting energies E_{ls} for $(p_{1/2}, p_{3/2})$ partners as a function of quantum number n in the left panel, and those for different $n=1$ doublets as a function of quantum number l in the right panel, for ^{42,52,62,72}Ca.

In the left panel of Fig. 16, spin-orbit splitting in doublets $(1p_{1/2}, 1p_{3/2})$ is much larger than in $(2p_{1/2}, 2p_{3/2})$ cases, and that in $(3p_{1/2}, 3p_{3/2})$ for ^{72}Ca is the smallest. It can be clearly seen in Eq. (32) that, besides the spin-orbit potential, the spin-orbit term also depends on the factor $E+2M-V(r)+S(r)$. However, this factor has little energy dependence. In fact, the energy E (varying from -40 to 10 MeV) is a small quantity compared with the potential $2M-V(r)+S(r)$, which is $2M$ ($\sim$ 1800 MeV) at the surface or about 1000 MeV in the nuclear interior. Thus, the energy dependence of spin-orbit splitting difference caused directly by the energy factor can be neglected.

It has been suggested that spin-orbit splitting differences mainly come from the overlap between the density distribution and the spin-orbit potential V_{ls}, as demonstrated for Sn isotopes in Ref. [**?**]. In Fig. 17 [Figure 17: see original paper], for ^{42}Ca, ^{52}Ca, ^{62}Ca, and ^{72}Ca, the derivative of the potential $V(r)-S(r)$, $d(V(r)-S(r))/dr$, and the density distributions of p-waves are given in the respective lower panels, while the spin-orbit potential V_{ls} multiplied by the density distributions for $p_{1/2}$ are given in the upper panels. The overlap between spin-orbit potential V_{ls} and particle distribution is represented by the curve in the corresponding upper panel. Their contribution to spin-orbit splitting is proportional to the area surrounded by the curve and the x-axis. It is clearly seen that the overlap of $1p_{1/2}$ is larger than that of $2p_{1/2}$ for all these isotopes, and that of $3p_{1/2}$ in ^{72}Ca is the smallest. The overlap values of $2p_{1/2}$ for ^{52,62,72}Ca are very close to each other and all are smaller than that for ^{42}Ca. These observations explain the features seen in the right panel of Fig. 16.

FIG. 17. Overlap between spin-orbit potential and density distributions for ^{42}Ca, ^{52}Ca, ^{62}Ca, and ^{72}Ca. In each nucleus, the spin-orbit potential V_{ls} multiplied by the density distribution of the $p_{1/2}$ orbit is given in the upper panel; the derivative of the anti-nucleon potential and the density distributions for the $p_{1/2}$ orbit (in arbitrary units) are given in the lower panel.

From the right panel of Fig. 16, spin-orbit splitting for $n = 1$ states shows similar features for ^{42}Ca, ^{52}Ca, ^{62}Ca, and ^{72}Ca. The splitting of doublets $(d_{1/2}, d_{3/2})$ is the largest. With increasing l, the splitting for $n = 1$ increases at first (from p to d) and then decreases (from d to f, g). This feature can also be understood by the overlap of the spin-orbit potential and density distribution of these orbits. Here we choose ^{72}Ca as an example (Fig. 18 [Figure 18: see original paper]); similar patterns appear in other Ca isotopes. The largest overlap for doublets $(d_{1/2}, d_{3/2})$ in ^{72}Ca is clearly seen in the upper panel of Fig. 18.

FIG. 18. Same as Fig. 17, but for $n = 1$ orbits in ^{72}Ca.

IV. THE PROSPECT OF GIANT HALOS

Nuclear halo phenomena have been studied by many nuclear theorists and experimentalists. More and more halo nuclei, including neutron halos, proton halos, and halos in excited states, have been identified and reported using different methods and advanced instruments. However, only nuclei with one or two halo nucleons have been found experimentally until now.

Using various theoretical models, theorists can quantitatively demonstrate halo density distributions. Furthermore, giant halo nuclei with more halo nucleons have been predicted in exotic Zr nuclei with $A > 122$ using the RCHB method [**?**]. This prediction of giant halos has aroused great interest among nuclear experimentalists. Unfortunately, exotic Zr nuclei are too heavy to produce in modern accelerators. Here we have investigated ground-state properties of the entire Ca chain by RCHB theory and predict that giant neutron halo phenomena would exist in exotic Ca nuclei with $A > 60$ as well as in exotic Zr nuclei. Compared with Zr isotopes, exotic Ca nuclei have smaller mass number and would be much easier to synthesize. The heaviest Ca isotope known experimentally is ^{57}Ca, so 5 more neutrons are needed to form the fringe giant halo nucleus ^{62}Ca, and 9 more neutrons to form the typical giant halo nucleus ^{66}Ca discussed above.

To confirm whether there exists a possibility of giant halo nuclei in a wider mass region, Fig. 19 [Figure 19: see original paper] shows two-neutron separation energies S_{2n} for even-neutron Ne, Na, Mg, and Al nuclei in the drip-line region. Open symbols represent values calculated from RCHB theory with the NL-SH parameter set, while solid symbols represent available data. We note that calculations were performed assuming spherical shape, so comparison with experimental data should be made with care.

FIG. 19. Two-neutron separation energies S_{2n} for even-neutron Ne, Na, Mg, and Al nuclei in the drip-line region. Open symbols represent values calculated with RCHB theory using the NL-SH parameter set, while solid symbols represent available data. The horizontal line at 2 MeV denotes the upper limit for possible halo occurrence.

FIG. 20 [Figure 20: see original paper]. Same as Fig. 19, but for even Ar, K, Ca, Sc, and Ti nuclei in the drip-line region.

In Fig. 20, we plot S_{2n} for exotic even-neutron Ar, K, Ca, Sc, and Ti isotopes. From these two figures, the two-neutron separation energies S_{2n} for all these isotope chains are almost parallel, with more than one line lying within 2 MeV in the drip-line region. Therefore, there is quite a large mass region where giant halos may exist.

Many nuclear physicists are making great efforts to search for the drip line for heavier elements. A new experiment [**?**, **?**] has reported that both ^{37}Na and ^{34}Ne are bound, while ^{33}Ne and ^{36}Na were not observed. Looking at Fig. 19, the ^{37}Na and ^{34}Ne nuclei lie in the drip-line region and approach the giant halo nuclei. Therefore, we suggest that much more experimental effort should be devoted to extending studies in this area and measuring masses to find possible giant halo nuclei. A similar situation can be seen in Fig. 20 for Ar, K, Ca, Sc, and Ti isotopes near the neutron drip line.

V. SUMMARY

We have investigated the ground-state properties of even-even proton magic O, Ca, Ni, Zr, Sn, and Pb isotopes with the relativistic continuum Hartree-Bogoliubov (RCHB) theory. We have found good agreement with available experimental data for binding energies and nuclear radii. We have shown binding energies E_b, two-neutron separation energies S_{2n}, and root mean square radii. The predicted neutron drip-line nuclei are ^{28}O, ^{72}Ca, ^{98}Ni, ^{136}Zr, ^{176}Sn, and ^{266}Pb, respectively. Particularly, giant halos in neutron-rich Ca and Zr isotopes close to the neutron drip line are predicted. Giant halo properties in exotic Ca isotopes with $A > 60$ are discussed in detail.

Giant neutron halos in exotic Ca isotopes have been studied through analysis of S_{2n}, radii, nucleon density distributions, single-particle energy levels, occupation probabilities, and contributions from the continuum. The spin-orbit splitting and potential diffuseness in Ca isotopes have also been investigated.

Summarizing the present investigation, we conclude:

1. Based on analysis of two-neutron separation energies S_{2n}, rms radii, single-particle level spectra, orbital occupation, and continuum contribution, giant halo phenomena are suggested to appear in Ca isotopes with $A > 60$. Similar phenomena can also be seen for nuclei near the Na or Ar isotopes near the neutron drip line.

2. The neutron drip-line nucleus for Ca is ^{72}Ca instead of ^{70}Ca, caused by the disappearance of the $N = 50$ magic number due to the halo effect of the $3s_{1/2}$ orbit.

3. The giant halos developed in these nuclei are due to pairing correlation and contribution from the continuum, e.g., the $3s_{1/2}$ orbit.

4. The spin-orbit splitting in Ca isotopes decreases monotonically from the proton drip line to the neutron drip line for most cases. This tendency mainly comes from the diffuseness of the nuclear potential with neutron number.

In this paper, proton magic even-even nuclei from the proton drip line to the neutron drip line are studied in detail using RCHB theory. The important contribution from the continuum due to pairing correlations has been taken into account. The power of the RCHB method has been demonstrated for proton magic nuclei using the assumption of spherical shape. Another important degree of freedom for exotic nuclei is deformation. The theoretical framework for exotic nuclei including deformation and continuum contributions is in progress and will be completed soon.

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Note: Figure translations are in progress. See original paper for figures.