A new barrier penetration formula and its application to -decay half-lives Postprint

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Abstract

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Full Text

Preamble

A New Barrier Penetration Formula and Its Application to -Decay Half-Lives

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Abstract: Starting from the WKB approximation, we propose a new barrier penetration formula for potential barriers containing a long-range Coulomb interaction. This formula is especially appropriate for barrier penetration with penetration energy much lower than the Coulomb barrier. The penetrabilities calculated from the new formula agree well with results from the WKB method.

As a first attempt, we use this new formula to evaluate -decay half-lives of atomic nuclei and obtain good agreement with experiment.

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Introduction

As a common quantum phenomenon, tunneling through a potential barrier plays a very important role in the microscopic world and has been studied extensively since the birth of quantum mechanics. One of the earliest applications of quantum tunneling was the explanation of decays in atomic nuclei. The quantum tunneling effect also governs many other nuclear processes such as fission and fusion. In particular, many new features are revealed in sub-barrier fusion reactions that are closely connected with tunneling phenomena [?, ?, ?, ?].

For most potential barriers, the penetrability cannot be calculated analytically [?]. Among those potentials for which analytical solutions can be obtained, the parabolic potential [?, ?] is the most commonly used in nuclear fusion studies. By approximating the Coulomb barrier as a parabola, Wong derived an analytic expression for the fusion cross section [?] that is widely adopted today in heavy-ion reaction studies (see, e.g., recent Refs. [?, ?]). The parabolic approximation works remarkably well for both penetrability and fusion cross section at energies around or above the Coulomb barrier [?].

However, the parabolic approximation breaks down at energies much smaller than the barrier height due to the long-range Coulomb interaction. One may calculate the penetration probability numerically using the path integral method or the WKB approximation. However, it is highly desirable to have an analytical expression for barrier penetrability when introducing an energy-dependent one-dimensional potential barrier [?] or barrier distribution functions [?, ?, ?, ?, ?].

In the present work, we derive a new barrier penetration formula based on the WKB approximation. The influence of the long Coulomb tail in the barrier potential is properly accounted for, making this formula especially applicable to barrier penetration with penetration energy much lower than the Coulomb barrier. As a first attempt and test study, we apply this new formula to evaluate -decay half-lives of atomic nuclei.

For decay, the penetrability is usually calculated with the WKB approach [?, ?, ?], which involves numerically integrating the wave number between two turning points where the interaction potential equals the Q-value of the decay. We will show that our analytical formula reproduces experimental results very well, especially for spherical nuclei.

The paper is organized as follows. In Sec. II we present the new barrier penetration formula. The validity of the new formula is investigated and its application to decays is given in Sec. III. Finally, we summarize our work in Sec. IV. The detailed derivation of the new penetration formula is provided in the Appendix.

Formalism

When the penetration energy is well below the Coulomb barrier, the barrier penetrability formula derived from the WKB approximation reads

$$P(E) = \exp\left[-2\int_{R_{\text{in}}}^{R_{\text{out}}} \sqrt{\frac{2\mu}{\hbar^2}(V(R)-E)}\, dR\right]$$

where the potential usually consists of three parts: the nuclear, Coulomb, and centrifugal potentials

$$V(R) = V_N(R) + V_C(R) + \frac{L(L+1)\hbar^2}{2\mu R^2}$$

with R_{in} and R_{out} being the inner and outer turning points determined by the relation $V(R) = E$.

By approximating $V(R)$ as a parabola with height V_B and width $\hbar\omega$, Eq. (1) reduces to

$$P(E) = \exp\left[-\frac{2\pi}{\hbar\omega}(V_B - E)\right]$$

which has been widely used in heavy-ion reaction studies.

Because of the long-range Coulomb interaction, the Coulomb barrier given in Eq. (2) has a long tail and is asymmetric. Thus for penetration well below the barrier, the parabolic approximation is not valid. We may divide the potential barrier into two parts at the barrier position R_B. The first part of $V(R)$ with $R_{\text{in}} < R < R_B$ could still be approximated by half of a parabola, and we need to evaluate the integration in Eq. (1) only in the range $R_B < R < R_{\text{out}}$. For S-wave, the integral in Eq. (1) is evaluated as

$$P(E) = \exp[-(x_1 + x_2)]$$

where

$$x_1 = \int_{R_{\text{in}}}^{R_B} \sqrt{\frac{2\mu}{\hbar^2}(V_B - \frac{1}{2}\mu\omega^2(R-R_B)^2 - E)}\, dR$$

under the parabolic approximation and

$$x_2 = \int_{R_B}^{R_{\text{out}}} \sqrt{\frac{2\mu}{\hbar^2}(V(R)-E)}\, dR = 2kR_B\left[\arcsin\sqrt{\tau} - \sqrt{\tau(1-\tau)}\right] - \ln[1 + e^{(R_0 - R_B)/a}]$$

with $k = \sqrt{2\mu E}/\hbar$ and $\tau = V_C(R_B)/E$. The details of the derivation of Eq. (6) are given in the Appendix. It should be mentioned that in the derivation of Eq. (6), a Woods-Saxon form is used for $V_N(R)$.

Results and Discussions

In this section, we use the new formula to study the typical barrier penetration problem of decays in atomic nuclei. The -decay half-life is related to the decay width Γ by [?, ?, ?]

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma}$$

The decay width Γ is calculated as [?]

$$\Gamma = \hbar \nu S P(Q) = \hbar \xi P(Q)$$

where ν is the assault frequency of the particle on the barrier, S the spectroscopic or preformation factor, and $P(Q)$ the penetrability with Q being the -decay Q-value. For spherical nuclei, ξ is parametrized as [?]

$$\xi = (6.1814 + 0.2988 A^{-1/6}) \times 10^{20}\,\mathrm{s}^{-1}$$

and the penetrability is calculated with Eqs. (4), (5), and (6).

For the -nuclear interaction, we adopt the Coulomb and Woods-Saxon potentials and parameters proposed in Ref. [?]:

$$V_C(R) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R_C}\left(3 - \frac{R^2}{R_C^2}\right) & \text{for } R \le R_C \\ \frac{Z_1 Z_2 e^2}{R} & \text{for } R > R_C \end{cases}$$

$$V_N(R) = \frac{V(A, Z, Q)}{1 + \exp[(R - R_m)/a]}$$

where A and Z are the mass and charge numbers of the daughter nucleus and Q the -decay energy. The parameters in these potentials and given in Eq. (9) were obtained by fitting -decay half-lives and cross-section data for several fusion reactions. It can be easily verified that the position of the Coulomb barrier R_B is larger than R_m, thus the use of the Coulomb force given in Eq. (A3) is valid.

A. Validity of the New Formula

Before applying the new formula to study decays, we investigate its validity in detail. First we examine how the effective potential connected with the new formula Eq. (6) compares to the exact potential. Two extreme examples are chosen for this purpose: ^{212}Po which has a quite long half-life 7.24×10^{-7} s and ^{144}Nd which has a very short half-life 3.02×10^{22} s [?]. The barrier potential $V(R)$ is shown in Fig. 1 for these two systems.

The effective potential is also shown for comparison. $R_V \approx 12$ fm is the radial position outside of which the nuclear part of the -nucleus potential could be neglected (see the Appendix for more details). In our calculations, the width of the parabolic potential is obtained by fitting the barrier potential from the inner turning point R_{in} to the position of the barrier R_B. Unlike the full parabolic approximation, the effective potential is asymmetric and coincides with the exact potential very well, especially on the outer side of the barrier which critically influences decays.

In order to examine more closely the accuracy of the new formula, we list the calculated penetration probabilities for decays of polonium isotopes in Table I. The values in the exponential of Eq. (4), x_1 and x_2, calculated from the WKB approach, the parabolic approximation, and the new formula are compared. We find good agreement between the results from the new formula and the WKB approach. For x_2, the average relative root-mean-square deviation is 0.28%. This demonstrates that the present formula could be used with satisfactory accuracy in studying barrier penetration well below the Coulomb barrier.

B. -Decay Half-Lives

The new barrier penetration formula is used to calculate -decay half-lives of 344 nuclei collected in Ref. [?]. The experimental values of -decay half-lives are also taken from Ref. [?], except for ^{215}Po. The experimental value is $\log(T_{1/2}^{\text{Exp}}/\text{s}) = 3.74$ for ^{215}Po in Ref. [?], while in Refs. [?, ?, ?] the experimental value is $\log(T_{1/2}^{\text{Exp}}/\text{s}) = 2.75$. We take the latter value in the present work. The experimental values of the -decay half-lives range from 10^{-7} to 10^{24} s.

The Q-values of the decays are also taken from Ref. [?], where these values were calculated from the Atomic Mass Evaluation by Audi et al. [?] or from the mass table by Möller et al. [?].

The angular momentum L carried by the emitted particle in a ground-state to ground-state transition of even-even nucleus is zero. In odd-A or odd-odd nuclei, L could be non-zero. Because information on L is absent, in the present work we assume $L = 0$ for all decays, as is usually done [?, ?, ?, ?, ?].

In Fig. 2, the calculated results and experimental values for -decay half-lives are compared. To show this clearly, these 344 nuclei are divided into four groups: 159 even-even, 72 even-odd (even-Z and odd-N), 66 odd-even, and

47 odd-odd nuclei. The ratios between the calculated and experimental values $S_\alpha = \log_{10}(T_{1/2}^{\mathrm{Cal}}/T_{1/2}^{\mathrm{Exp}})$ are presented in Fig. 3. Two dashed lines are drawn to guide the eye. We find that most calculated results are of the same order of magnitude as the experimental values.

A statistical analysis of the agreement between calculation and experiment is presented in Table II. Among all 344 nuclei, only seven have calculated -decay half-lives that deviate by more than two orders of magnitude from the corresponding experimental values, and 93.90% agree with experimental values within one order of magnitude.

Our results are particularly good for even-even nuclei: the calculated half-lives for 97.48% of the 159 even-even nuclei deviate from experiment by less than one order of magnitude. The ratio S_α is less than one for 95.83% of 72 even-Z and odd-N nuclei, 90.91% of 66 odd-even nuclei, and 82.98% of 47 odd-odd nuclei. The angular momentum carried by the emitted particle might not be zero for odd-A or odd-odd nuclei, which introduces some errors for these nuclei in our calculation because the centrifugal potential is ignored in the present study.

Deformation influences the -decay lifetime both in the preformation mechanism and in the penetration process [**?**, **?**, **?**, **?**]. In the present work, we have assumed the barrier potential to be spherical. In 68 of these 344 nuclei, the spherical potential assumption is well met (with $|\beta_2| < 0.01$ for the daughter nucleus [**?**]). In Table III, the calculated and experimental values of the -decay half-lives for these nuclei are given. The statistical summary is also shown in the last line of Table II. We find that the new formula gives very good results for these spherical nuclei. In most cases, the differences between the calculated and experimental values of $\log_{10} T_{1/2}$ are smaller than 0.5. The root-mean-square deviation between $\log_{10}[T_{1/2}^{\mathrm{Cal}}/\mathrm{s}]$ and $\log_{10}[T_{1/2}^{\mathrm{Exp}}/\mathrm{s}]$ is 0.34.

Conclusion

In the study of barrier penetration in nuclear physics, the parabolic approximation is usually adopted because an analytical solution exists for the penetrability of a parabolic barrier potential. The parabolic approximation works indeed well for both penetrability and fusion cross section at energies around or above the Coulomb barrier, but it fails at energies much smaller than the barrier height due to the long-range Coulomb interaction.

In the present work, we derived a new barrier penetration formula, Eq. (6), based on the WKB approximation. We properly accounted for the influence of the long Coulomb tail in the barrier potential. Therefore, this formula is especially applicable to barrier penetration with penetration energy much lower than the Coulomb barrier. We have shown that the present analytical formula reproduces the WKB results very well.

This new penetration formula is used to calculate -decay half-lives of 344 nuclei with the -nucleus potential given in Ref. [**?**]. Satisfactory agreement between the present calculation and experiment is achieved. For spherical and even-even nuclei, the results are particularly good. Therefore, the new formula could be used in the study of barrier penetration at energies much smaller than the barrier height. Furthermore, we expect that the new formula will facilitate the study of barrier penetrability where one has to introduce an energy-dependent one-dimensional potential barrier or a barrier distribution function.

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Appendix A: Derivation of the New Penetration Formula

In order to evaluate the integration x_2 in Eq. (4), we divide the potential between the position of the barrier R_B and the outer turning point R_{out} into two parts, $R_B \leq R \leq R_V$ and $R_V \leq R \leq R_{\text{out}}$. R_V should be large enough so that the nuclear potential vanishes for $R \geq R_V$ and $R_V \leq R_{\text{out}}$.

For S-wave, x_2 is written as

$$x_2 = \frac{2}{\hbar}\int_{R_B}^{R_{\text{out}}} \sqrt{2\mu[V_N(R) + V_C(R) - E]}\, dR$$

$$= \frac{2}{\hbar}\int_{R_B}^{R_V} \sqrt{2\mu[V_N(R) + V_C(R) - E]}\, dR + \frac{2}{\hbar}\int_{R_V}^{R_{\text{out}}} \sqrt{2\mu[V_C(R) - E]}\, dR$$

Since the Coulomb potential outside the barrier ($R \geq R_B$) is well described by [c.f. Eq. (10)]

$$V_C(R) = \frac{Z_1 Z_2 e^2}{R}$$

the first term in the above equation can be evaluated easily as

$$\frac{2}{\hbar}\int_{R_V}^{R_{\text{out}}} \sqrt{2\mu[V_C(R) - E]}\, dR = 2kR_B \left[\arcsin\sqrt{\tau} - \sqrt{\tau(1-\tau)}\right]$$

with $k = \sqrt{2\mu E}/\hbar$ and $\tau = V_C(R_B)/E$.

For the evaluation of the second term in Eq. (A2), we adopt a Woods-Saxon form for the nuclear part of the barrier potential

$$V_N(R) = \frac{V_0}{1 + \exp[(R - R_0)/a]}$$

and replace $V_C(R_B) - V_C(R) - E$ in the denominator by $V_N(R) + V_C(R) - E \approx V_N(R) + V_C(R_B) - E$. This leads to

$$\frac{2}{\hbar} \int_{R_B}^{R_V} \sqrt{2\mu[V_N(R) + V_C(R) - E]}\, dR \approx -\ln[1 + e^{(R_0 - R_B)/a}]$$

In the above derivation, we have used the fact that $\exp[(R_V - R_0)/a] \gg 1$ for decay and penetration well below the Coulomb barrier. Finally, we have an analytical expression for x_2:

$$x_2 = 2kR_B \left[\arcsin\sqrt{\tau} - \sqrt{\tau(1-\tau)}\right] - \ln[1 + e^{(R_0 - R_B)/a}]$$

It has been verified that when R_V is not very close to R_{out}, $V_N(R)/(V_C(R) - E) \ll 1$, therefore

$$\sqrt{V_C(R) - E + V_N(R)} \approx \sqrt{V_C(R) - E} + \frac{V_N(R)}{2\sqrt{V_C(R) - E}}$$

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Note: Figure translations are in progress. See original paper for figures.