Electricity Market Theory Based on Continuous-Time Commodity Models

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Abstract

Real-time pricing theory serves as the theoretical foundation for spot market design in many countries, yet numerous issues have emerged in electricity market practice. Real-time pricing has two major deficiencies: first, it remains based on traditional time-of-use dispatch models, neglecting the crucial characteristic of temporal continuity in electricity production and consumption, and fails to properly address inter-temporal constraints; second, it assumes that electricity commodities within the same time period are homogeneous, unable to distinguish the distinct technical characteristics and cost structures of base-load, mid-merit, and peak-load generating units. To overcome these deficiencies, this paper proposes a continuous-time electricity commodity model, including electricity commodity models under both real-time pricing and load-duration-based pricing schemes, and maps the market optimization problems under these two pricing schemes to the mathematical concepts of Riemann integration and Lebesgue integration, respectively, establishes a functional extremum optimization model, derives the market equilibrium solution based on the Euler-Lagrange equation, and proves the feasibility of load-duration-based pricing through rigorous mathematical derivation. Numerical examples demonstrate that load-duration-based pricing can reduce total market procurement costs while achieving a more equitable distribution of profits among power plants. The theories and methods proposed in this paper can provide novel insights and a theoretical foundation for electricity market development both domestically and internationally.

Full Text

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Abstract

The theory of spot pricing serves as the foundation for electricity spot market design in many countries, yet numerous problems have emerged in practice. Spot electricity pricing suffers from two major drawbacks: First, it remains based on traditional hourly scheduling/dispatch models, ignoring the crucial time continuity characteristic of electric power production and consumption and failing to properly handle intertemporal constraints. Second, it assumes electricity products are homogeneous within the same dispatch period, making it impossible to distinguish the clearly different technical characteristics and cost structures of base-load, shoulder-load, and peak-load generators.

To overcome these deficiencies, this paper proposes a continuous-time electricity commodity model, including models under both spot pricing and load-duration-based pricing. We establish functional extremum optimization models that correspond mathematically to Riemann and Lebesgue integrals for the two pricing approaches, respectively, and derive market equilibrium solutions based on the Euler-Lagrange equation. Through rigorous mathematical derivation, we prove the feasibility of load-duration-based pricing. Numerical examples demonstrate that this pricing method reduces total market procurement costs while achieving fairer profit distribution among power plants. The theory and methods proposed in this paper provide entirely new ideas and theoretical foundations for electricity market development both domestically and internationally.

Keywords: Electricity markets, Spot pricing, Functional optimization
Classification Codes: TM73; F045.32

Since the issuance of "Several Opinions on Further Deepening Power System Reform" (Document No. 9 [2015] of the CPC Central Committee), electricity market reform has become a hot topic in the industry, with numerous debates surrounding target models and implementation pathways for China's electricity market construction. On November 26, 2015, the National Development and Reform Commission and the National Energy Administration issued supporting Document No. 2 for power system reform, "Implementation Opinions on Promoting Electricity Market Construction," which stipulated certain provisions regarding electricity market models and trading mechanisms. On December 29, 2016, the NDRC and NEA issued the "Basic Rules for Medium- and Long-term Electricity Trading (Interim)"(hereinafter referred to as the "Basic Rules"), representing a relatively comprehensive market rule covering different trading cycles (multi-year, annual, quarterly, monthly, weekly, and beyond-daily) and both intra-provincial and inter-provincial/cross-regional medium- to long-term electricity quantity trading varieties. This became the first highly implementable market rule issued at the national level in the new round of power system reform. Currently, with the deepening of electricity market reform, accelerating spot market construction has become an important measure for relevant authorities to promote power system reform.

Regardless of the electricity market model and trading mechanism adopted, the definition of electricity commodity models, cost structures, and price formation mechanisms constitute the most fundamental issues. Due to the physical homogeneity of electricity commodities (once electricity from all power plants enters the grid, it cannot be distinguished) and the existence of complex physical networks (power systems), electricity has become one of the world' s most complex commodities. Consequently, the definition of electricity commodities and electricity pricing theory are not self-evident matters. Professor F. C. Schweppe of MIT and colleagues published *Spot Pricing of Electricity* in 1988, which became the classic literature on spot pricing theory and the theoretical foundation for electricity spot market design in multiple countries [1]. Research on electricity pricing theory should be divided into two components: electricity cost analysis (i.e., what constitutes reasonable electricity prices) and price formation mechanisms in electricity markets. Electricity cost analysis forms the basis for measuring price reasonableness, but prices must ultimately be formed through market mechanisms. In an ideal electricity market, clearing prices should equal the marginal production cost of electricity in the power system (as well as the marginal utility of electricity for consumers).

While spot pricing theory is mathematically elegant, we argue that it does not align with the physical characteristics of power production and consumption. In practice, spot markets designed based on spot pricing theory have encountered various problems: real-time market prices fluctuate dramatically, creating significant financial risks for market participants and necessitating additional financial hedging instruments; most electricity users lack the capacity (or willingness) to respond to rapidly changing real-time prices, forcing them to rely on retail companies to convert wholesale market real-time prices into simple procurement packages. This fails to achieve the theoretical design objective of enhancing supply-demand interaction through real-time pricing, allows intermediaries such as retail companies to capture the benefits of power reform, and prevents these benefits from reaching end-users, contradicting the original intent of reform. Furthermore, real-time pricing cannot fully cover fixed costs, leading to inadequate generation investment capacity and requiring separate capacity markets.

2 Principles of Real-Time Electricity Pricing

Real-time electricity prices depend on power supply and demand conditions in a particular hour, specifically: - Load (total load and zonal load) - Generation adequacy and costs (including purchases from other companies) - Transmission and distribution network adequacy and losses

Real-time price is defined as the electricity price for each user in each time period (1h/0.5h/0.25h/···), comprising the following components: generation marginal fuel cost, generation marginal maintenance cost, generation power quality cost, generation revenue reconciliation cost, network marginal loss cost, network power quality cost, and network revenue reconciliation cost.

When ignoring revenue reconciliation costs, real-time price is determined by marginal cost, where ρ_{kt} represents the real-time price for user k in hour t (\$/kWh), and d_{kt} represents the electricity demand of user k in hour t (kWh).

The derivative in the above equation must satisfy the following constraints: - Power balance: Total generation equals total load plus losses - Generation limits: Total demand in hour t cannot exceed the sum of available capacity of all power plants in that hour - Kirchhoff's laws: Power flows and losses must satisfy circuit laws - Line flow limits: Power flow on any line must not exceed its transmission limit

Real-time pricing is formed based on the principle of social welfare maximization in classical microeconomics. While its original theory comprehensively considered short- and long-term aspects, operation and planning, real-time prices in actual electricity markets are often derived from short-term operational optimization models such as Security Constrained Unit Commitment (SCUC) and Security Constrained Economic Dispatch (SCED). However, real-time pricing calculation models still adopt traditional time-period-based power balance models, dividing the entire study interval into a series of cycles, each further divided into periods. Calculation then proceeds period-by-period using an energy (electricity quantity) balance model. Since power in each period is assumed constant, the energy balance model becomes equivalent to a power balance model. As shown in [Figure 1: see original paper], the electricity commodity model in this process is actually defined as follows:

The area under the entire load curve is cut into several "strips" by time period, with each strip further divided into several "segments." In one time period, each winning generator takes one "segment" (i.e., one commodity), and under marginal price clearing, all segments in the same period (all commodities) settle at the same price (the highest price among winning units in that period).

[Figure 1: see original paper] Electricity commodity model in spot pricing theory

Due to space limitations, this paper does not provide a detailed introduction to spot pricing theory; interested readers may refer to references [1-3].

3 Defects of Spot Markets Based on Real-Time Pricing and the Continuous-Time Electricity Commodity Model

Spot markets designed based on real-time pricing theory exhibit the following defects: 1. Real-time pricing is calculated based on traditional time-period-based scheduling (or time-period-based power balance) models, which do not seriously address intertemporal constraints (mentioned but not deeply explored in [1]) and consequently ignore the crucial time continuity characteristic of electricity production and consumption. This problem becomes particularly severe with large-scale integration of wind, solar, and other renewable energy sources and rapidly increasing demand for power system flexibility. 2. They assume electricity commodities in the same time period are homogeneous. By failing to consider the temporal dynamic characteristics of different types of generators, they cannot distinguish the clearly different technical features and cost structures of base-load, shoulder-load, and peak-load units. 3. Although real-time pricing theoretical models include resource optimization across long time scales from operation to planning [2], such ultra-large-scale optimization problems are impractical in reality. Actual markets often use SCUC or SCED models to calculate clearing prices, which do not include economic signals for long-term generation capacity investment and cannot ensure generation capacity adequacy. 4. Both electricity production and consumption have time continuity. For both generators and loads, the commodity sold or purchased is an "energy block" with certain duration. Energy (electricity quantity) is the primary concern for power producers and consumers, while power balance mainly serves as a physical constraint for power system operation (primarily reflected in power flow equations). In real-time pricing theory, since temporal factors are not incorporated into the commodity model, power balance becomes equivalent to energy balance, with physical constraints directly serving as market supply-demand balance conditions—an inappropriate approach. 5. Energy-type commodities (electricity quantity), particularly those in medium- to long-term trading, differ from power-type commodities. Energy-type commodities are more similar to ordinary goods, requiring only dynamic supply-demand balance over longer periods. In medium- to long-term trading, energy-type commodities are actually storable, primarily in the form of coal (or other fuels) and reservoir water storage, contrasting sharply with non-storable power-type commodities. 6. As a foundational product and means of production (particularly in China), ensuring continuous and stable electricity supply is the most critical consideration, without one-sided pursuit of "maximum efficiency" in the electricity market itself (especially absolute maximum efficiency in short-term spot markets).

To emphasize the role of temporal factors in electricity commodities, this paper redefines the continuous-time electricity commodity model composed of (power, time) pairs, where the area under the power curve represents electricity quantity, as shown in [Figure 2: see original paper]. When $t_2 = t_1 + 1$, this degenerates into the time-period-based electricity commodity model in spot pricing theory. Since power can be viewed as a function of time, this continuous-time electricity commodity model can also be written as $(P(t), t)$ for $t_1 \leq t \leq t_2$. The electricity quantity point in real-time pricing definition becomes a function defined over the time interval $[t_1, t_2]$, requiring analysis using mathematical theories in infinite-dimensional spaces (such as functional analysis and calculus of variations). With the introduction of the continuous-time electricity commodity model, the social welfare maximization problem for market clearing transforms from a multi-stage static optimization problem into a continuous-time functional optimization problem, with solution methods shifting from Lagrange methods to solving Euler-Lagrange equations.

It is worth noting that the MIT research team had already recognized the time continuity issue in electricity production and utilization. Appendix 4 of reference [2] (pp. 247-257) employs a continuous-time model to study optimal electricity consumption behavior of power users and derives optimality conditions based on Hamiltonian functions. Other scholars domestically and internationally have also proposed bidding models considering time continuity of electricity commodities, such as the horizontal auction mechanism in [5] and the piecewise bidding mechanism in [6], though none have conducted in-depth research on the foundational theory of electricity markets based on continuous-time commodity models.

[Figure 2: see original paper] Continuous-time electricity commodity model

4 Electricity Market Modeling Based on Continuous-Time Commodity Model

We continue to define the electricity market using the microeconomic model based on social welfare maximization from reference [3]. For ease of reference, we adopt that paper' s variable symbols and expressions. Since this paper' s purpose is primarily illustrative, the market model is simplified based on that foundation.

Let $B_j(\phi)$ denote the variable cost (benefit) of market participant j over a market cycle, with the cost (benefit) function expressed as:

$$B_j(\phi) = \int_0^T B_j(P_j(t), t)dt, \quad j \in \phi$$

Where: - ϕ is the set of all market participants - $P_j(t)$ is the power curve function of market participant j - When j is a power plant, $B_j \leq 0$ (representing generation cost) - When j is a power consumer, $B_j \geq 0$ (representing consumption benefit) - T is the transaction period duration

The objective function is the standard social welfare maximization model in microeconomics:

$$\text{Maximize Social Welfare} = \text{Electricity Use Value} - \text{Electricity Production Cost}$$

The objective function can be written as:

$$\max_{P_j(t), j \in \phi} W = \sum_{j \in \phi} \int_0^T B_j(P_j(t), t)dt \tag{1}$$

Considering only energy balance constraints:

$$\text{Energy Balance} = \text{Generation} - \text{Network Loss} - \text{Consumption} = 0$$

The energy balance constraint can be written as:

$$\sum_{j \in \phi_e} P_j(t) - L_0(P(t), t) - L_t = 0, \quad t \in [0, T] \tag{2}$$

where $L_0(P(t), t) > 0$ represents active power losses. Note that in this equation, time t is a continuous variable, no longer discretized. When j is a generator, $P_j(t)$ is positive; otherwise, it is negative.

With the continuous-time electricity commodity model, the profit maximization model for market participants becomes an integral (or functional of the power curve) form. The profit maximization model under real-time pricing corresponds mathematically to the Riemann integral, while the profit maximization model under load-duration-based pricing introduced in this paper corresponds to the Lebesgue integral. Note that the basic idea of load-duration-based pricing was already described in references [5, 6]. Basic knowledge about Riemann and Lebesgue integrals is provided in the Appendix.

1) Participant Profit Maximization Model Under Real-Time Pricing (Riemann Integral)

Assume participant j receives revenue (for generators) or purchases electricity (for consumers) according to the time-varying real-time price $\pi_j(t)$. The participant' s objective is to maximize net revenue under production (consumption) capacity constraints:

$$\max_{P_j(t)} \pi_j = \int_0^T \pi_j(t) P_j(t) dt - B_j(P_j(t), t)$$

subject to:

$$P_j^{\min} \le P_j(t) \le P_j^{\max}$$

For a specific participant, $\pi_j(t)$ is often constant (not varying with time).

2) Participant Profit Maximization Model Under Load-Duration-Based Pricing (Lebesgue Integral)

Assume participant j receives revenue (for generators) or purchases electricity (for consumers) according to the price function $\pi_j(y)$ determined by load duration (measure). This paper assumes the load curve is monotonically increasing, so this price function can also be written as a function of load (see numerical examples). The participant' s objective is to maximize net revenue under production (consumption) capacity constraints:

$$\max_{P_j(t)} \pi_j = \int_{y_{\min}}^{y_{\max}} \pi_j(y) m_j(y) dy - B_j(P_j(t), t)$$

subject to:

$$P_j^{\min} \leq P_j(t) \leq P_j^{\max}$$

Where $m_j(y) = \mu\{t : P_j(t) > y\}$ is the measure corresponding to function value y. When j is a generator, the first term takes a positive sign; otherwise, it takes a negative sign.

The electricity commodity models also differ under the two pricing approaches. Under real-time pricing, the power curve $P_j(t)$ of participant j over time interval $[t_1, t_2]$ is considered "one" commodity sold (or purchased), with total price:

$$\int_{t_1}^{t_2} \pi_j(t) P_j(t) dt$$

We can further define the unit energy price of this commodity as:

$$\pi_j = \frac{\int_{t_1}^{t_2} \pi_j(t) P_j(t) dt}{\int_{t_1}^{t_2} P_j(t) dt}$$

Under load-duration-based pricing, the "energy block" within a certain power range $[P_1, P_2]$ of participant j is considered "one" commodity sold (or purchased), i.e.:

$$\{P_j(t) : P_1 \leq P_j(t) \leq P_2, t \in [t_1, t_2]\}$$

with total price:

$$\int_{P_1}^{P_2} \pi_j(y) m_j(y) dy$$

and unit energy price:

$$\pi_j = \frac{\int_{P_1}^{P_2} \pi_j(y) m_j(y) dy}{\int_{P_1}^{P_2} m_j(y) dy}$$

The overall market optimization objective is to maximize social welfare expressed in equation (1) under constraints expressed in equation (2). Clearly, this problem is a variational problem concerning $P_j(t), j \in \phi$. According to its optimality conditions, we obtain the dual variable (shadow price) $\lambda(t)$.

In the market mechanism based on real-time pricing, $\lambda(t)$ is directly used as the market price $\pi_j(t)$ for participant j, i.e., $\pi_j(t) = \lambda(t)$. This represents the market equilibrium price, where individual optimality (3) is consistent with overall social optimality (1).

In the market mechanism based on load-duration-based pricing, we must seek a market price $\pi_j(y)$ for participant j that makes all $\pi_j(y), j \in \phi$ equal, which becomes the market equilibrium price. In this case, individual optimality (4) is consistent with overall social optimality (1). The following specific model solution and numerical analysis illustrate this.

5 Solving the Electricity Market Model Based on Continuous-Time Commodity Model

For solution and analysis convenience, this paper temporarily considers a single-sided bidding model on the generation side, assuming generator cost functions take quadratic form and ignoring network losses. Under these assumptions, the social welfare maximization problem (1) and (2) becomes the following generation cost minimization problem, where $P_d(t)$ is the system load at time t.

First, solve the variational problem (7) with one constraint. The Euler-Lagrange equation is:

$$\begin{cases} C_j'(P_j(t)) - \lambda(t) = 0, & j = 1, 2, \dots, n \\ \sum_{j=1}^{n} P_j(t) - P_d(t) = 0 \end{cases}$$

From the j-th equation above, we can obtain $P_j(t) = C_j'^{-1}(\lambda(t))$. Substituting this into the final power balance equation yields $\lambda(t)$, and subsequently all $P_j(t)$.

Next, solve the variational problem (3) in the Riemann integral sense. Temporarily ignoring power constraint limitations, treat variational problem (3) as an unconstrained variational problem and write its Euler-Lagrange equation as:

$$\pi_j(t) - C_j'(P_j(t)) = 0$$

Comparing (8) and (9), we obtain:

$$\pi_j(t) = \lambda(t)$$

which represents the market equilibrium price.

When cost functions take quadratic form, we can solve for:

$$\lambda(t) = \frac{P_d(t) + \sum_{j=1}^{n} \frac{b_j}{2a_j}}{\sum_{j=1}^{n} \frac{1}{2a_j}}$$

This shows that real-time price $\lambda(t)$ is related to the load curve $P_d(t)$.

Finally, consider the variational problem (4) in the Lebesgue integral sense. Under the assumptions of monotonically increasing load curve and single-sided bidding on the generation side, (4) can be written as:

$$\max_{P_j(t)} \pi_j = \int_{P_{\min}}^{P_{\max}} \pi_j(y) m_j(y) dy - \int_0^T C_j(P_j(t)) dt$$

where $m_j(y) = \mu\{t : P_j(t) > y\}$ is the measure corresponding to function value y, with $P_{\max} = \max_{t\in[0,T]} P_d(t)$ and $P_{\min} = \min_{t\in[0,T]} P_d(t)$.

When cost functions take quadratic form, from (11) we know $C_j'(P_j)$ is a strictly monotonically increasing function, and $m_j(y) = m(y)$. We can perform variable substitution $y = P_j(t)$, transforming (12) into the following variational problem:

$$\max_{P_j(t)} \pi_j = \int_0^T \pi_j(P_j(t))P_j(t)dt - \int_0^T C_j(P_j(t))dt$$

The Euler-Lagrange equation for this problem is:

$$\frac{d}{dt}\left[\pi_j(P_j(t)) + P_j(t)\pi_j'(P_j(t)) - C_j'(P_j(t))\right] = 0$$

This yields a first-order linear ordinary differential equation (14) for the price function $\pi_j(t)$. On the other hand, from (11) we know:

$$\sum_{j=1}^{n} P_j(t) = P_d(t)$$

Thus, the coefficients and nonlinear terms in (14) are independent of j, meaning the solution to (14) is independent of j. Consequently, all power plants face the same price, i.e.:

$$\pi_j(t) = \pi(t)$$

This represents the market equilibrium price under load-duration-based pricing.

Assume a monotonically increasing load curve:

$$P_d(t) = 350(1 + 2t), \quad t \in [0, 1]$$

Assume the market has three power plants with costs differing by factors of 2, representing low-cost (base-load), medium-cost (shoulder-load), and high-cost (peak-load) plants, with cost functions (taking $a = 0.001$, $b = 0.07$, $c = 0.2$):

$$\begin{cases} C_1(P_1) = aP_1^2 + bP_1 + c \\ C_2(P_2) = 2aP_2^2 + 2bP_2 + 2c \\ C_3(P_3) = 4aP_3^2 + 4bP_3 + 4c \end{cases}$$

From equation (8), the real-time price $\lambda(t)$ and each plant’ s load $P_i(t)$ can be solved as:

$$\begin{cases} \lambda(t) = 0.32 + 0.4t \\ P_1(t) = 250 + 400t \\ P_2(t) = 90 + 200t \\ P_3(t) = 10 + 100t \end{cases}$$

The generation costs for the three plants are calculated as:

$$\begin{cases} C_1 = 107.92 \\ C_2 = 39.43 \\ C_3 = 8.86 \end{cases}$$

1) Revenue and Profit Analysis Under Real-Time Pricing

Under real-time pricing, the sales revenues for the three plants are:

$$\begin{cases} R_1 = 247.32 \\ R_2 = 105.52 \\ R_3 = 34.48 \end{cases}$$

The profits for each plant are: 107.69, 39.06, and 8.04, with profit margins of 77%, 59%, and 30% (profit/cost × 100%), respectively. The total market procurement cost is 387.32, total generation cost for the three plants is 232.53, total profit is 154.79, and the overall market profit margin is 67% (total profit/total cost × 100%).

2) Market Price, Revenue, and Profit Analysis Under Load-Duration-Based Pricing

Substituting the data into the corresponding Euler-Lagrange equation (14) yields:

$$\pi_j'(t) - 1.2t\pi_j(t) + 0.48t = 0, \quad j = 1, 2, 3$$

This simplifies to the ordinary differential equation:

$$\pi_j'(t) - 1.2t\pi_j(t) + 0.48t = 0$$

From the example data, the corresponding measure function is:

$$m(y) = \begin{cases} \frac{650-y}{400}, & y \in [250, 650] \\ \frac{290-y}{200}, & y \in [90, 290] \\ \frac{110-y}{100}, & y \in [10, 110] \end{cases}$$

Therefore, under load-duration-based pricing, the sales revenues for the three plants are:

$$\begin{cases} R_1 = 224.80 \\ R_2 = 98.80 \\ R_3 = 39.16 \end{cases}$$

The profits for each plant are: 84.37, 34.38, and 12.72, with profit margins of 60%, 52%, and 48%, respectively. The total market procurement cost is 364, total generation cost is 232.53, total profit is 131.47, and the overall market profit margin is 57%.

These results demonstrate that real-time pricing, unable to distinguish among base-load, shoulder-load, and peak-load electricity, leads to vastly different profits among the three plants, with base-load plants receiving excessive profits. Load-duration-based pricing reduces total market procurement costs while achieving fairer profit distribution among plants. Compared to the real-time pricing mechanism, it reduces base-load plant profits and increases peak-load plant profits. Since both pricing approaches have their respective advantages and disadvantages, future research will consider market mechanisms combining both methods.

Current electricity market reform urgently requires foundational theoretical research on electricity commodities and market trading. Building upon analysis of defects in foreign spot markets based on spot pricing theory, this paper proposes a continuous-time electricity commodity model, including models under both real-time pricing and load-duration-based pricing. We establish a social welfare maximization-based electricity market model, further corresponding market optimization models under the two pricing approaches to Riemann and Lebesgue integrals in mathematics. Market equilibrium solutions are obtained based on respective Euler-Lagrange equations, with rigorous mathematical derivation particularly proving the feasibility of load-duration-based pricing. Finally, a numerical example validates the rationality of the proposed theory and methods. The theory and methods presented in this paper are expected to provide entirely new ideas and theoretical foundations for electricity market construction both domestically and internationally.

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Author Contributions Statement
Chen Haoyong: Proposed the electricity market theory based on continuous-time commodity model and research framework, established market optimization models, drafted and revised the final manuscript.

Han Lijia: Solved the electricity market model based on continuous-time commodity model, performed formula derivation and numerical validation.

Appendix: Basic Concepts of Riemann and Lebesgue Integrals

1. Riemann Integral

The conventional integral introduced in advanced mathematics and calculus is the Riemann integral, defined as follows:

Let $f(x)$ be defined on $[a,b]$. Arbitrarily partition $[a,b]$ into n subintervals $[x_{i-1}, x_i]$ using division points $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$, with each subinterval length $\Delta x_i = x_i - x_{i-1}$. In each subinterval, arbitrarily select a point ξ_i where $x_{i-1} \leq \xi_i \leq x_i$ $(i = 1, 2, \ldots, n)$. If the limit of the sum $\sum_{i=1}^{n} f(\xi_i)\Delta x_i$ exists as $\lambda = \max_{1\leq i\leq n}\{\Delta x_i\} \to 0$, then $f(x)$ is said to be integrable on $[a,b]$, and this limit value is called the definite integral of $f(x)$ over $[a,b]$, denoted as:

$$\int_a^b f(x)dx = \lim_{\lambda\to 0}\sum_{i=1}^{n} f(\xi_i)\Delta x_i$$

Here, $f(x)$ is the integrand, $f(x)dx$ is the integrand expression, x is the integration variable, the symbol "$\int$" is the integral sign, $[a,b]$ is the integration interval, and a and b are the lower and upper limits of integration, respectively. The sum $\sum_{i=1}^{n} f(\xi_i)\Delta x_i$ is called the integral sum.

From the definition of the Riemann integral, we see that calculating a Riemann integral employs an approximation method involving three steps: partition, approximate summation, and taking the limit. The Riemann integration concept divides the interval into subintervals, constructs corresponding approximate sums, and obtains the definite integral value through limits.

According to Riemann integration theory, continuous functions are integrable. The physical meaning of the Riemann integral is the area of the planar region bounded by the continuous function curve and the x-axis, making it an important tool for area calculation.

2. Lebesgue Integral

Since the class of functions integrable under Riemann integration is too limited, the Lebesgue integral modifies the Riemann integral definition appropriately to make more functions integrable.

First, we provide the definition of Lebesgue measure [1]:

We first agree that a set $I = \{x = (x_1, x_2, \ldots, x_n) : a_i < x_i < b_i, i = 1, 2, \ldots, n\}$ is called an open interval in $\mathbb{R}^n$, denoted as $I = (a_1, b_1; a_2, b_2; \ldots; a_n, b_n)$. The "volume" of I is defined as $|I| = \prod_{i=1}^{n}(b_i - a_i)$.

For any set $E \subset \mathbb{R}^n$, if there exists at most a countable collection of open intervals $\{I_i\}$ such that $E \subset \bigcup_i I_i$, then $\inf \sum_i |I_i|$ is called the Lebesgue outer measure of E, denoted as $m^*(E)$. Thus, outer measure can be described as the infimum of measures of open sets containing E. Similarly, by filling the interior of E with closed sets (whose complements are open sets and therefore must also have measure), the supremum of measures of these inscribed closed sets is called the inner measure $m_*(E)$ of E. When $m^*(E) = m_*(E)$, E is said to be Lebesgue measurable.

Next, we examine the definition of the Lebesgue integral [1]:

For the Riemann integral of a bounded function on a finite interval $[a, b]$, the definition first partitions the interval $[a, b]$. The unique feature of the Lebesgue integral definition is that it partitions the function' s range.

Let E be a measurable set with $m(E) < +\infty$, and $f(x)$ be a bounded function on E. Partition the segment between the infimum A and supremum B of $f(x)$' s values on $[a, b]$ into n subintervals:

$$A = y_0 < y_1 < y_2 < \cdots < y_{n-1} < y_n = B$$

Let $E_i = \{x \in E : y_{i-1} \leq f(x) \leq y_i\}$ $(i = 1, 2, \ldots, n)$, and arbitrarily select $\xi_i \in [y_{i-1}, y_i]$. Form the sum:

$$S = \sum_{i=1}^{n} f(\xi_i) m(E_i)$$

If the limit of S exists as $\lambda = \max_{1 \leq i \leq n}\{y_i - y_{i-1}\} \to 0$, then $f(x)$ is said to be Lebesgue integrable on E, and this limit value is called the Lebesgue integral.

The Lebesgue integration process can be vividly described by Figure A.1: The area of the curved region bounded by $y = f(x)$, $y = 0$, $x = a$, and $x = b$ can be approximated by the sum of rectangular areas formed through horizontal partitioning of the range:

$$S = \sum_{i=1}^{n} y_i m(E_i)$$

As the range partition becomes increasingly fine, the sum of rectangular areas approaches the curved region' s area. If the limit of this sum exists, it is the Lebesgue integral of $f(x)$. This integration method contrasts with Riemann integration, which partitions the domain.

Figure A.1 Schematic diagram of Lebesgue integration

[1] Cheng Qixiang, Zhang Dianzhou, Wei Guoqiang, et al. *Foundations of Real Variable Functions and Functional Analysis* (3rd Edition) [M]. Beijing: Higher Education Press, 2010.

Note: Figure translations are in progress. See original paper for figures.