Relativistic Continuum Hartree Bogoliubov Theory for Ground State Properties of Exotic Nuclei (Postprint)

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Full Text

Preamble

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Abstract

The Relativistic Continuum Hartree-Bogoliubov (RCHB) theory, which properly takes into account the pairing correlation and the coupling to (discretized) continuum via Bogoliubov transformation in a microscopic and self-consistent way, has been reviewed together with its new interpretation of the halo phenomena observed in light nuclei as the scattering of particle pairs into the continuum, the prediction of the exotic phenomena—giant halos in nuclei near neutron drip line, the reproduction of interaction cross sections and charge-changing cross sections in light exotic nuclei in combination with the Glauber theory, better restoration of pseudospin symmetry in exotic nuclei, predictions of exotic phenomena in hyper nuclei, and new magic numbers in superheavy nuclei, etc. Recent investigations on new effective interactions, the density dependence of the interaction strengths, the RMF theory on the Woods-Saxon basis, the single particle resonant states, and the resonant BCS (rBCS) method for the pairing correlation, etc. are also presented in some details.

Key words: Relativistic mean field theory, continuum, pairing correlation, Bogoliubov transformation, relativistic continuum Hartree-Bogoliubov, exotic nuclei, halo, giant halo, hyperon halo, interaction cross section, charge-changing cross section, pseudospin symmetry, hyper nuclei, magic number, superheavy nuclei.

Contents

5 Superheavy nuclei

5.1 Two nucleon separation energies

5.3 Shell correction energies

5.4 Magic numbers

5.4.1 Magic numbers for spherical superheavy nuclei

5.4.2 Stability of the doubly magic superheavy nuclei against deformation

5.5 Alpha decay and the half-lives

Introduction

Currently nuclear physics is undergoing a renaissance as evidenced by the fact that worldwide there are numerous Radioactive Ion Beam (RIB) facilities operating, being upgraded, under construction, or planned for construction. Examples include the Cooling Storage Ring (CSR) project at HIRFL in China, which was completed in 2005 [2]; the RIB Factory at RIKEN in Japan, which began operation at the end of 2006 [3]; the FAIR project at GSI in Germany, which was approved in 2003 [4]; and the construction of the Rare Isotope Accelerator (RIA), which is considered to be of the highest priority across all physics disciplines in the US [5]. These new facilities, together with developments in detection techniques, have fundamentally changed the nuclear physics landscape, making it possible to produce and study nuclei far from the stability line—so-called "EXOTIC NUCLEI" [6–12].

New exciting discoveries have been made by exploring hitherto inaccessible regions of the nuclear chart [6–12]. For example, a novel phenomenon called halo—a state in which nucleons spread like a thin mist around the nucleus—was first discovered in ^{11}Li with RIB in 1985 [13] and later observed in many other light exotic nuclei. Exotic nuclei also exhibit other interesting phenomena such as the disappearance of traditional shell gaps and the emergence of new ones, which result in new magic numbers [14]. Furthermore, exotic nuclei play important roles in nuclear astrophysics, as their properties are crucial for understanding stellar nucleosynthesis.

Current nuclear models are primarily based on knowledge obtained from nuclei near the β-stability line. The cornerstones of modern nuclear physics include the shell model [15] and the collective model [16, 17], which are respectively based on magic numbers (the enhanced stability of certain nuclei compared to their neighbors) and the incompressibility of nuclear matter. Therefore, the change of magic numbers and the unprecedented low-density nuclear matter in halo nuclei have shaken the foundations of nuclear physics. New innovative nuclear models are needed to describe exotic nuclei characterized by weak binding and low density.

The relativistic mean field (RMF) theory [18] has received wide attention due to its successful description of numerous nuclear phenomena over the past years [19–23]. In the framework of RMF theory, nucleons interact via the exchange of mesons and photons. The representations with large scalar and vector fields in nuclei, of order a few hundred MeV, provide simpler and more efficient descriptions than nonrelativistic approaches that hide these scales. Dominant evidence includes the spin-orbit splittings, while other evidence encompasses the density dependence of the optical potential, the observation of approximate pseudospin symmetry, correlated two-pion exchange strength, QCD sum rules, and more [24]. The relativistic Brueckner-Hartree-Fock theory and RMF theory with density-dependent coupling constants extracted from it can reproduce nuclear saturation properties (the Coester line) in nuclear matter [25, 26]. Furthermore, RMF theory can better reproduce measurements of isotopic shifts in the Pb region [27], more naturally provides the spin-orbit potential and the origin of pseudospin symmetry [28, 29] as a relativistic symmetry [30–32] and spin symmetry in the anti-nucleon spectrum [33], and is more reliable for nuclei far from the β-stability line. Obviously, RMF is one of the best candidates for describing exotic nuclei.

To describe exotic nuclei, the pairing correlation and coupling to the continuum—which are extremely crucial for describing drip line nuclei—must be properly taken into account [34]. The continuum effect is commonly incorporated in Hartree-Fock-Bogoliubov (HFB) [35] or relativistic-Hartree-Bogoliubov (RHB) [21] approaches. In most calculations, the continuum is replaced by a set of positive energy states determined by solving the HFB or RHB equations in coordinate space with box boundary conditions [36–38]. Recently, the HFB equations were also solved with exact boundary conditions for the continuum spectrum, both for zero-range [39] and finite-range pairing forces [40]. These resonant continuum HFB or RHB results with exact boundary conditions are generally close to those with box boundary conditions [39, 41]. The extension of RMF theory to account for both bound states and (discretized) continuum via Bogoliubov transformation in a microscopic and self-consistent way—i.e., the Relativistic Continuum Hartree-Bogoliubov (RCHB) theory—has been developed in Refs. [36, 42]. The RCHB theory has been very successful in describing ground state properties of nuclei both near and far from the β-stability line. Using a density-dependent zero-range interaction, the halo in ^{11}Li has been successfully reproduced in this self-consistent picture. Remarkable successes of the RCHB theory include the new interpretation of the halo in ^{11}Li [36], the prediction of exotic phenomena such as giant halos in Zr ($A > 122$) [43] and Ca ($A > 60$) [44] isotopes, the reproduction of interaction cross sections and charge-changing cross sections in light exotic nuclei in combination with Glauber theory [45, 46], better restoration of pseudospin symmetry in exotic nuclei [31, 32], and predictions of exotic phenomena in hypernuclei [47] and new magic numbers in superheavy nuclei [48].

The main purpose of the present manuscript is to review the RCHB theory and its applications to exotic and superheavy nuclei in the spherical case, with

some other related topics covered briefly. In Section 2, the formalism, numerical solutions, and effective interactions for the RMF theory are presented. In Section 3, the pairing correlations and approaches for pairing are sketched, followed by discussion of continuum states where the resonant BCS method and several methods to obtain single particle resonant states are briefly reviewed. Then the detailed formalism of the RCHB theory together with discussion on effective pairing interactions is given. In Section 4, applications of the RCHB theory to properties of exotic nuclei are presented, such as binding energies, particle separation energies, radii and cross sections, single particle levels, shell structure, restoration of pseudo-spin symmetry, halo and giant halo, and halos in hypernuclei. In Section 5, predictions of new magic numbers in superheavy nuclei are presented. Finally, a brief summary and perspectives are given in Section 6.

2 Relativistic mean field theory

In this section, we present the formalism, numerical solutions, and effective interactions for the RMF theory, as well as its application to nuclear matter. In the first subsection, the effective Lagrangian density and equations of motion for the nucleon and mesons are given. The numerical solutions of the Dirac equation for finite nuclei are discussed in the second subsection. Subsequently, effective interactions with nonlinear self-coupling meson fields and density-dependent meson-nucleon couplings, and their influences on properties of nuclear matter are discussed.

2.1 The general formalism

The RMF theory describes the nucleus as a system of Dirac nucleons that interact in a relativistic covariant manner via meson fields. The meson fields are treated as classical fields. In the simplest RMF version—the σ-ω model [18]—the mesons do not interact among themselves, which leads to too large incompressibility in nuclear matter. Therefore, a nonlinear self-coupling of the σ-field was proposed [49].

To reproduce the density dependence of the vector and scalar potentials from Dirac-Brueckner calculations [26], nonlinear self-coupling of the ω-meson was found to be necessary [50]. Recently, nonlinear self-coupling of the isovector ρ-meson was also introduced to improve the density-dependence of the isospin-dependent part of the potentials [51]. Within this scheme, the isoscalar-scalar σ-meson provides the mid- and long-range attractive part of the nuclear interaction, whereas the short-range repulsive part is provided by the isoscalar-vector ω-meson. The photon field $A_\mu(x)$ accounts for the Coulomb interaction, while the isospin dependence of the nuclear force is described by the isovector-vector ρ-meson. The π-meson field is not included because it does not contribute at the Hartree level. In principle, other mesons apart from σ, ω, and ρ may also contribute to the nuclear interaction, e.g., the isovector scalar δ-meson, which was suggested in Ref. [52] and also on the basis of relativistic Brueckner theory

in Refs [53–56], has been used in some structure calculations for exotic nuclei [55], nuclear matter [57], and stellar matter [58]. However, as the RMF theory is only an effective theory, it is expected that contributions from other mesons can be effectively taken into account by adjusting model parameters to properties of nuclear matter and finite nuclei. With various versions of nonlinear self-couplings of meson fields, the RMF theory has been used to describe many nuclear phenomena over the past years with great success [19–21]. To avoid instability problems for nonlinear interactions at high densities, RMF theories with density-dependent couplings have also been developed [26, 51, 59–62].

The Lagrangian density of the RMF theory can be written as:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + g_\sigma\bar{\psi}\sigma\psi + g_\omega\bar{\psi}\gamma^\mu\omega_\mu\psi + g_\rho\bar{\psi}\gamma^\mu\vec{\tau}\cdot\vec{\rho}_\mu\psi - e\bar{\psi}\gamma^\mu A_\mu\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - U_\sigma(\sigma) - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m$$

where M and m_i (g_i) ($i = \sigma, \omega, \rho$) are the masses (coupling constants) of the nucleon and mesons respectively, and $\Omega_{\mu\nu}$, $\vec{R}_{\mu\nu}$, and $F_{\mu\nu}$ are the field tensors of the vector mesons and the electromagnetic field. We adopt arrows to indicate vectors in isospin space and bold types for space vectors. Greek indices μ and ν run over 0, 1, 2, 3 or t, x, y, z, while Roman indices i, j, etc. denote spatial components. The field tensors are defined as:

$$\begin{aligned}
\Omega_{\mu\nu} &= \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \\
\vec{R}_{\mu\nu} &= \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu
\end{aligned}$$

The nonlinear self-coupling terms $U_\sigma(\sigma)$, $U_\omega(\omega_\mu)$, and $U_\rho(\vec{\rho}_\mu)$ for the σ-meson, ω-meson, and ρ-meson in the Lagrangian density (1) respectively have the following forms:

$$\begin{aligned}
U_\sigma(\sigma) &= \frac{1}{2}g_2\sigma^3 + \frac{1}{3}g_3\sigma^4 \\
U_\omega(\omega_\mu) &= \frac{1}{2}c_3(\omega^\mu\omega_\mu)^2 \\
U_\rho(\vec{\rho}_\mu) &= \frac{1}{2}d_3(\vec{\rho}^\mu\cdot\vec{\rho}_\mu)^2
\end{aligned}$$

From the Lagrangian density (1), the Hamiltonian operator can be obtained by the general Legendre transformation, where the conjugate momenta of the field operators ϕ_i ($\phi_i = \psi, \sigma, \omega_\nu, \rho_\nu, A_\nu$) are defined as:

$$\pi_i(x) = \frac{\partial\mathcal{L}}{\partial(\partial_0\phi_i(x))}$$

Then the Hamiltonian density of the system can be easily obtained as:

$$\mathcal{H} = \sum_i \pi_i(x)\partial_0\phi_i(x) - \mathcal{L}$$

The relativistic mean field theory is formulated on the basis of the effective Lagrangian (1) with the mean field approximation, i.e., the meson fields are treated as classical c-numbers. With this approximation, all quantum fluctuations of the meson fields are removed and the nucleons are described as independent particles moving in the effective meson and photon fields. Therefore, the nucleon field operator can be expanded on a complete set of single-particle states as:

$$\psi(x) = \sum_a \psi_a(x)c_a$$

where c_a is the annihilation operator for a nucleon in state a of the Dirac fields and ψ_a is the corresponding single-particle spinor. The operators c_a and their conjugates $c_a^\dagger$ satisfy the anticommutation rules:

$$\{c_a, c_b^\dagger\} = \delta_{ab}, \quad \{c_a, c_b\} = \{c_a^\dagger, c_b^\dagger\} = 0$$

Confined to single-particle states i with positive energies (the no-sea approximation), the ground state of the nucleus can be constructed as:

$$|\Phi_0\rangle = \prod_{i=1}^{A} c_i^\dagger|0\rangle$$

where $|0\rangle$ is the physical vacuum.

With the ground state (9) and the mean field approximation, the energy functional—the expectation value of the Hamiltonian (6)—is obtained as:

$$E_{\mathrm{RMF}}[\rho,\phi] = \int d^3x \left\{ \mathrm{Tr}[(-i\alpha\cdot\nabla + \beta M + g_\sigma\sigma + g_\omega\gamma^\mu\omega_\mu + g_\rho\vec{\tau}\cdot\vec{\rho}_\mu\gamma^\mu + e(1+\tau_3)A_\mu\gamma^\mu)\rho] + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + U_\sigma(\sigma$$

where $\phi = \{\sigma, \omega_\nu, \rho_\nu, A_\nu\}$ and the density matrix ρ is defined as:

$$\rho_{ij} \equiv \langle\Phi_0|c_j^\dagger c_i|\Phi_0\rangle = \psi_i^\dagger(x')\psi_j(x)\delta_{ij}$$

For systems with time reversal symmetry, the space-like components of the vector fields vanish. Furthermore, one can assume that in all nuclear applications the nucleon single-particle states do not mix isospin, i.e., the single-particle

states are eigenstates of τ_3, so only the third component of ρ_ν survives. Stationarity implies that the nucleon single-particle wave function can be written as:

$$\psi_i(x) = e^{-i\epsilon_i t}\psi_i(\mathbf{r})$$

where ϵ_i is the single-particle energy. Accordingly, the density matrix reduces to:

$$\rho_{ij} = \psi_j^\dagger(\mathbf{r}')\psi_i(\mathbf{r})\delta_{ij}$$

Altogether, only the meson fields $\sigma, \omega_0, \rho_3^0, A_0$ remain, which are time-independent.

The equations of motion for the nucleon and mesons can be obtained by requiring that the energy functional (10) be stationary with respect to variations of ρ and ϕ. More explicitly, the stationary condition reads:

$$\frac{\delta E_{\mathrm{RMF}}[\rho,\phi]}{\delta\rho} = 0, \quad \frac{\delta E_{\mathrm{RMF}}[\rho,\phi]}{\delta\phi} = 0$$

Using the variation $\delta\rho$ with respect to ψ_i, the stationary condition (14) leads to the Dirac equation for the nucleon and the Klein-Gordon equations for sigma, omega, rho, and the photon:

$$[-i\alpha\cdot\nabla + \beta(M + S(r)) + V(r)]\psi_i(\mathbf{r}) = \epsilon_i\psi_i(\mathbf{r})$$

$$\nabla^2\sigma + U'_\sigma(\sigma) = -g_\sigma\rho_s$$

$$\nabla^2\omega_0 + U'_\omega(\omega_0) = -g_\omega\rho_v$$

$$\nabla^2\rho_3^0 = -g_\rho\rho_3$$

$$\nabla^2 A_0 = -e\rho_c$$

The scalar potential S and vector potential V in equation (15) are respectively:

$$S(\mathbf{r}) = g_\sigma\sigma(\mathbf{r})$$

$$V(\mathbf{r}) = g_\omega \omega_0(\mathbf{r}) + g_\rho \tau_3 \rho_3^0(\mathbf{r}) + e\frac{(1+\tau_3)}{2} A_0(\mathbf{r})$$

While the scalar density (ρ_s), baryon density (ρ_v), isovector density (ρ_3), and charge density (ρ_c) in the Klein-Gordon equations (16a–16d) are respectively:

$$\rho_s(\mathbf{r}) = \mathrm{Tr}[\beta\rho] = \sum_{i=1}^{A} |\psi_i(\mathbf{r})|^2$$

$$\rho_v(\mathbf{r}) = \mathrm{Tr}[\rho] = \sum_{i=1}^{A} \psi_i^\dagger(\mathbf{r})\psi_i(\mathbf{r})$$

$$\rho_3(\mathbf{r}) = \mathrm{Tr}[\tau_3\rho] = \sum_{i=1}^{A} \psi_i^\dagger(\mathbf{r})\tau_3\psi_i(\mathbf{r})$$

$$\rho_c(\mathbf{r}) = \mathrm{Tr}\left[\frac{(1+\tau_3)}{2}\rho\right] = \sum_{i=1}^{A} \psi_i^\dagger(\mathbf{r})\frac{(1+\tau_3)}{2}\psi_i(\mathbf{r})$$

The total energy of the system can be obtained from the energy functional (10) as:

$$E = \mathrm{Tr}[(-i\alpha\cdot\nabla+\beta M)\rho]+\frac{1}{2}g_\sigma\int d^3r\sigma(\mathbf{r})\rho_s(\mathbf{r})+\frac{1}{2}g_\omega\int d^3r\omega_0(\mathbf{r})\rho_v(\mathbf{r})+\frac{1}{2}g_\rho\int d^3r\rho_3^0(\mathbf{r})\rho_3(\mathbf{r})+\frac{1}{2}e\int d^3rA_0(\mathbf{r})\rho_c(\mathbf{r})$$

In the density-dependent RMF approach, where the nonlinear self-couplings for the σ, ω, and ρ mesons in the Lagrangian density are respectively replaced by density dependence of the coupling constants $g_\sigma(\rho), g_\omega(\rho)$, and $g_\rho(\rho)$, an additional term—the rearrangement term—appears in the Dirac equation (15) [26, 51, 59–62].

2.2 Numerical algorithm for spherical nuclei

The harmonic oscillator basis has served as a very useful tool in nuclear structure studies. Normally, the equations of motion for nucleons moving in a mean field are solved by expanding them on the harmonic oscillator (HO) basis [15–17, 35, 63]. However, for exotic nuclei with large spatial extension, e.g., halo nuclei, it is not justified to work in the conventional harmonic oscillator basis due to its localization [36, 64–66]. Instead, one can choose to work either in coordinate space, improve the asymptotic behavior of the HO wave function, or adopt other bases that have correct asymptotic behavior, such as the Woods-Saxon basis.

In this subsection, we focus on the numerical solution of the RMF for spherical nuclei. Due to the special spatial symmetry, both the Dirac equation for the

nucleon and the Klein-Gordon equations for the mesons and photon become radially dependent only, thus greatly facilitating the solution of the coupled equations. The formalism for the spherical relativistic Hartree (SRH) theory is briefly presented. We then review the SRH theory in different bases, including the Finite Element Method (FEM) in coordinate space, transformed harmonic oscillator basis, and the Woods-Saxon basis. The application of the SRH theory to doubly magic nuclei follows.

2.2.1 Spherical relativistic Hartree theory For spherical nuclei, starting from Eqs. (15) and (16a–16d) given in the previous subsection, one derives the coupled radial equations—the radial Dirac equation and radial Klein-Gordon equations. The Dirac spinor, which is the expansion coefficient $\psi_a(r)$ $(a = \alpha, \kappa, m)$ in Eq. (12) (the coordinate has been changed from x to r to reflect spherical symmetry), is characterized by the angular momentum quantum numbers $\kappa(l, j), m$, parity, isospin $t = +1/2$ for neutrons and $-1/2$ for protons, and the radial quantum number α, and has the form:

$$\psi_{\alpha\kappa m}(\mathbf{r}, t) = \begin{pmatrix} \frac{G_{\alpha\kappa}(r)}{r} Y_{ljm}(\theta, \phi)\chi_t \\ i\frac{F_{\alpha\kappa}(r)}{r} Y_{\tilde{l}jm}(\theta, \phi)\chi_t \end{pmatrix}$$

where $G_{\alpha\kappa}(r)$ and $F_{\alpha\kappa}(r)$ are the radial wave functions for the upper and lower components, $Y_{ljm}(\theta, \phi)$ are the spinor spherical harmonics [67], and χ_t is the isospin spinor.

Substituting Eq. (20) into the Dirac equation (15), one obtains the radial Dirac equations:

$$\frac{dG_{\alpha\kappa}(r)}{dr} + \frac{\kappa}{r} G_{\alpha\kappa}(r) - [\epsilon_\alpha - (M + S(r) + V(r))]F_{\alpha\kappa}(r) = 0$$

$$\frac{dF_{\alpha\kappa}(r)}{dr} - \frac{\kappa}{r} F_{\alpha\kappa}(r) + [\epsilon_\alpha + (M + S(r) - V(r))]G_{\alpha\kappa}(r) = 0$$

with scalar and vector potentials:

$$S(r) = g_\sigma \sigma(r), \quad V(r) = g_\omega \omega_0(r) + g_\rho \tau_3 \rho_3^0(r) + e\frac{(1 + \tau_3)}{2} A_0(r)$$

The meson field equations (16a–16d) become radial Laplace equations of the form:

$$\left(\frac{d^2}{dr^2} - m_\phi^2\right) \phi(r) = s_\phi(r)$$

where m_ϕ are the meson masses for $\phi = \sigma, \omega, \rho$ and zero for the photon. The source terms are:

$$s_\phi(r) = \begin{cases} -g_\sigma \rho_s(r) - g_2\sigma^2(r) - g_3\sigma^3(r) & \text{for the } \sigma \text{ field} \\ -g_\omega \rho_v(r) - c_3\omega_0^3(r) & \text{for the } \omega \text{ field} \\ -g_\rho \rho_3(r) - d_3[\rho_3^0(r)]^3 & \text{for the } \rho \text{ field} \\ -e\rho_c(r) & \text{for the Coulomb field} \end{cases}$$

The densities are calculated as:

$$\rho_s(r) = \sum_i \frac{2j_i+1}{4\pi r^2}[G_i^2(r) - F_i^2(r)]$$

$$\rho_v(r) = \sum_i \frac{2j_i+1}{4\pi r^2}[G_i^2(r) + F_i^2(r)]$$

$$\rho_3(r) = \sum_i \frac{2j_i+1}{4\pi r^2}[G_i^2(r) + F_i^2(r)]\tau_{3i}$$

$$\rho_c(r) = \sum_i \frac{2j_i+1}{4\pi r^2}[G_i^2(r) + F_i^2(r)]\frac{(1+\tau_{3i})}{2}$$

The procedure for solving these coupled equations is as follows: (a) with an initial set of estimated meson and photon fields, the scalar and vector potentials in Eqs. (22) are calculated and the radial Dirac equation is solved; (b) the obtained nucleon wave functions are used to construct the source term of each radial Laplace equation for mesons and the photon; and (c) the new meson and photon fields obtained from solving these Laplace equations are used to replace the fields in step (a). This procedure is iterated until the desired accuracy is achieved.

With the above procedure, there are two methods to solve the coupled equations (21–25). One is in coordinate space with the shooting method [68]—spherical relativistic Hartree theory in r-space (SRHR)—and the finite element method [42, 69–71] (SRHFEM). Another is in configuration space, e.g., the harmonic oscillator basis [72] (SRHHO), the transformed harmonic oscillator basis [64] (SRHTHO), and the Woods-Saxon basis [73] (SRHWS). Readers are referred to Ref. [68] and Ref. [72] for SRHR and SRHHO, respectively.

In the following, numerical techniques for solving the Dirac equation using the finite element method, the transformed harmonic oscillator basis, and Woods-Saxon basis are briefly introduced.

2.2.2 Finite element method A convenient procedure for coordinate space discretization of the Dirac equations (21) is provided by the Finite Element Method (FEM) [42, 69–71]. Similar to the r-space method [68], the Dirac equations (21) are solved in a box with box size R and proper boundary conditions. The radial wave functions $G_{\alpha\kappa}$ and $F_{\alpha\kappa}$ are discretized at $N+1$ points: $r_1 = 0, r_2 = \Delta r, ..., r_{N+1} = N\Delta r$, where $\Delta r = R/N$. The wave functions thus become tabulated values: $G_{\alpha\kappa}(r_1), ..., G_{\alpha\kappa}(r_{N+1})$ and $F_{\alpha\kappa}(r_1), ..., F_{\alpha\kappa}(r_{N+1})$.

The idea of FEM is as follows: if $\Delta r = R/N$ is small enough (or equivalently N is large enough), the wave function $G_{\alpha\kappa}(r)$ in the element $[r_i, r_{i+1}]$ can be well approximated by $G_{\alpha\kappa}(r_i)$ and $G_{\alpha\kappa}(r_{i+1})$ together with some simple analytic function, e.g., linear, quadratic, cubic, or 4th-order shape functions [69].

Taking the linear shape functions $\eta_a(r) = 1 - \rho$ and $\eta_b(r) = \rho$, with $\rho = (r - r_i)/\Delta r$ and $r \in [r_i, r_{i+1}]$, as an example, the wave function $G_{\alpha\kappa}(r)$ can be approximated by:

$$G_{\alpha\kappa}(r) = G_{\alpha\kappa}(r_i)\eta_a(r) + G_{\alpha\kappa}(r_{i+1})\eta_b(r), \quad r \in [r_i, r_{i+1}]$$

A similar expression can be obtained for $F_{\alpha\kappa}(r)$. For $r \in [r_i, r_{i+1}]$, the Dirac equations (21) can now be written as:

$$\begin{pmatrix} M + S(r) + V(r) & -\frac{d}{dr} + \frac{\kappa}{r} \\ \frac{d}{dr} + \frac{\kappa}{r} & -[M + S(r) - V(r)] \end{pmatrix} \begin{pmatrix} G_{\alpha\kappa}(r_i)\eta_a(r) + G_{\alpha\kappa}(r_{i+1})\eta_b(r) \\ F_{\alpha\kappa}(r_i)\eta_a(r) + F_{\alpha\kappa}(r_{i+1})\eta_b(r) \end{pmatrix} = \epsilon_\alpha \begin{pmatrix} G_{\alpha\kappa}(r_i)\eta_a(r) + G_{\alpha\kappa}(r_{i+1}) \\ F_{\alpha\kappa}(r_i)\eta_a(r) + F_{\alpha\kappa}(r_{i+1}) \end{pmatrix}$$

Multiplying from the left by $(G_{\alpha\kappa}(r_i)\eta_a(r) + G_{\alpha\kappa}(r_{i+1})\eta_b(r), F_{\alpha\kappa}(r_i)\eta_a(r) + F_{\alpha\kappa}(r_{i+1})\eta_b(r))$ and integrating with r^2dr, algebraic equations for $G_{\alpha\kappa}(r_i), G_{\alpha\kappa}(r_{i+1})$ and $F_{\alpha\kappa}(r_{i+1})$ are obtained. Repeating this procedure for all N elements, the Dirac equations (21) are transformed into a generalized eigenvalue problem. This generalized eigenvalue problem can be solved by standard diagonalization algorithms, yielding all energies ϵ_α and wave functions discretized at $r = r_1, r_2, ..., r_{N+1}$.

It has been shown that FEM can provide very accurate solutions for the relativistic eigenvalue problem in the self-consistent mean-field approximation [42, 69–71].

Compared with the shooting method in r-space [68], FEM has the advantage that one can obtain all energies ϵ_α and wave functions $G_{\alpha\kappa}, F_{\alpha\kappa}$ by a single diagonalization, and it is straightforward to generalize to cases with nonlocal interactions in the pairing channel. However, to obtain the same accuracy as the shooting method, a huge matrix must be constructed, making it more time-consuming.

One can also replace the linear shape functions $\eta_a(r)$ and $\eta_b(r)$ in Eq. (26) with quadratic, cubic, or 4th-order shape functions; the procedures are the same,

and in principle the same accuracy can be achieved with linear shape functions using larger N.

2.2.3 Transformed harmonic oscillator basis To modify the asymptotic behavior of the harmonic oscillator wave function at large r, the local-scaling transformation method [74–76] has been introduced to construct the so-called local-scaling transformed harmonic oscillator basis (THO) in Refs. [64, 65].

A local-scaling point coordinate transformation (LST) is defined as:

$$\mathbf{r}' = f(r)\hat{\mathbf{r}}$$

where the transformed radius vector has the same direction $\hat{\mathbf{r}}$, while its magnitude $r' = f(r)$ depends on the scalar LST function $f(r)$. The function $f(r)$ is assumed to be an increasing function of r, with $f(0) = 0$. The corresponding LST wave function can be expressed as:

$$\Psi_f(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A) = \left[\prod_{i=1}^{A} f^2(r_i)\frac{\partial f(r_i)}{\partial r_i}\right]^{1/2} \bar{\Psi}(f(r_1), f(r_2), ..., f(r_A))$$

where $\bar{\Psi}(r_1, r_2, ..., r_A)$ is an A-particle wave function normalized to unity. The local one-body density corresponding to an A-body wave function Ψ is:

$$\rho(\mathbf{r}) = A\int d^3r_2...d^3r_A|\Psi(\mathbf{r}, \mathbf{r}_2, ..., \mathbf{r}_A)|^2$$

There exists a simple relation between the local density $\rho_f(r)$ associated with the LST function Ψ_f and the density $\bar{\rho}(r)$ corresponding to the prototypical model function $\bar{\Psi}$:

$$\rho_f(r) = f^2(r)\frac{\partial f(r)}{\partial r}\bar{\rho}(f(r))$$

When the form of the density $\rho_f(r)$ is known, Eq. (32) becomes a first-order nonlinear differential equation for the LST function f and can be solved easily. For a system with spherical symmetry, $\rho_f, \bar{\rho}$, and f depend only on $r = |\mathbf{r}|$, and Eq. (32) can be reduced to a nonlinear algebraic equation:

$$\int_0^{f(r)} \bar{\rho}(u)u^2du = \int_0^r \rho_f(u)u^2du$$

The solution can be found subject to the boundary condition $f(0) = 0$.

For shell-model or mean-field applications, one must consider the case where the model many-body wave function is a Slater determinant:

$$\bar{\Psi}(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A) = \frac{1}{\sqrt{A!}} \det[\bar{\phi}_i(\mathbf{r}_j)]$$

The single-particle functions $\bar{\phi}_i(r)$ form a complete set. The LST wave function is defined by transformation (30) and written as a product state of the transformed basis states:

$$\Psi_f(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A) = \frac{1}{\sqrt{A!}} \det[\phi_i(\mathbf{r}_j)]$$

with:

$$\phi_i(\mathbf{r}) = f^{3/2}(r)\bar{\phi}_i(f(r)\hat{\mathbf{r}})$$

In Refs. [64, 65], the transformed harmonic oscillator basis (THO) has been derived by a local scaling-point transformation of the spherical harmonic-oscillator radial wave functions. The unitary scaling transformation produces a basis with improved asymptotic properties. The THO basis is employed in the solution of the relativistic Hartree-Bogoliubov (RHB) equations in configurational space [64]. As shown in Fig. 1, an expansion of nucleon spinors and mean-field potentials in the THO basis reproduces the asymptotic properties of neutron densities calculated by FEM in coordinate space.

2.2.4 Woods-Saxon basis Woods-Saxon basis from the Schrödinger equation (SWS basis)

For the Schrödinger equation with a spherical Woods-Saxon potential:

$$V_{\rm WS}(r) = \frac{V_0}{1 + e^{(r-R_0)/a_0}}$$

for $r < R_{\rm max}$, and $V_{\rm WS}(r) = 0$ for $r \geq R_{\rm max}$, where $R_{\rm max}$ is introduced for practical reasons to define the box boundary. The eigenfunction can be written as $\phi_{nlml}(r) = R_{nl}(r)Y_{lml}(\theta, \phi)$, and its radial Schrödinger equation is:

$$\left[-\frac{1}{2m}\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right) + \frac{l(l+1)}{2mr^2} + V_{\rm WS}(r)\right] R_{nl}(r) = E_{nl}R_{nl}(r)$$

Equation (38) is solved on a discretized radial mesh with mesh size Δr. $R_{\rm max}$ (Δr) is chosen large (small) enough to ensure that final results do not depend on it. The radial wave functions thus obtained form a complete basis $\{R_{nl}(r); n =$

$0, 1, ...; l = 0, 1, ...\}$ in terms of which the radial parts of the upper and lower components of the Dirac spinor in Eq. (21) are expanded respectively as:

$$G_{\alpha\kappa}(r) = \sum_{n=0}^{n_{\max}} g^{\kappa}_{\alpha n} R_{nl}(r)$$

$$F_{\alpha\kappa}(r) = \sum_{\tilde{n}=0}^{\tilde{n}_{\max}} f^{\kappa}_{\alpha\tilde{n}} R_{\tilde{n}\tilde{l}}(r)$$

The radial Dirac equation (21) is transformed into the WS basis as:

$$\sum_{n} A_{mn} g^{\kappa}_{\alpha n} + \sum_{\tilde{n}} B_{m\tilde{n}} f^{\kappa}_{\alpha\tilde{n}} = \epsilon_{\alpha} g^{\kappa}_{\alpha m}$$

$$\sum_{n} C_{\tilde{m}n} g^{\kappa}_{\alpha n} + \sum_{\tilde{n}} D_{\tilde{m}\tilde{n}} f^{\kappa}_{\alpha\tilde{n}} = \epsilon_{\alpha} f^{\kappa}_{\alpha\tilde{m}}$$

where the matrix elements are calculated as:

$$A_{mn} = \int r^2 dr R_{ml}(r)[V(r) + S(r) + M]R_{nl}(r)$$

$$B_{m\tilde{n}} = -\int r^2 dr R_{ml}(r) \left[\frac{d}{dr} - \frac{\kappa}{r}\right] R_{\tilde{n}\tilde{l}}(r)$$

$$C_{\tilde{m}n} = \int r^2 dr R_{\tilde{m}\tilde{l}}(r) \left[\frac{d}{dr} + \frac{\kappa}{r}\right] R_{nl}(r)$$

$$D_{\tilde{m}\tilde{n}} = \int r^2 dr R_{\tilde{m}\tilde{l}}(r)[V(r) - S(r) - M]R_{\tilde{n}\tilde{l}}(r)$$

In practical calculations, an energy cutoff E_{cut} (relative to the nucleon mass M) is used to determine the cutoff of the radial quantum number $n_{\max}$ [73].

Woods-Saxon basis from the Dirac equation (DWS basis)

The radial Dirac equation (21) may be solved in r-space [68] with Woods-Saxon-like potentials for $V_0(r)$ and $S_0(r)$ [77] within a spherical box of size $R_{\max}$, together with spherical spinors that give a complete WS basis $\{\psi^0_{n\kappa m}(r, s, t)\}$ with $n = 0, 1, ..., = ..., and $m = -j_{\kappa}, ..., j_{\kappa}$. In such cases, states both in the Fermi sea and in the Dirac sea should be included in the basis for completeness. The nucleon wave function (20) can be expanded in terms of this basis as:

$$\psi_{\alpha\kappa m}(\mathbf{r},s,t)=\sum_{n=0}^{n_{\max}}c_{\alpha n}\psi^{0}_{n\kappa m}(\mathbf{r},s,t)$$

where $n_{\max}=n^{+}_{\max}+n^{-}_{\max}+1$ and the summation runs over positive energy levels in the Fermi sea for $n^{+}_{\max}$ and over negative energy levels in the Dirac sea for $n^{-}_{\max}$. The negative energy states are obtained with the same method as the positive energy ones. In this WS basis, the Dirac equation (21) becomes:

$$\sum_{n=1}^{n_{\max}}c_{\alpha n}H'_{mn}=\epsilon_{\alpha}c_{\alpha m},\quad m=1,...,n_{\max}$$

where:

$$H'_{mn}=\epsilon^{0}_{m}\delta_{mn}+\int drF^{0}_{m}(r)[\Delta V(r)+\beta\Delta S(r)]G^{0}_{n}(r)+\int drG^{0}_{m}(r)[\Delta V(r)+\Delta S(r)]F^{0}_{n}(r)$$

with $\Delta V(r)=V(r)-V_0(r)$, $\Delta S(r)=S(r)-S_0(r)$, and the angular, spin, and isospin quantum numbers omitted for brevity.

In the expansion (44) of the nucleon wave function in the SRHDWS theory, one must take into account not only states in the Fermi sea but also those in the Dirac sea, as these states form a complete basis together. The contribution from negative energy states for ^{16}O is given in Table 1. It is found that without including negative energy levels, the calculated results depend on the potentials for the basis. The contribution of negative energy states in the Dirac sea to the wave function can be calculated by $\sum_{i\in\text{Dirac sea}}|c_i|^2$ in the expansion (44), which is around 10^{-4}–10^{-5} (note that the nucleon wave function is normalized to one). However, such a small component from negative energy states in the wave functions contributes to physical observables such as E/A and r_{rms} by magnitudes of 1–10%, as seen from Table 1.

Comparisons between r-space, harmonic oscillator basis, and Woods-Saxon basis

It is found that for stable nuclei, the SRHR, SRHSWS, SRHDWS, and SRHHO approaches are all valid, and results from them are in excellent agreement [73]. However, for unstable nuclei near the neutron drip line, these methods differ from each other to some extent.

In Fig. 2, the neutron density distribution of ^{72}Ca from different SRH approaches is compared. With the same box size, the density distribution from SRHR is almost identical to that from SRHWS, indicating equivalence between SRHWS and SRHR. For brevity, only $\rho_n(r)$ from SRHR with $R_{\max}=35$ fm is displayed, which covers the curve corresponding to $\rho_n(r)$ from SRHWS with

$R_{\max} = 35$ fm in Fig. 2. On the other hand, $\rho_n(r)$ from SRHHO even with $N_{\max} = 43$ fails to reproduce the SRHR result due to the well-known localization property of the HO potential [64].

These results indicate that even the long tail (or halo) behavior in neutron density distribution for nuclei near the drip line can be reproduced quite well by the expansion method in the Woods-Saxon basis in the Schrödinger framework (SRHSWS) and that in the Dirac framework (SRHDWS). This can be seen in that the neutron density distribution obtained in SRHR is reproduced well in both SRHSWS and SRHDWS when a sufficiently large box size is taken.

2.3 Effective interactions

In the Lagrangian density (1), meson masses $m_\sigma, m_\omega, m_\rho$ and meson-nucleon coupling constants $g_\sigma, g_\omega, g_\rho$ remain to be determined, along with the nonlinear self-couplings of the meson fields. These represent the nucleon-nucleon interactions in RMF theory and should in principle be determined either by more fundamental theories or by experiments. However, as relativistic mean field theory is formulated based on the effective Lagrangian in connection with mean-field and no-sea approximations, it is difficult to determine these interactions microscopically. Instead, the masses and coupling strengths of the mesons and the nonlinear self-couplings are determined by reproducing properties of nuclear matter and a few doubly magic nuclei. They are effective interactions in a similar sense as their conventional counterparts. The effective interactions in RMF theory can be determined by minimizing the least-square error:

$$\chi^2(\mathbf{a}) = \sum_{i=1}^{N} \left(\frac{y(\mathbf{x}_i; \mathbf{a}) - y_i^{\exp}}{\sigma_i} \right)^2$$

where $\mathbf{a}$ is the ensemble of meson masses $m_\sigma, m_\omega, m_\rho$ and meson-nucleon coupling constants $g_\sigma, g_\omega, g_\rho$ together with nonlinear self-couplings to be fitted, and $y_i^{\exp}$ and σ_i are experimental observables and corresponding weights. Generally, masses and charge radii of spherical nuclei near the β-stability line are adopted as observables in the least-square fitting procedure.

Among existing effective interactions for RMF theory, the most frequently used are NL1 [78], NLSH [79], TM1 [50], and NL3 [80] with nonlinear self-couplings of mesons. Along the β-stability line, NL1 gives excellent results for binding energies and charge radii and provides an excellent description of superdeformed bands [81, 82]. However, when moving away from the stability line, results are less satisfactory, partly attributable to the large asymmetry energy $J \approx 44$ MeV. Moreover, neutron skin thicknesses calculated with NL1 show systematic deviations from experimental data [83].

In NLSH, this problem was treated better, and improved isovector properties were obtained with an asymmetry energy of $J \approx 36$ MeV. Furthermore, NLSH seems to describe deformation properties better than NL1. However, NLSH

produces slight over-binding along the β-stability line and fails to reproduce superdeformed minima in Hg-isotopes in constrained calculations for the energy landscape. A remarkable difference between these effective interactions is the quite different values predicted for nuclear matter incompressibility [84]: $K = 212$ MeV for NL1 while $K = 355$ MeV for NLSH [85, 86]. As an improvement, NL3 and TM1 provide reasonable compression modulus ($K_{\rm NL3} = 271.7$ MeV, $K_{\rm TM1} = 281.16$ MeV) and asymmetry energy ($J_{\rm NL3} = 37.42$ MeV, $J_{\rm TM1} = 36.89$ MeV) but fairly small baryonic saturation density ($\rho_{\rm TM1} = 0.145$ fm^{-3}).

To improve description of these quantities, two nonlinear self-coupling effective interactions—PK1 with nonlinear σ- and ω-meson self-couplings and PK1R with nonlinear σ-, ω-, and ρ-meson self-couplings—were developed [51] (see Table 2).

To better reproduce experimental quantities such as binding energies and nuclear radii, an additional correction should be added in calculating the energy: the center-of-mass correction. Conventionally, a phenomenological center-of-mass correction of $41A^{-1/3}$ MeV is used. Microscopically, the correction can be calculated by projection-after-variation in first-order approximation [87]:

$$E_{\rm c.m.}^{\rm mic} = \frac{\langle\Phi_0|\mathbf{P}_{\rm c.m.}^2|\Phi_0\rangle}{2mA}$$

where the center-of-mass momentum $\mathbf{P}_{\rm c.m.} = \sum_i \mathbf{p}_i$ and the expectation value of its square reads:

$$\langle\Phi_0|\mathbf{P}_{\rm c.m.}^2|\Phi_0\rangle = \sum_{ab} \mathbf{p}_{ab}\cdot\mathbf{p}_{\bar{a}\bar{b}} v_a u_a v_b u_b$$

accounting for pairing effects, where a, b denote BCS states (see the following section). It should be mentioned that prescription (48) is based on non-relativistic considerations. It does not preserve Lorentz invariance and breaks the complete self-consistency of the variational scheme. However, as it is not included in the self-consistent procedure and only presents an additional correction term to the binding energy, it is satisfactory for current purposes. Compared with binding energy, this center-of-mass correction is sizable in light nuclei (about 9% in ^{16}O) but much less important in medium and heavy nuclei (about 0.4% in ^{208}Pb), as seen in Fig. 3.

For nuclear radii, effects from center-of-mass motion can also be taken into account. Because of its fairly small effects, a rather rough correction is adopted for protons:

$$\mathbf{R}_{\rm c.m.} = \frac{1}{A}\sum_i \mathbf{r}_i$$

where $\mathbf{R}_{\rm c.m.}$ is the center-of-mass coordinate. Then one obtains:

$$R_p^2 = R_M^2 - \frac{N}{A}(R_n^2 - R_p^2)$$

where R_p and R_M denote the proton and matter radii. Here we only consider direct-term contributions to conform to the spirit of RMF theory. For neutron radii, one can follow the same procedure as for protons. The charge radius is obtained from the proton radius combining with proton and neutron sizes and the center-of-mass correction (51) included in R_p^2:

$$R_{\rm ch}^2 = R_p^2 + (0.862\,{\rm fm})^2 - (0.336\,{\rm fm})^2\frac{N}{Z}$$

With the microscopic center-of-mass motion, a multi-parameter fitting can be performed using the Levenberg-Marquardt method [88]. In Ref. [51], masses of ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{68}Ni, ^{90}Zr, ^{116}Sn, ^{132}Sn, ^{194}Pb, and ^{208}Pb and bulk quantities of nuclear matter were chosen as observables to determine the effective interactions. Radii were excluded because proper values of compression modulus K and baryonic saturation density ρ_0 are sufficient to give good description of radii. For a fixed K, a large ρ_0 gives small charge radius and vice versa. Therefore, proper description of both masses and radii of finite nuclei can be obtained by carefully adjusting K and ρ_0. To give a fairly precise description of masses, center-of-mass correction is essential for both light and heavy nuclei. As seen in Fig. 3, deviation between microscopic and phenomenological results is considerably large not only for light nuclei but also for heavy ones, and there exist remarkable shell effects in microscopic results impossible to obtain with phenomenological methods. The microscopic center-of-mass correction [87] is therefore chosen to treat center-of-mass motion.

Because contribution to nuclear masses from the nonlinear ρ-meson term is fairly small, effective interaction PK1R was obtained by fixing the nonlinear self-coupling constant d_3 to 350.0 and adjusting other parameters.

In RMF theory with density-dependent meson-nucleon couplings, the density-dependence of coupling constants g_σ and g_ω can be parameterized as:

$$g_i(\rho_v) = g_i(\rho_{\rm sat})f_i(x), \quad \text{for } i = \sigma, \omega$$

where $f_i(x) = a_i(1 + b_i(x + d_i)^2)/(1 + c_i(x + d_i)^2)$ is a function of $x = \rho_v/\rho_{\rm sat}$, and $\rho_{\rm sat}$ denotes baryonic saturation density of nuclear matter. For the ρ meson, an exponential dependence is utilized as:

$$g_\rho = g_\rho(\rho_{\rm sat})\exp[-a_\rho(x - 1)]$$

For functions $f_i(x)$, there are five constraint conditions: $f_i(1) = 1, f_i'(1) = 0, f_i''(0) = 0$. Then 8 parameters related to density dependence for σ-N and

ω-N couplings are reduced to 3 free parameters. In general, nucleon and ρ-meson masses are fixed, and nonlinear self-coupling constants g_2, g_3, c_3, and d_3 are set to zero. With 4 free parameters for density dependence, there are 8–9 parameters left free in the Lagrangian density (1) for density-dependent meson-nucleon coupling RMF theory. A density-dependent meson-nucleon coupling effective interaction PKDD has also been obtained in Ref. [51] (see Tables 2 and 3).

Tables 2 and 3 tabulate new effective interactions PK1, PK1R, and PKDD [51] in comparison with older ones TM1 [50], NL3 [80], TW99 [61], and DD-ME1 [62]. The newly obtained ones reproduce experimental masses [89] better. PK1, PK1R, and PKDD also describe charge radii very well, especially for Pb isotopes. More comprehensive comparisons between Hartree-Fock-Bogoliubov, extended Thomas-Fermi model with Strutinski integral, RMF, and macroscopic-microscopic approaches with different forces have been performed for description of nuclear masses and charge radii of spherical even-even nuclei (116 nuclides) from light ($A = 16$) to heavy ($A = 220$) in Ref. [90].

Table 4 lists nuclear matter quantities calculated with newly obtained effective interactions PK1, PK1R, and PKDD, in comparison with other interactions. All new effective interactions give proper values for compression modulus K.

2.4 Density and isospin dependence of effective interactions

There are quite a number of effective interactions: PK1, PK1R, PKDD [51] together with NL1, NL2 [91], NL3 [80], NLSH [79], TM1, TM2 [50], GL-97 [92], and density-dependent effective interactions TW-99 [61], DD-ME1 [62], etc. It is very interesting to investigate density and isospin dependence of interaction strengths of various effective interactions in RMF theory and study their effects on nuclear matter [93].

Table 4 shows nuclear matter properties calculated with PK1, PK1R, and PKDD [51] compared with TM1 [50], NL3 [80], TW99 [61], and DD-ME1 [62]. Although for nonlinear self-coupling effective interactions, density dependencies are only embodied in the Klein-Gordon equations, it is still worthwhile to obtain quantitative understanding of coupling constants.

In Fig. 4, density dependencies of interaction strengths for g_σ (top), g_ω (middle), and g_ρ (bottom) in nuclear matter as functions of nuclear density are shown. Curves in the figures from top to bottom are labeled in order from left to right. The shadowed area corresponds to empirical saturation point in nuclear matter (Fermi momentum $k_F = 1.35$ fm^{-1} or density $\rho = 0.166$ fm^{-3}). For nonlinear effective interactions, the "equivalent" density dependence of interaction strengths for σ, ω, and ρ is extracted from meson field equations according to [51]:

$$g_\sigma^{\rm eff}(\rho) = g_\sigma + \frac{g_2\sigma(\rho) + g_3\sigma^2(\rho)}{\rho_v}$$

$$g_{\omega}^{\text{eff}}(\rho) = g_{\omega} + \frac{c_3\omega_0^3(\rho)}{\rho_v}$$

$$g_{\rho}^{\text{eff}}(\rho) = g_{\rho} + \frac{d_3(\rho_3^0(\rho))^3}{\rho_3}$$

For the σ-meson, TW-99 and DD-ME1 exhibit quite different behaviors for g_σ compared to other effective interactions in either magnitude or slope. In particular, strengths in TW-99 and DD-ME1 for the density interval in Fig. 4 are almost two times larger than that of GL-97. Quite different results can also be seen at empirical nuclear matter densities. For the ω-meson, except for TW-99, DD-ME1, TM1, and GL-97, strengths in all other effective interactions are density-independent. However, strengths are closer to each other at empirical saturation density than those of the σ-meson, although large differences can be seen at low densities. For the ρ-meson which describes isospin asymmetry, strengths for TW-99 and DD-ME1 show strong density dependence in contrast to other effective interactions, while those of PK1, PK1R, and PKDD are just in between [51].

In Fig. 5, binding energy per particle E/A as a function of baryonic density ρ is shown for symmetric nuclear matter (left) and neutron matter (right). All density-dependent meson-nucleon coupling effective interactions give softer results than nonlinear self-coupling ones, especially for neutron matter. Behaviors predicted by PK1, PK1R are much softer than NL3 and slightly harder than TM1. Results from PKDD are slightly softer than DD-ME1 and much harder than TW99 at high densities. These behaviors can be explained in the density-dependent meson-nucleon coupling framework.

As mentioned in expressions (56), meson-nucleon coupling constants in nonlinear self-coupling mesons can be expressed as some kind of density dependence. Fig. 4 shows density dependence of coupling constants for nonlinear self-coupling effective interactions and density-dependent versions, where almost all density-dependent coupling constants decrease with increasing density except for g_σ of NL3, NLSH, NL1, which have strong σ^4 self-couplings. On the other hand, coupling constants g_σ and g_ω of TM1, which has relatively weak σ self-coupling ($g_3 = 0.6183$) and strong ω self-coupling ($c_3 = 71.3075$), are smaller than others, meaning TM1 provides relatively weaker scalar and vector potentials. This is why TM1 presents softer behavior than other nonlinear self-coupling effective interactions.

In Fig. 5, TW-99 predicts the softest results because of its relatively small g_ω compared with DD-ME1, PKDD, and NL3, and large g_σ compared with PK1, PK1R, and TM1 in Fig. 4. As is known, repulsive potential dominates at high densities. In Fig. 5, NL3 gives the hardest results because of its constant and large g_ω even t

Note: Figure translations are in progress. See original paper for figures.