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Abstract

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Full Text

Neutron Halo in Deformed Nuclei

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Abstract

Halo phenomena in deformed nuclei are investigated within a deformed relativistic Hartree-Bogoliubov (DRHB) theory. These weakly bound quantum systems present interesting examples for the study of the interdependence between the deformation of the core and the particles in the halo. Contributions of the halo, deformation effects, and large spatial extensions of these systems are described in a fully self-consistent way by the DRHB equations in a spherical Woods-Saxon basis with the proper asymptotic behavior at large distance from the nuclear center. Magnesium and neon isotopes are studied and detailed results are presented for the deformed neutron-rich and weakly bound nucleus ^{24}Mg . The core of this nucleus is prolate, but the halo has a slightly oblate shape. This indicates a decoupling of the halo orbitals from the deformation of the core. The generic conditions for the occurrence of this decoupling effect are discussed.

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Introduction

The “shape” provides an intuitive understanding of spatial density distributions in quantum many-body systems, such as molecules [?], atoms [?], atomic nuclei [?], or mesons [?]. Quadrupole deformations play an important role in this context. The interplay between quadrupole deformation and weak binding can result in new phenomena, such as “quadrupole-bound” anions [?].

Halo phenomena in nuclei are driving forces for the development of physics with radioactive ion beams. They are threshold effects [?] and have been first observed in the weakly bound system ^{11}Li [?]. Considering that most open-shell nuclei are deformed, the interplay between deformation and weak binding raises interesting questions, such as whether or not there exist halos in deformed nuclei and, if yes, what are their new features.

Calculations in a deformed single-particle model [?] have shown that valence particles in specific orbitals with low projection of the angular momentum on the symmetry axis can give rise to halo structures in the limit of weak binding. The deformation of the halo is in this case solely determined by the intrinsic structure of the weakly bound orbitals. Indeed, halos in deformed nuclei were investigated in several mean-field calculations in the past [?, ?, ?]. However, in Ref. [?], it has been concluded that in the neutron orbitals of an axially deformed Woods-Saxon potential the lowest- Ω component becomes dominant at large distances from the origin and therefore all $\Omega = 1/2$ levels do not contribute to deformation for binding energies close to zero. Such arguments raise doubt about the existence of deformed halos.

In addition, a three-body model study [?] suggests that it is unlikely to find halos in deformed drip-line nuclei because the correlations between the nucleons and those due to static or dynamic deformations of the core inhibit the formation of halos.

Theoretical Framework

Therefore, a model which provides an adequate description of halos in deformed nuclei must include in a self-consistent way the continuum, deformation effects, large spatial distributions, and the coupling among all these features. In addition, it should be free of adjustable parameters that make predictions unreliable. Density functional theory fulfills all these requirements. Spherical nuclei with halos have been described in the past successfully in this way by the solution of either the non-relativistic Hartree-Fock-Bogoliubov (HFB) [?, ?, ?] or the relativistic Hartree-Bogoliubov (RHB) equations [?, ?, ?] in coordinate (r) space. However, for deformed nuclei the solution of HFB or RHB equations in r space is a numerically very demanding task. In the past, considerable effort has been made to develop mean-field models either in r space or in a basis with an improved asymptotic behavior at large distances [?, ?, ?, ?, ?, ?, ?]. In particular, an expansion in a Woods-Saxon (WS) basis was shown to be fully equivalent to calculations in r space [?].

In the present investigation, we therefore study halo phenomena in deformed exotic nuclei within a DRHB model using a spherical WS basis. The RHB equations for the nucleons read [?, ?]

$$hD - \mu - h^* = E_k !$$

where E_k is the quasiparticle energy, μ the chemical potential, and hD is the Dirac Hamiltonian [?, ?, ?, ?, ?]

$$hD = \boldsymbol{\alpha} \cdot \mathbf{p} + V(r) + (M + S(r)).$$

Neglecting here for simplicity spin and isospin degrees of freedom, the pairing potential reads

$$V_{pp}(r_1, r_2) = V_{pp}(r_1, r_2) \delta(r_1 - r_2),$$

with a density-dependent force of zero range in the particle-particle channel

$$V_{pp}(r_1, r_2) = V_0 \delta(r_1 - r_2) \text{sat} \left(\frac{1 - P}{\rho} \right). \quad (4)$$

and the pairing tensor which is defined in the conventional way.

For axially deformed nuclei with spatial reflection symmetry, we represent the potentials and densities in terms of the Legendre polynomials

$$f(r) = f(r)P(\cos \theta), \quad = 0, 2, 4, \dots$$

For fixed quantum numbers Ω , the deformed quasiparticle wave functions $U_k(r)$ and $V_k(r)$ in Eq. (1) are expanded in a spherical WS basis (for details see Ref. [?]).

Computational Details

The calculations are based on the density functional NL3 [?] and the pp interaction (4) with the parameters $\text{sat} = 0.152 \text{ fm}^{-3}$, $V_0 = 380 \text{ MeV} \cdot \text{fm}^3$, and a

cut-off energy $E_{\text{p. cut}} = 60$ MeV in the quasi-particle space. These parameters reproduce the proton pairing energy of the spherical nucleus ^{24}Mg obtained from a spherical RHB calculation with the Gogny force D1S. A spherical box of size $R_{\text{max}} = 20$ fm and mesh size $r = 0.1$ fm are used for generating the spherical Dirac WS basis. An energy cutoff $E_{\text{+ cut}} = 100$ MeV is applied to truncate the positive energy states in the WS basis, and the number of negative energy states in the Dirac sea is taken to be the same as that of positive energy states in each (i, j) -block.

Results for ^{24}Mg

In the present study of Mg isotopes, the last nucleus within the neutron drip line is ^{24}Mg . Of course, it is difficult to predict the position of the drip line precisely for nuclei so far from the experimentally known area, and therefore the results discussed in the following have to be taken as generic results. In this study, ^{24}Mg is an almost spherical nucleus. The neighboring nucleus ^{26}Mg is well deformed ($\beta = 0.32$) and weakly bound with the two-neutron separation energy $S_n = 0.44$ MeV. Therefore, this nucleus is taken here as an example for a detailed investigation.

The density distributions of all protons and all neutrons in ^{24}Mg are shown in Fig. 1a [Figure 1: see original paper]. Due to the large neutron excess, the neutron density not only extends much farther in space but also shows a halo structure. The neutron density is decomposed into the contribution of the core in Fig. 1b and that of the halo in Fig. 1c. Details of this decomposition are given further down. We find that the core of ^{24}Mg is prolate and that the halo has a slightly oblate deformation. This indicates a decoupling between the deformations of core and halo.

Single-Particle Structure

Weakly bound orbitals or those embedded in the continuum play a crucial role in the formation of a nuclear halo. For an intuitive understanding of the single-particle structure, we keep in mind that HB-functions can be represented by BCS-functions in the canonical basis. The single-particle energies in the canonical basis $\epsilon_k = \langle k | h_D | k \rangle$ shown in Fig. 2 are expectation values of the Dirac Hamiltonian (2) for the eigenstates $|k\rangle$ of the single-particle density matrix $\hat{\rho}$ with eigenvalues v^2_k . The spectrum of $\hat{\rho}$ has a discrete part with $v^2_k = 0$. Obviously, only the first part contributes to the HB-wave function, and only this part is plotted in Fig. 2 [Figure 2: see original paper]. This part of the spectrum ϵ_k is discrete even for levels in the continuum. This is only possible because the wave functions $|k\rangle$ are not eigenfunctions of the Hamiltonian. As long as the chemical potential μ is negative, the corresponding density $\rho(r)$ is localized [?] and the particles occupying the levels in the continuum are bound.

The orbitals in Fig. 2 are labeled by the conserved quantum numbers Ω and π . The character n numbers the different orbitals appearing in this figure according

to their energies. The neutron Fermi level lies within the pf shell, and most of the single-particle levels have negative parities. Since the chemical potential $\mu_n = -230$ keV is relatively small, orbitals above the threshold have noticeable occupation due to pairing correlations. For example, the occupation probabilities of the 5th ($\Omega = 7/2$) and the 6th ($\Omega = 1/2$) orbitals are 27.2% and 14.3%.

Decomposition Analysis

As we see in Fig. 2, there is a considerable gap between the two levels with numbers $n = 2$ and $n = 3$. The levels with $\epsilon_n < -2.5$ MeV contribute to the “core,” and the remaining weakly bound and continuum orbitals with $\epsilon_n > -1$ MeV naturally form the “halo.” Therefore, we decompose the neutron density $n(r)$ into two parts: one part coming from orbitals with canonical single-particle energies $\epsilon_n < -2.5$ MeV (called “core”) and the other from the remaining weakly bound and continuum orbitals (called “halo”).

The spherical components of these densities [i.e., the contribution of $l = 0$ in Eq. (5)] are plotted together with that of the total neutron density in Fig. 3a [Figure 3: see original paper]. It is seen that the tail part of the neutron density originates mainly from orbitals with $\epsilon_n > -1$ MeV. The average number of neutrons that are weakly bound or in the continuum is around 4.34. On average, 2.92 of these neutrons are in the weakly bound orbits 3 and 4, and the others are in the continuum. The rms radii of the core and the halo are 3.72 fm and 5.86 fm, respectively. A further decomposition shows that the two weakly bound orbitals—the 3rd ($\Omega = 1/2$) and the 4th ($\Omega = 3/2$)—contribute mostly to the halo. This is more clearly seen in Fig. 3b, where we represent the relative contributions of weakly bound and continuum orbitals to the total neutron density, which is indicated in arbitrary units by the shaded area. The two continuum orbitals—the 6th ($\Omega = 1/2$) and the 8th ($\Omega = 1/2$)—also contribute to the tail.

Shape Analysis

If we decompose the deformed wave functions of the two weakly bound orbitals (the 3rd ($\Omega = 1/2$) and the 4th ($\Omega = 3/2$)) in the spherical WS basis, it turns out that in both cases the major part comes from p waves, as indicated on the right-hand side of Fig. 2. The p-wave components for the 3rd and 4th orbitals are 66% and 80%, respectively. Considering that the occupation probabilities of these two orbitals are 0.736 and 0.693 and each orbital has degeneracy 2, there are about 2 neutrons in weakly bound p states. The low centrifugal barrier for p waves gives rise to the formation of the halo. Having a small p-wave component, the 6th orbital ($\Omega = 1/2$) contributes less to the halo, though it is in the continuum and the occupation probability is rather large. The contribution of the 8th orbital ($\Omega = 1/2$) to the tail of the density is even smaller because its main components are d waves. The large centrifugal barrier of f states strongly hinders the spatial extension of the wave functions of the other two continuum orbitals—the 5th ($\Omega = 7/2$) and the 7th ($\Omega = 3/2$)).

Multipole Decomposition

In Fig. 4 [Figure 4: see original paper], the densities of the core and the halo are decomposed into spherical, quadrupole, and hexadecapole components. As seen in Fig. 4a, the quadrupole component of the core is positive, consistent with the prolate shape of ^{24}Mg . However, for the halo, the quadrupole component has a negative sign, which means the halo has an oblate deformation. The quadrupole moments of the neutron core and the halo are 160 and -27 fm^2 , respectively. This explains the decoupling between the quadrupole deformations of the core and the halo, as seen in Figs. 1b and 1c. There is also a noticeable hexadecapole component in the density distribution of the halo.

The slightly oblate shape of the halo originates from the intrinsic structure of the weakly bound and continuum orbitals. As shown in Fig. 2, the main WS components of the two weakly bound orbitals—the 3rd ($\Omega = 1/2$) and the 4th ($\Omega = 3/2$)—are p states. We know that the angular distribution of $|Y_{\Lambda}(\theta)|^2 \propto \cos^2$ with a projection of the orbital angular momentum on the symmetry axis $\Lambda = 0$ is prolate and that of $|Y_{\Lambda}(\theta)|^2 \propto \sin^2$ with $\Lambda = 1$ is oblate. It turns out that in the 3rd ($\Omega = 1/2$) orbital, both $\Lambda = 0$ and $\Lambda = 1$ components contribute, and the latter dominates. Therefore, this orbital has a slightly oblate shape. For the 4th ($\Omega = 3/2$) state, there is only the $\Lambda = 1$ component from the p / wave, so an oblate shape is also expected.

Neon Isotopes

To show that these results depend crucially on the single-particle structure near the Fermi surface, we also investigate weakly bound nuclei in the neighboring chain of Ne isotopes. In Fig. 5a [Figure 5: see original paper], the density distributions of all protons and all neutrons in the prolate deformed nucleus ^{30}Ne are shown ($\beta = 0.52$). Again, as in ^{24}Mg , due to the large neutron excess, the neutron density not only extends much farther in space but also shows a halo structure. The neutron density is decomposed into the contribution of the core in Fig. 5b and that of the halo in Fig. 5c. In contrast to the nucleus ^{24}Mg , we observe now a prolate halo because the essential level of the halo has a large contribution from the prolate $\Lambda = 0$ (p wave) component.

In Fig. 6 [Figure 6: see original paper], we show similar density distributions for the oblate deformed nucleus ^{30}Ne ($\beta = -0.24$), which is the last nucleus within the neutron drip line in the present calculation. In this case, the Fermi level is again within the pf shell. However, the levels dominated by p waves are either less occupied or not so weakly bound, and therefore we do not find a pronounced halo. From these examples, it is clear that the existence and deformation of a possible neutron halo depend essentially on the quantum numbers of the main components of the single-particle orbits in the vicinity of the Fermi surface: s levels with $\Lambda = 0$ produce spherical halos, p levels with $\Lambda = 0$ prolate halos, and p levels with $\Lambda = 1$ oblate halos [?].

Summary

In summary, the very neutron-rich deformed nucleus ^{11}Mg is investigated within deformed relativistic Hartree-Bogoliubov theory in the continuum. In contrast to several expectations [?, ?], a pronounced deformed neutron halo is found. It is formed by several orbitals close to the threshold (either weakly bound or in the continuum). They have large components of low l -values and therefore feel only a small centrifugal barrier. Although ^{11}Mg and its core are well deformed and prolate, the deformation of the halo is slightly oblate. This implies a decoupling between the deformations of core and halo.

This mechanism is investigated by studying the details of the neutron densities for core and halo, the single-particle levels in the canonical basis, and the decomposition of the halo orbitals. We also studied weakly bound nuclei in Ne isotopes and discussed the conditions for the occurrence of a halo and its shape. It is shown that the existence and deformation of a possible neutron halo depend essentially on the quantum numbers of the main components of the single-particle orbits in the vicinity of the Fermi surface.

Final Remarks

Finally, we note that besides the “quadrupole-bound” molecule [?] and the nuclear halo in deformed nuclei, similar coupling effects between deformation and the weakly bound part of the system could also exist in other quantum many-body systems, such as Rydberg atoms in which the electron(s) can be extremely weakly bound and where the quadrupole moment is sizable [?].

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