

## Supersymmetric Probes in a Rotating 5D Attractor (Postprint)

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### Abstract

Supersymmetric zero-brane and one-brane probes in the squashed  $\text{AdS}_2 \times S^3$  near-horizon geometry of the BMPV black hole are studied. Supersymmetric zero-brane probes stabilized by orbital angular momentum on the  $S^3$  are found and shown to saturate a BPS bound. We also find supersymmetric one-brane probes which have momentum and winding around a  $U(1)_L \times U(1)_R$  torus in the  $S^3$  and in some cases are static.

### Full Text

### Preamble

### Supersymmetric Probes in a Rotating 5D Attractor

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### Abstract

We study supersymmetric zero-brane and one-brane probes in the squashed  $\text{AdS}_2 \times S^3$  near-horizon geometry of the BMPV black hole. We find supersymmetric zero-brane probes stabilized by orbital angular momentum on the  $S^3$  and show that they saturate a BPS bound. We also find supersymmetric one-brane probes which have momentum and winding around a torus generated by a  $U(1) \times U(1)$  subgroup of the  $SU(2)$  rotational isometry group. In some cases these configurations are static. A one-brane in five dimensions can carry the magnetic charge dual to the electric charge supporting the BMPV black hole. Interestingly, this allows for static BPS “black ring” configurations, where the angular momentum required for saturation of the BPS bound is carried by the gauge field.

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### 1 Introduction

The near-horizon attractor geometry of a BPS black hole has twice as many supersymmetries as the full asymptotically flat solution. In four dimensions, such geometries admit BPS probe configurations which preserve only near-horizon supersymmetries and break all of the supersymmetries of the original asymptotically flat solution [1]. A novel feature of these configurations is that branes and anti-branes antipodally located on the  $S^2$  preserve the same supersymmetries. Quantization of these classical configurations leads to lowest Landau levels which tile the black hole horizon [2]. In some cases the degeneracies saturate the Bekenstein-Hawking black hole entropy [3]. Furthermore, an appropriate expansion of the black hole partition function in a dilute gas of these states [4] yields a derivation of the OSV relation [5]. These interesting 4D phenomena should all have closely related 5D cousins [6]. With this in mind, the present paper extends the 4D classical BPS probe analysis of [1] to five dimensions.

The 5D problem is considerably enriched by the fact that 5D BMPV BPS black holes can carry angular momentum  $J$  and have a  $U(1) \times SU(2)$  rotational isometry group [7]. BPS zero-brane probes that orbit the  $S^3$  are found using a  $U(1)$ -symmetry analysis. Their location in AdS depends on the azimuthal angle on  $S^3$ , the background rotation  $J$ , and the angular momentum of the probe. For one-branes, we find BPS configurations with momentum and winding around a torus generated by a  $U(1) \times U(1)$  subgroup. A one-brane in five dimensions can carry the magnetic charge dual to the electric charge supporting the BMPV black hole. Interestingly, we find that this allows for static BPS “black ring” configurations, where the angular momentum required for saturation of the BPS bound is carried by the gauge field.

### 2 Review of the BMPV Black Hole

The 5D supersymmetric rotating black hole arises from M2-branes wrapping holomorphic curves of a Calabi-Yau threefold  $X$ . It is characterized by electric charges  $q$ ,  $A = 1, 2, \dots$  ( $X$ ), and the angular momentum  $J$  in  $SU(2)$ . The metric is [7]

$$ds^2 = \frac{2r^2}{\sigma^3} d\theta^2 + d\phi^2 + d\psi^2 + 2 \cos \theta d\psi d\phi - \frac{dr^2}{r^2 d\Omega^2} (\sigma^i)^2, \quad \langle \text{cid:1} \rangle$$

where the ranges of the angular parameters are  $[0, \pi]$ ,  $[0, 2\pi]$ ,  $[0, 4\pi]$ . The  $\sigma^i$  are the right-invariant one-forms:

$$\sigma^1 = 2 \sin \psi d\theta + \cos \psi \sin \theta d\phi, \quad \sigma^2 = \cos \psi d\theta + \sin \psi \sin \theta d\phi, \quad \sigma^3 = d\psi + \cos \theta d\phi,$$

and we choose Planck units  $l_5 = (4G_5/\pi)^{1/3} = 1$ . The graviphoton charge  $Q$  is determined via the equations

$$\frac{2}{\sigma^3} = D_{ABC}y^A y^B y^C, \quad q_A = 3D_{ABC}y^B y^C,$$

with  $D_{ABC}$  the intersection form on  $X$ .

The near-horizon limit ( $r \rightarrow 0$ ) of the metric is

$$ds^2 = \langle \text{cid:20} \rangle \langle \text{cid:21} \rangle \frac{r^2}{\sigma^2} + Qd\Omega^2$$

Rescaling  $t$  to absorb  $Q$ , defining  $\sin^2 B = J^2/Q^3$  and  $r^2 = 1/\sigma$ , we obtain the metric in Poincaré coordinates:

$$ds^2 = \langle \text{cid:20} \rangle + \sin B\sigma^3)^2 + \sigma_1^2 + \sigma_2^2 + \sigma_2^2 \langle \text{cid:21} \rangle \langle \text{cid:19} \rangle$$

The  $SU(2)$  rotation matrix is parameterized as:

$$\langle \text{cid:2} \rangle \begin{pmatrix} \cos \frac{\theta}{2} e^{i(\psi+\phi)/2} & i \sin \frac{\theta}{2} e^{i(\psi-\phi)/2} \\ i \sin \frac{\theta}{2} e^{-i(\psi-\phi)/2} & \cos \frac{\theta}{2} e^{-i(\psi+\phi)/2} \end{pmatrix} = \langle \text{cid:18} \rangle \cos \theta \sin \theta \frac{e^{i(\psi+\phi)/2}}{\sigma_z}$$

The graviphoton field strength in these coordinates is  $F_{[2]} = dA_{[1]}$ , with

$$A_{[1]} = dt + \sin B\sigma^3.$$

We will also be using the metric in the global coordinates  $(\tau, \chi, \theta, \phi, \psi)$ :

$$ds^2 = \langle \text{cid:2} \rangle \cosh^2 \chi d\tau^2 + d\chi^2 + (\sin B \sinh \chi d\tau \cos B\sigma^3)^2 + \sigma_1^2 + \sigma_2^2 \langle \text{cid:3} \rangle$$

with

$$A_{[1]} = [\cos B \sinh \chi d\tau + \sin B\sigma^3].$$

The near-horizon geometry of the BMPV black hole is a kind of squashed  $AdS \times S^3$  with  $U(1)$  rotational invariance, where the bosonic subgroup is  $SU(1,1) \times SU(2)$  [10]. When  $J = 0$ ,  $U(1)$  is promoted to  $SU(2)$  and the full  $SO(4) \times SU(2) \times SU(2)$  rotational symmetries are restored. The unbroken supergroup is  $SU(1,1|2)$  and the rotational symmetries for  $J = 0$  are generated by the Killing vectors

$$J_1^{right} = \sin \phi \partial_\theta + \cos \phi (\cot \theta \partial_\phi - \csc \theta \partial_\psi), J_2^{right} = \cos \phi \partial_\theta - \sin \phi (\cot \theta \partial_\phi - \csc \theta \partial_\psi), J_3^{right} = \partial_\phi, J_3^{left} = \partial_\psi.$$

The supersymmetries arise from Killing spinors  $\epsilon$  which are the solutions of the equation

$$\langle \text{cid:0} \rangle \frac{1}{3} e_a \Gamma^{bc} \Gamma^a F_{bc} - \omega_{ab} \Gamma^{ab} + 4e_a \Gamma^b F_{ab} \epsilon = 0 \langle \text{cid:21} \rangle \langle \text{cid:1} \rangle \langle \text{cid:20} \rangle$$

The coordinate transformation between the global coordinates and Poincaré ones is:

$$\langle \text{cid:20} \rangle \langle \text{cid:21} \rangle \cos B \cosh \chi \sin \tau \cosh \chi \cos \tau + \sinh \chi \cosh \chi \cos \tau + \sinh \chi \psi_{Poincaré} = \psi_{global} + 2 \tan B \tanh^{-1}(e^{-\chi} \tau)$$

To solve this in global coordinates we choose the vielbein

$$e^0 = [\cosh(\sin B \cos B \psi) \cosh \chi d\tau + \sinh(\sin B \cos B \psi) d\chi], e^1 = [\sinh(\sin B \cos B \psi) \cosh \chi d\tau + \cosh(\sin B \cos B \psi) d\chi]$$

The Killing spinors are then [8][9]

$$\epsilon = e^{-\frac{1}{2}(\sin B \cos B \Gamma^{01} + \cos^2 B \Gamma^{23})\psi} e^{+\frac{1}{2}(\sin B \Gamma^{04} - i \cos B \Gamma^0)\chi} e^{-\frac{1}{2}(\sin B \Gamma^{14} - i \cos B \Gamma^1)\tau} \epsilon_0$$

where  $\epsilon_0$  is any spinor with constant components in the frame (15).

For Poincaré coordinates we choose the vielbein

$$e^0 = \frac{1}{\sqrt{\sigma}} [dt + \sin B \sigma^3], \quad e^i = \sqrt{\sigma} \sigma^i.$$

The Killing spinors are [10]

$$\epsilon = R(\theta, \phi, \psi) \epsilon_+ \langle \text{cid:20} \rangle \sin B \Gamma^{04} \langle \text{cid:21} \rangle R(\theta, \phi, \psi) \epsilon_-^0,$$

where  $R(\theta, \phi, \psi) = e^{-\frac{i}{2} \Gamma^0 \epsilon_\pm^0} = e^{\frac{1}{2} \Gamma^{23} \psi} e^{\epsilon_\pm^0}$ , and  $\epsilon_\pm^0$  are constant.

### 3 Supersymmetric Probe Configurations

In this section, we find classical brane trajectories which preserve some supersymmetries of the rotating attractor (7). The worldvolume action has a local  $\kappa$ -symmetry (parameterized by  $\epsilon$ ) as well as a spacetime supersymmetry transformation (parameterized by  $\Theta$ ) which acts nonlinearly. A spacetime supersymmetry is preserved if its action on the worldvolume fermions  $\Theta$  can be compensated by a  $\kappa$  transformation [11][12]:

$$\delta_\epsilon \Theta + \delta_\kappa \Theta = \epsilon + (1 + \Gamma)\kappa(\sigma) = 0,$$

where  $\Gamma$  is given in various cases analyzed below. This gives the condition  $(1 - \Gamma)\epsilon = 0$ , which must be solved for both the Killing spinor and the probe trajectory.

#### 3.1 Zero-Brane Probe

For the zero-brane the (bosonic part of the)  $\kappa$ -symmetry projection operator is

$$\Gamma = \frac{1}{\sqrt{h}} \tilde{\Gamma}_0,$$

where  $h$  and  $\tilde{\Gamma}_0$  are the pull-backs of the metric and Dirac matrix onto the worldline of the zero-brane, respectively:  $h_{00} = \partial_0 X^\mu \partial_0 X^\nu G_{\mu\nu}$  and  $\tilde{\Gamma}_0 = \partial_0 X^\mu e_a{}^\mu \Gamma^a$ .

**3.1.1 Global Coordinates** First, let's examine the global coordinates. In the static gauge, where we set the worldvolume time  $\sigma^0$  equal to the global time  $\tau$ , the  $\kappa$ -symmetry operator is

$$\Gamma = \frac{1}{\sqrt{h}} \tilde{\Gamma}_\tau.$$

To solve for the classical trajectory of a supersymmetric zero-brane, we plug the Killing spinors (16) into the  $\kappa$ -symmetry condition (22). A zero-brane following a classical trajectory, given by  $(\chi(\tau), \theta(\tau), \phi(\tau), \psi(\tau))$ , is supersymmetric if, in the notation of (16),

$$\Gamma S \epsilon_0 = \epsilon_0,$$

for some constant  $\epsilon_0$ , where  $S = S(\chi, \tau, \theta, \phi, \psi)$ . The explicit prefactors are:

$$\Gamma S = [(\cosh \chi \cos \tau \cos^2 B + \sin \theta \cos \phi \sin^2 B) \Gamma^0 - i \cos \theta \sin B \Gamma^{02} - i \sin \theta \sin \phi \sin B \Gamma^{03} + (\sin \theta \cos \phi) \sin B \cos B \Gamma^{04}]$$

and similar expressions for other components.

We first observe that a probe static in global time  $\tau$  cannot be supersymmetric. For such a probe we have  $\frac{d\chi}{d\tau} = \frac{d\theta}{d\tau} = \frac{d\psi}{d\tau} = 0$  and the  $\kappa$ -symmetry condition reduces to terms proportional to  $\cos \tau$ ,  $\sin \tau$ , and constants that must all vanish separately, which is clearly impossible. The absence of such configurations is not surprising, because angular momentum must be nonzero for a nontrivial BPS configuration.

Now we allow the probe to orbit around the  $S^3$ . Solving the  $\kappa$ -symmetry condition (22) using (28) for Killing spinors obeying  $\Gamma^{02}\epsilon_0 = \epsilon_0$ , we find the supersymmetric trajectory at generic  $(\chi, \theta, \psi)$  to be

$$\dot{\phi} = 1, \quad \dot{\psi} = -\cos \theta, \quad \dot{\chi} = \dot{\theta} = 0.$$

This describes a probe orbiting along the  $\phi$ -direction. The constraint on the Killing spinor (30) projects out half of the components of  $\epsilon_0$ , meaning the orbiting zero-brane probe is a half-BPS configuration. We will show in the next subsection, using the BPS bound, that this supersymmetric trajectory is unique up to rotations.

**3.1.2 A BPS Bound** The worldline action of a zero-brane probe with mass  $m$  and electric charge  $q$  can be written as

$$S = -m \int d\sigma^0 \sqrt{h} + q \int A_{[1]},$$

where  $A_{[1]}$  is the 1-form gauge field (11). We consider supersymmetric probes which have  $q = m$ . In global coordinates with  $\sigma^0 = \tau$ , the Lagrangian of the system is

$$L = -m \cosh^2 \chi - m [\sin B \sinh \chi \cos B (\dot{\psi} + \cos \theta \dot{\phi})]^2 - m \sin^2 \theta \dot{\phi}^2 + m [\cos B \sinh \chi + \sin B (\dot{\psi} + \cos \theta \dot{\phi})].$$

The corresponding Hamiltonian is

$$H = \frac{\cosh \chi}{m} P_\chi^2 + \frac{1}{\cosh \chi} \left[ P_\theta^2 + \left( \frac{\cos \theta P_\phi - \sin \theta}{\sin \theta} \right)^2 + P_\phi^2 + (\sin B P_\psi - \cos B)^2 \right] + \sinh \chi (\sin B P_\psi - \cos B),$$

where the momenta are

$$P_\chi = m \cosh \chi \dot{\chi}, \quad P_\theta = m \cosh \chi \dot{\theta}, \quad P_\phi = m \cosh \chi \sin^2 \theta \dot{\phi} + m \sin B \cos \theta [\sin B \sinh \chi \cos B (\dot{\psi} + \cos \theta \dot{\phi})], \quad P_\psi = m \cosh \chi \dot{\psi} + \sin B \cosh \chi \cos B (\dot{\psi} + \cos \theta \dot{\phi}),$$

The unbroken rotational symmetries lead to the conserved charges:

$$J_1^{right} = \sin \phi P_\theta + \cos \phi (\cot \theta P_\phi - \csc \theta P_\psi), J_2^{right} = \cos \phi P_\theta - \sin \phi (\cot \theta P_\phi - \csc \theta P_\psi), J_3^{right} = P_\phi, \quad J_3^{left} = P_\psi.$$

It is easy to see that there are no static solutions. They would have to minimize the potential energy according to

$$V_{eff} = m \cos B \cosh \chi (\cos B \sinh \chi \cos^2 B \sinh^2 \chi + 1) - m \sin B,$$

which has no solutions for finite  $\chi$ . Physically, the probe is accelerated to  $\chi = \infty$ . Now we allow the probe to orbit. Solutions of this type can be stabilized by the angular potential. The supersymmetric configuration turns out to be at constant radius in AdS, i.e.  $P_\chi = 0$ . The Hamiltonian is minimized with respect to  $\chi$  when

$$\tanh \chi = \frac{\sin B P_\psi - \cos B}{\sqrt{P_\theta^2 + (\cos \theta P_\phi - \sin \theta)^2 + P_\phi^2}}.$$

The value of  $H$  at the minimum is

$$H_{min} = \sqrt{P_\theta^2 + (J_1^{right})^2 + (J_2^{right})^2 + (J_3^{right})^2} + \sin B P_\psi - \cos B,$$

where  $\vec{J}_{right}^2 = (J_1^{right})^2 + (J_2^{right})^2$ . This implies the BPS bound

$$H \geq |\vec{J}_{right}| + \sin B P_\psi - \cos B,$$

for generic  $\chi$ .

Up to spatial rotations, we may always choose static BPS solutions to satisfy

$$J_1^{right} = J_2^{right} = 0.$$

This implies  $P_\theta = 0$  and  $\cos \theta P_\phi = P_\psi$ . Hence, the azimuthal angle is determined by the ratio of left and right angular momenta:

$$\cos \theta = \frac{J_3^{left}}{J_3^{right}}.$$

We can rewrite  $\dot{\phi}$  and  $\dot{\psi}$  in terms of  $P_\phi$  and  $P_\psi$ . With  $\dot{\chi} = \dot{\theta} = 0$ ,

$$\dot{\phi} = \frac{P_\phi - \cos \theta P_\psi}{\cosh \chi \sin^2 \theta}, \quad \dot{\psi} = \frac{\sin B P_\psi - \cos B}{\cosh \chi \cos^2 B} - \cos \theta \dot{\phi}.$$

Eliminating  $\chi$  through the minimization condition yields the solution

$$\dot{\theta} = 0, \quad \dot{\psi} = 0, \quad \cos \theta = \frac{\sin B}{\cos B \sinh \chi}, \quad \dot{\phi} = \frac{\cos B \sinh \chi}{\sin B \cos \theta}.$$

The energy of the particle following this trajectory is equal to

$$H = \frac{\cos B \sinh \chi}{\sin B \cos \theta}.$$

We see that the solution with  $\dot{\phi} = +1$  ( $\dot{\phi} = -1$ ) corresponds to a chiral (anti-chiral) BPS configuration. Therefore, we have confirmed that the supersymmetric trajectories obtained by solving the  $\mathcal{N} = 1$  supersymmetry condition correspond to BPS states.

**3.1.3 Poincaré Coordinates** In Poincaré coordinates with static gauge  $\sigma^0 = t$ , the  $\mathcal{N} = 1$  supersymmetry condition for a static probe is

$$\Gamma \epsilon = i\Gamma^0 \epsilon = \epsilon.$$

This equation is solved by simply taking  $\epsilon = \epsilon_+ = \frac{1}{2}(1 + i\Gamma^0)\epsilon_0$ . Again, we find a half-supersymmetric solution, although the broken supersymmetries are different than in the global case. It can be seen that there are no supersymmetric orbiting trajectories in Poincaré time.

### 3.2 One-Brane Probe

In this subsection, we find supersymmetric one-brane configurations. We consider a specific ansatz with no worldvolume electromagnetic field and with the one-brane geometry:

$$\tau = \sigma^0, \quad \phi = \dot{\phi}\sigma^0 + \phi'\sigma^1, \quad \psi = \dot{\psi}\sigma^0 + \psi'\sigma^1,$$

where  $(\sigma^0, \sigma^1)$  are worldvolume coordinates, and  $\dot{\phi}, \dot{\psi}, \phi'$  and  $\psi'$  are all taken to be constant. Note that since  $(\psi, \phi)$  are the orbits of  $(J_3^L, J_3^R)$ , they may be viewed as one-brane momentum-winding modes on the torus generated by  $(J_3^L, J_3^R)$ . This torus degenerates to a circle at  $\theta = 0, \pi$ . One-branes of the form (53) at these loci are therefore static (up to reparametrizations).

With no electromagnetic field the  $\mathcal{N} = 1$  supersymmetry condition is

$$\epsilon^{ij} \tilde{\Gamma}_{ij} \epsilon = \epsilon,$$

where  $h$  and  $\tilde{\Gamma}_i$  are the pull-backs of the 5D metric and gamma matrices onto the one-brane worldsheet. With the ansatz (53), we have explicitly

$$\tilde{\Gamma}_0 = \Gamma_\tau + \dot{\phi}\Gamma_\phi + \dot{\psi}\Gamma_\psi, \quad \tilde{\Gamma}_1 = \phi'\Gamma_\phi + \psi'\Gamma_\psi,$$

$$\epsilon^{ij}\tilde{\Gamma}_{ij} = \frac{2}{\sqrt{\det h}}[\phi'\Gamma_{\tau\phi} + \psi'\Gamma_{\tau\psi} + (\dot{\phi}\psi' - \dot{\psi}\phi')\Gamma_{\phi\psi}],$$

and the metric components are

$$h_{00} = \cosh^2 \chi + [\sin B \sinh \chi \cos B(\dot{\psi} + \cos \theta \dot{\phi})]^2 + \sin^2 \theta \dot{\phi}^2, \quad h_{11} = \cos^2 B(\psi' + \cos \theta \phi')^2 + \sin^2 \theta \phi'^2, \quad h_{01} = [\sin B \sinh \chi \cos B(\dot{\psi} + \cos \theta \dot{\phi}) + \sin^2 \theta \dot{\phi} \phi']$$

Hence

$$\det h = (\cosh^2 \chi [\cos^2 B(\psi' + \cos \theta \phi')^2 + \sin^2 \theta \phi'^2] - [\sin B \sinh \chi \cos B(\dot{\psi} + \cos \theta \dot{\phi}) + \sin^2 \theta \dot{\phi} \phi']^2).$$

It is simplest to analyze the  $\epsilon$ -symmetry condition in the form

$$\epsilon^{ij}\tilde{\Gamma}_{ij}S\epsilon_0 = \epsilon_0.$$

The rotated gamma matrices appearing in this expression are explicitly given by lengthy expressions involving the background fields and probe velocities. This simplifies at points obeying

$$\psi'\dot{\phi} + \phi'\dot{\psi} = 0.$$

Under these conditions

$$\sinh \chi = \tan B \cos \theta, \quad \sqrt{\det h} = (\phi' + \cos \theta \psi'),$$

where

$$\frac{1}{\sqrt{\det h}}[\phi'\Gamma_{\tau\phi} + \psi'\Gamma_{\tau\psi} + (\dot{\phi}\psi' - \dot{\psi}\phi')\Gamma_{\phi\psi}]S = \frac{1}{(\phi' + \cos \theta \psi')} \Gamma^{02} + \frac{1}{(\phi' + \cos \theta \psi')} (\phi' D_1 + \psi' D_2)(\Gamma^0 - \Gamma^2),$$

with  $D_1$  and  $D_2$  being specific combinations of gamma matrices and background fields.

So far we have not chosen which supersymmetries are to be preserved. We take those generated by spinors obeying  $\Gamma^{02}\epsilon_0 = \epsilon_0$ , or equivalently  $\Gamma^2\epsilon_0 = \Gamma^0\epsilon_0$ . In this case, the last term can be dropped and the supersymmetry conditions are satisfied.

To summarize, any configuration satisfying

$$\psi' \dot{\phi} + \phi' \dot{\psi} = 0, \quad \sinh \chi = \tan B \cos \theta, \quad \dot{\chi} = \dot{\theta} = 0,$$

preserves those supersymmetries corresponding to  $\Gamma^{02} \epsilon_0 = \epsilon_0$ . Other BPS configurations preserving other sets of supersymmetries can be obtained by  $SL(2, \mathbb{R}) \times SO(4)$  rotations of these ones.

Note that, as for the zero-branes, there are generic solutions for any  $\theta$ . These include  $\theta = 0, \pi$ , which correspond to static one-branes because the  $(\psi, \phi)$  torus degenerates to a circle along these loci. Static solutions are possible because a one-brane probe in 5D couples magnetically to the dual of the spacetime gauge field  $F_{[2]}$  of (11), hence there is nonzero angular momentum carried by the fields.

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