Two component dark matter with multi-Higgs postprint

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Abstract

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Full Text

Preamble

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Two component dark matter with multi-Higgs portals

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Abstract

With the assistance of two additional symmetries—an extra hidden gauge group $SU(2)_D$ and a global U(1) group—we propose a two-component dark matter (DM) model. After the spontaneous breaking of $SU(2)_D\times$ U(1), we obtain both vector and scalar DM candidates. These two DM components communicate with the Standard Model (SM) via three Higgs bosons that serve as multi-Higgs portals. The three Higgs fields are mixing states of the SM Higgs, the hidden sector Higgs, and the real part of an additional complex scalar singlet. We study the relic density and direct detection prospects of DM in three scenarios. The resonance behaviors and interplay between the two DM components are revealed through investigations of the relic density in the parameter spaces of the DM masses. The electroweak precision observables constrain the two Higgs portal couplings (λ_m and δ_2). The relevant vacuum stability and naturalness problems in the λ_m-δ_2 parameter space are also studied. The model can alleviate these two problems in certain parameter regions under the constraints from electroweak precision observables and Higgs indirect searches.

Contents

Introduction

The discovery of a Higgs particle with mass around 126 GeV at the Large Hadron Collider (LHC) [1, 2], roughly consistent with Standard Model (SM) predictions, appears to complete the SM. However, the naturalness problem remains unsolved. To alleviate this problem, as is well known and extensively studied, additional bosonic fields are needed [3–7]. Recently, it has been noted that Higgs field-strength renormalization could be enhanced by these new bosonic fields [8–10], and that after Higgs field normalization, modifications to Higgs couplings could be detected indirectly [9, 11]. Thus, this sheds light on testing mechanisms that alleviate the naturalness problem. Secondly, the SM must be extended to accommodate cold dark matter (CDM) and the baryon asymmetry of the universe (BAU). To prevent the BAU from being washed out after its generation, a strong first-order electroweak phase transition (SFOEWPT) is necessary, which is beyond the capacity of the SM and requires new scalar fields [12-14]. Simultaneously, the method for testing mechanisms that alleviate the naturalness problem could also be used to test SFOEWPT, which is one of the primary aims of the International Linear Collider (ILC) [8]. Thirdly, vacuum instability can be remedied by introducing vector and scalar fields [3, 17-22], thus making inflation with a not-too-small top quark mass compatible with BICEP and Planck data [23, 24].

All these considerations require extending the SM with new bosonic degrees of freedom, for which we choose vector and scalar fields. There have been many studies on multi-component DM scenarios [3, 25-35], as well as the dynamical DM scenario [36] whose phenomenological consequences are often quite distinct and can be applied to a much broader variety of multi-component DM scenarios [37]. The primary purpose of this work is to explain the CDM relic density, where the two different DM components interact with each other in addition to interacting with the three Higgs bosons, thus affecting the evolution of the DM component number densities through coupled Boltzmann equations [3, 38].

Instead of introducing vector dark matter through an effective Lagrangian approach that yields strong constraints on the parameter space from unitarity bounds [3, 39, 40], we consider a scenario where the vector dark matter fields (V^μ) respect an additional non-Abelian gauge symmetry $\mathrm{SU}(2)_D$. After $\mathrm{SU}(2)_D$ is broken via a complex doublet (ϕ), an SO(3) custodial symmetry is induced that V^μ respects, thereby making V^μ stable. An additional complex scalar singlet S is added to the model.

After the global U(1) symmetry, under which $S \to e^{i\alpha} S$ and respected by the potential $V(H, \phi, S)$, is broken both spontaneously and softly, the real part of S (denoted as S) acquires a vacuum expectation value (VEV), while the imaginary part of S (denoted as A) respects a residual Z_2 symmetry, which stabilizes it as the other DM candidate. Following the breaking of $\mathrm{SU}(2)_D$ by two scalar fields—the real parts of ϕ (η') and S—the two DM components interact with SM particles and with each other through three Higgs fields, leading to three possible resonance enhancement effects. This results in significant enhancement of the annihilation cross sections for both DM-to-DM and DM-to-SM particle processes.

The magnitudes of the relic density around the three resonances decrease, as it is robustly inversely proportional to the annihilation cross sections of DM to SM particles. The annihilation cross sections of scalar DM fields (A) and vector DM fields (V) to SM particles are enhanced around the three Higgs masses due to t-channel, u-channel, and seagull diagrams.

The spin-independent (SI) DM-nucleon scattering cross sections are determined by t-channel interactions between nucleons and the two DM components through the exchange of three Higgs fields. The model can be distinguished from models without interactions between the two DM components, since the opening of the channel $AA \leftrightarrow VV$ affects the evolution of the DM number density and thus the magnitude of the relic density. These effects are explored and illustrated in the DM relic density analysis section of this work. Higgs indirect searches at the LHC require small mixing between h and η' (and S). Electroweak precision observable experiments impose stringent constraints on our parameter space. The vacuum stability of the model can be improved under these considerations. We also explore ways to alleviate the naturalness problem and its indirect signatures.

The paper is organized as follows. We first construct the model and explore the relic density and direct detection prospects of the DM components. We then investigate Higgs indirect searches and electroweak precision constraints on the model. Subsequently, we study the vacuum stability of the model. We also consider the possibility of alleviating the naturalness problem and how to trace its footprint. Finally, we conclude with a discussion and summary.

2 The Model

To construct a model that includes stable vector and scalar fields, we introduce these fields as follows. The vector field ${V'}^{\mu}$ is introduced as the gauge field of the $\mathrm{SU}(2)_D$ symmetry, which couples to the SM through a doublet ϕ that is a singlet under SM gauge interactions but charged under $\mathrm{SU}(2)_D$. Note that mixing between ${V'}^{\mu}$ and SM gauge bosons through kinetic mixing is absent due to the non-Abelian nature of $\mathrm{SU}(2)_D$. We add one additional complex scalar singlet S, which requires an extra global U(1) symmetry beyond $\mathrm{SU}(2)_D$ and the SM gauge group. This complex singlet interacts with the SM and $\mathrm{SU}(2)_D$ sectors through Higgs portal and ϕ portal terms in the scalar potential, while transforming trivially under the SM and $\mathrm{SU}(2)_D$ gauge groups.

The Lagrangian of our model is given by:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{scalar}} + \mathcal{L}_{\mathrm{Yukawa}}$$

where the scalar potential is

$$V_{H,\phi,S} = V_{H,\phi} + V_{H,S} + V_{\phi,S}$$

with

$$V_{H,\phi} = \lambda_m \phi^\dagger \phi H^\dagger H$$

$$V_{H,S} = \frac{\delta_2}{2} H^\dagger H S^2 + \frac{a_1}{\sqrt{2}} e^{i\phi_{a_1}} S + \text{c.c.}$$

$$V_{\phi,S} = \frac{\delta_1}{2}\phi^\dagger\phi S^2 + \frac{b_1}{2}e^{i\phi_{b_1}}S^2 + \text{c.c.} + \lambda_\phi(\phi^\dagger\phi)^2 + d_2|S|^4$$

Hereafter, the Lorentz index μ of the vector field V^μ will be omitted for simplicity in some cases. The covariant derivative is $D_\mu\phi = \partial_\mu\phi - ig_\phi V'_\mu\phi$. The hidden gauge coupling $g_\phi < 4\pi$, required by the unitarity bound, must hold for any thermal particle whose relic density arises from freeze-out of its annihilations [39, 45, 46]. In particular, the b_1 and a_1 terms explicitly break the global U(1) symmetry. In the SM Lagrangian, the Higgs potential is defined as

$$V_H = -\frac{m^2}{2}H^\dagger H + \lambda(H^\dagger H)^2$$

with $H = (0, v+h)/\sqrt{2}$, where v is the VEV of the Higgs field.

After $\mathrm{SU}(2)_D$ is spontaneously broken, both ϕ and the singlet S acquire VEVs:

$$\phi = \left(0, \frac{v_\phi + \eta'}{\sqrt{2}}\right), \quad S = \frac{v_s + S + iA}{\sqrt{2}}$$

In unitary gauge, Eq. (2.1) becomes

$$\mathcal{L} \supset -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}(g_\phi v_\phi)^2 V_\mu V^\mu + g_\phi^2 v_\phi V_\mu V^\mu \eta' + \frac{1}{2}g_\phi^2 V_\mu V^\mu \eta a'^2 + \frac{1}{2}(\partial_\mu\eta')^2 + V(h, \eta', S, A)$$

where $V^\mu = UV'^\mu U^{-1} - \frac{i}{g_\phi}[\partial_\mu U]U^{-1}$ with $U = \exp(i\xi/v_\phi)$. The vector DM mass is given by $m_V = g_\phi v_\phi/2$, and the tree-level potential $V_0(h, \eta', S, A)$ becomes:

$$\begin{aligned} V_0(h,\eta',S,A) &= \frac{\lambda}{4}(h+v)^4 + \frac{\lambda_\phi}{4}(\eta'+v_\phi)^4 + \frac{\lambda_m}{2}(h+v)^2(\eta'+v_\phi)^2 \\ &+ \frac{\delta_2}{4}(h+v)^2(S+v_s)^2 + \frac{\delta_1}{4}(\eta'+v_\phi)^2(S+v_s)^2 + \frac{d_2}{4}(S+v_s)^4 \\ &+ \frac{b_2+b_1}{4}(S+v_s)^2A^2 + \frac{a_1}{\sqrt{2}}(S+v_s) + \frac{d_2}{4}A^4 + \text{c.c.} \end{aligned}$$

Here, η' lives in the fundamental representation of $\mathrm{SU}(2)_D$ and exhibits a custodial SO(3) symmetry in the V^μ component space, which makes the three components degenerate in mass and thus stable [47]. The explicit Z_2-breaking term proportional to a_1 is introduced to avoid the cosmological domain wall problem [48–50]. After choosing $\phi_{a_1} = \phi_{b_1} = \pi$, the potential retains a Z_2 symmetry for Im(S), thereby ensuring the stability of the particle A [51, 52].

Requiring that the potential in Eq. (2.9) has a minimum at $\langle h\rangle = v/\sqrt{2}$, $\langle\eta'\rangle = 0$, $\langle S\rangle = v_s$, and $\langle A\rangle = 0$, the following minimization conditions are obtained:

$$\left.\frac{\partial V_0}{\partial h}\right|_{\min} = 0, \quad \left.\frac{\partial V_0}{\partial \eta'}\right|_{\min} = 0, \quad \left.\frac{\partial V_0}{\partial S}\right|_{\min} = 0$$

where all derivatives are evaluated at $(h, \eta', S, A) = (v, 0, v_s, 0)$. These minimization conditions allow the Higgs VEV v and the singlet VEV v_s to replace m^2 and b_2 according to:

$$m^2 = \lambda v^2 + \lambda_m v_\phi^2 + \frac{\delta_2 v_s^2}{2}$$

$$b_2 = -\frac{a_1}{\sqrt{2} v_s} - \frac{\delta_2 v^2}{4} - \frac{\delta_1 v_\phi^2}{4} - \frac{d_2 v_s^2}{2}$$

Thus, at the minimum, the mass matrix in the basis (h, η', S) is obtained as:

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda v^2 & \lambda_m v v_\phi & \delta_2 v v_s \\ \lambda_m v v_\phi & 2\lambda_\phi v_\phi^2 & \delta_1 v_\phi v_s \\ \delta_2 v v_s & \delta_1 v_\phi v_s & 2a_1/v_s + d_2 v_s^2 \end{pmatrix}$$

To work in the mass eigenstates h_1, h_2, h_3, we diagonalize the mass matrix Eq. (2.13) through $R\mathcal{M}^2 R^T = \mathcal{M}^2_{\text{diag}}$, with the rotation matrix R given by:

$$R = \begin{pmatrix} c_1 c_2 & c_1 s_2 s_3 & s_1 s_2 c_3 - c_2 s_1 s_3 & s_1 c_3 + c_2 s_1 s_3 \\ -s_1 c_2 & -s_1 s_2 s_3 + c_1 c_3 & c_1 s_2 c_3 + s_1 s_3 & c_1 c_3 - s_1 s_2 \\ s_2 & -c_2 s_3 & -c_2 c_3 & \end{pmatrix}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for $i = 1, 2, 3$ represent the mixing angles θ_{12}, θ_{23}, and θ_{13} respectively.

3 Dark matter analysis

In our model, we have two DM components. To properly present the novel features of the model, we use the coupled Boltzmann equations explored in [3], with annihilation channels depicted in Fig. 1 [Figure 1: see original paper], and details of the annihilation cross sections provided in Appendix A.

The relic density of each DM component can be obtained through

$$\Omega_{A,V} h^2 = 2.755 \times 10^8 \frac{M_{A,V}}{\text{GeV}} Y_{A,V}(T_0) [53, 54]$$

after numerically calculating $Y_{A,V}(T_0)$. The total relic density is the sum of the two components:

$$\Omega h^2 = \Omega_A h^2 + \Omega_V h^2$$

For the numerical calculations of $Y_{A,V}(T_0)$, we refer to Eqs. (B.1), (B.2), (B.3), and (B.4).

To investigate how the interplay between the two DM components affects the evolution of the DM abundance and the dependence of the DM relic density on various parameters, we scan the parameter space according to the following three scenarios:

1. Scan in the M_A-M_V plane for the case $M_A > M_V$, with other parameters given in Table 1 .
2. Scan in the M_A-M_V plane for the case $M_V > M_A$, with other parameters given in Table 1.
3. Scan in the λ_m-δ_2 plane, with DM masses fixed as $M_V = 120$ GeV, $M_A = 150$ GeV, and other parameters given in Table 2 .

3.1 Relic density analysis

Setting the free parameters as in Table 1, the three Higgs masses are obtained as $m_{h_1} = 124.8$ GeV, $m_{h_2} = 233.8$ GeV, and $m_{h_3} = 71.1$ GeV in ascending order.

1. The first group with $M_A > M_V$ Figure 2 [Figure 2: see original paper] shows the cross sections for the channels $AA \to XX$ and $AA \to VV$ (left panel) and the corresponding relic density $\Omega_A h^2$ (right panel). Figure 3 [Figure 3: see original paper] shows the cross sections for the channel $VV \to XX$ (left panel) and the corresponding relic density $\Omega_V h^2$ (right panel) for $M_A > M_V$.

The annihilation channel $AA \leftrightarrow VV$ opens up and is shown by the green line in the left panel of Fig. 2. The width of the green band arises from the variation of the vector DM mass M_V, and this width becomes increasingly larger as the scalar DM mass M_A increases, since a larger scalar DM mass leaves a wider viable range for M_V.

Figure 2 depicts that the channel $AA \to XX$ dominates when $M_A > 70$ GeV, and the variation of the relic density $\Omega_A h^2$ is shown in the right panel. Three peaks in the annihilation cross section for $AA \to VV$ appear at $M_A = m_{h_1}/2$, $m_{h_2}/2$, and $m_{h_3}/2$, corresponding to three resonances at these mass values, as can also be seen from Eq. (A.6). Simultaneously, we find three additional peaks in the annihilation cross section for $AA \to XX$, besides the three Higgs resonance peaks, due to the opening of new partial t-channel, u-channel, and seagull channels $AA \to h_i h_i$ through the exchange of the scalar DM particle itself around these mass values. These peaks manifest in the right panel of Fig. 2 as color transitions along the M_A axis. When a peak in the cross section exceeds approximately 1×10^{-9} pb (left panel), a corresponding decrease in the relic density appears (right panel).

Finally, we emphasize that the opening of the channel $AA \to VV$ causes a decrease in the magnitude of $\Omega_A h^2$. Because $\Omega_A h^2 \propto 1/\langle\sigma v\rangle_{AA\to XX}$ when the channel $AA \to VV$ is closed, one might expect···However, the right panel of Fig. 2 shows a greater decrease in $\Omega_A h^2$ around $M_A = m_{h_1,h_2}/2$, demonstrating the effects of the annihilation process $AA \to VV$.

For the case $M_A > M_V$, as shown in Fig. 3, the channel $VV \to AA$ is closed when analyzing $\Omega_V h^2$. The cross sections of the channels $AA \to XX$ and $VV \to XX$ affect the corresponding relic density $\Omega_V h^2$, with the latter dominating the value of $\Omega_V h^2$. Only the narrow peak caused by the resonance at $M_V \approx m_{h_2}/2$ brings a sizable decrease in $\Omega_V h^2$. The viable region of M_V that matches the experimental value $\Omega h^2 = 0.1189$ [55] is very small, as shown in the right panel.

2. The second group with $M_V > M_A$ Figure 4 [Figure 4: see original paper] shows the annihilation cross sections for the channel $AA \to XX$ (left panel) and the corresponding relic density $\Omega_A h^2$ (right panel) for $M_V > M_A$. In this case, the channel $AA \leftrightarrow VV$ is forbidden due to $M_V > M_A$.

As M_A increases, the first peak caused by the resonance effect leaves no trace in the relic density (right panel of Fig. 4) because the cross section magnitude is too small. The second peak arises from the resonance effect at $M_A = m_{h_1}/2$, and the slight increase in the cross section induces a small decrease in the corresponding relic density. The third peak demonstrates the opening of partial t- and u-channels $AA \to h_3 h_3$ and brings a very large decrease in the relic density around $M_A = m_{h_3}$ as expected. The fourth peak, caused by the resonance at $M_A = m_{h_2}/2$, also brings a sizable decrease in the relic density magnitude. We also notice small wiggles around $M_V = 160$ GeV in the right panel of Fig. 4. This phenomenon can be explained by Eq. (B.4), which shows that the behavior of $VV \to AA$ affects the calculation of $\Omega_A h^2$, and the variation of the cross section $\langle\sigma v\rangle_{VV\to AA}$ generates these wiggles.

Figure 5 [Figure 5: see original paper] shows the cross sections for annihilation channels $VV \to XX$ and $VV \to AA$ (left panel) and the dominant relic density $\Omega_V h^2$ (right panel) for $M_V > M_A$. The cross sections are small in most regions, but large decreases in $\Omega_V h^2$ occur in two regions of M_V. For the first decrease, both channels contribute comparably. For the second decrease, $VV \to XX$ dominates the magnitude of $\Omega_V h^2$, especially when the partial t-, u-, and seagull channels $VV \to h_2 h_2$ open up for $M_V > m_{h_2}$. These effects produce too large a $VV \to XX$ cross section, and the window for generating the correct relic density $\Omega_V h^2$ is almost closed.

Finally, the two panels in Fig. 6 [Figure 6: see original paper] present the total relic density for $M_A > M_V$ and $M_V > M_A$, respectively. With the parameter set given in Table 1, a sufficiently large relic density can be easily obtained.

3. The third group: varying λ_m and δ_2 Figure 7 [Figure 7: see original paper] depicts the relationships among δ_2, λ_m, and the cross sections. Figure

8 [Figure 8: see original paper] illustrates that the magnitude of $\Omega_A h^2$ ($\Omega_V h^2$) reaches its critical value around $\delta_2 \approx 0.01$ and $\lambda_m \approx 0.09$. The DM relic density for each component depends primarily on its own coupling parameters, as all relevant annihilation cross sections are proportional to either δ_2 or λ_m.

3.2 Direct detection

In our model, the two DM components—the scalar A and vector V—interact with SM particles through the exchange of three Higgs bosons. Thus, the DM-nucleon scattering cross section is spin-independent (SI). For each individual component, the SI DM-nucleon cross sections are calculated to be:

$$\sigma_A^{\rm SI} = \frac{m_N^4 f_N^2}{16\pi v^2 (M_A + m_N)^2} \left| \sum_{i=1}^{3} \frac{R_{i1} R_{i2}}{m_{h_i}^2} \right|^2$$

$$\sigma_V^{\rm SI} = \frac{m_N^4 f_N^2}{16\pi v^2 (M_V + m_N)^2} \left| \sum_{i=1}^{3} \frac{R_{i1} R_{i3}}{m_{h_i}^2} \right|^2$$

where R_{ij} comes from Eq. (2.15), m_N is the nucleon mass, and $f_N = \sum_q f_L^q + \frac{3}{27} f_H^q$ is the effective Higgs-nucleon coupling, which includes contributions from light quarks (f_L) and heavy quarks (f_H). We take $f_N = 0.326$ [56] in our numerical analysis.

Since current experiments assume the local DM density is provided by a single DM species, the situation where both A and V components contribute to the local DM density prevents us from using current experimental results directly. Assuming the contribution of each DM component to the local density is proportional to its contribution to the relic density, the SI scattering cross section should be rescaled by a factor $\Omega_{A,V} h^2 / \Omega_{\rm DM} h^2$. Thus, the corresponding upper limit on the SI DM-nucleon cross section for each component is [38, 52]:

$$\sigma_A^{\rm SI} < \left(\frac{\Omega_{\rm DM} h^2}{\Omega_A h^2} \right) \sigma_{\rm exp}$$

$$\sigma_V^{\rm SI} < \left(\frac{\Omega_{\rm DM} h^2}{\Omega_V h^2} \right) \sigma_{\rm exp}$$

Similar to the previous section, we perform analyses using LUX experiment results [57] for the three scenarios:

1. $M_A > M_V$: The left and right panels of Fig. 9 [Figure 9: see original paper] show that, with M_V unconstrained, M_A should be greater than 30 GeV.

2. $M_V > M_A$: The left and right panels of Fig. 10 [Figure 10: see original paper] show that both M_A and M_V should be no smaller than 50 GeV.
3. **Parameter space of δ_2 and λ_m**: Figure 11 [Figure 11: see original paper] shows the parameter regions that survive the direct detection constraints on the model coefficients. The figure depicts that all parameter spaces satisfy the experimental constraints.

4 Higgs indirect search and electroweak precision constraints

In this section, we study the effects of mixing among the three fields h, η', and S using Higgs indirect searches and constraints from electroweak precision observables.

4.1 Higgs indirect search

Current LHC measurements of Higgs couplings constrain the matrix element R_{11}, which describes the deviation of h_1 from the SM Higgs. The couplings of h_1 to all SM particles are rescaled versions of the SM couplings, taking the form:

$$g_{h_1 XX} = R_{11} g^{\rm SM}_{h_1 XX}$$

For the first two parameter groups in the previous section, additional decay channels $h_1 \to AA(VV)$ exist, but these hardly change the total width since they are suppressed by factors of R_{12}^2 (R_{13}^2) in $\Gamma(h_1 \to AA(VV))$. The same logic applies to the analysis with the third parameter group. Thus, signal strengths μ_{XX} associated with Higgs measurements are functions of R_{11}:

$$\mu_{XX} = \frac{\sigma}{\sigma_{\rm SM}} \frac{\rm BR}{\rm BR_{SM}} = R_{11}^2$$

where σ and BR (with superscript SM) are the production cross section and branching ratios of h_1 (and the SM Higgs). Therefore, the extent to which the model differs from SM Higgs measurements is determined by the value of R_{11}. The value of R_{11} for the parameter setup given in Table 1 is 0.9973, and we refer to Fig. 12 [Figure 12: see original paper] for the R values corresponding to the benchmark scenario given in Table 2.

4.2 Electroweak precision observables constraints

The presence of the scalar h_3 with mass $m_{h_3} < 114$ GeV is subject to tight bounds from LEP [58]. However, these bounds are easily satisfied in all our parameter choices, since the mixing between h_1 and h_3 is very small, approximately 0.0573 for the benchmark scenario given in Table 1, as can be seen in Fig. 12 for the benchmark scenario given in Table 2.

As for electroweak precision observables, the oblique parameters S and T, which parameterize potential new physics contributions to electroweak radiative corrections, are computed following Refs. [47, 60]. In our model, they are:

$$S = \frac{1}{2\pi}\left[\sum_{i=1}^{3} R_{i1}^2 \log\frac{m_{h_i}^2}{m_Z^2} - \log\frac{m_h^2}{m_Z^2} + \cdots\right]$$

$$T = \frac{1}{16\pi \sin^2\theta_W}\left[\sum_{i=1}^{3} R_{i1}^2 G(m_{h_i}^2, m_Z^2) - G(m_h^2, m_Z^2) + \cdots\right]$$

where R_{AB}, $G(m_A^2, m_B^2)$, and $f(R_{AB})$ are given by:

$$R_{AB} = \frac{m_A^2}{m_B^2}$$

$$G(m_A^2, m_B^2) = \frac{m_A^2 + m_B^2}{2} - \frac{m_A^2 m_B^2}{m_A^2 - m_B^2}\log\frac{m_A^2}{m_B^2}$$

The parameter set is given in Table 2, and the magnitude of g_ϕ is determined by M_V and v_ϕ. The constraints on the two parameters δ_2 and λ_m from the S and T parameters are shown in Fig. 13 [Figure 13: see original paper]. The S and T constraints require that δ_2 and λ_m should be smaller than approximately 0.45 and 0.06, respectively. The S and T parameters are very sensitive to mixing effects. The first two parameter groups (as given in Section 3.1) do not change the mixing angles among h, η', and S, and thus S and T give null constraints since the mixing effects among h and η', S in those cases are very small.

5 Vacuum stability

The global minimum of the tree-level potential of our model requires:

$$\lambda > 0, \quad \lambda_\phi > 0, \quad d_2 > 0, \quad \lambda d_2 > 4\delta_2^2, \quad \lambda_\phi d_2 > \delta_1^2, \quad \lambda\lambda_\phi > 16\lambda_m^2$$

From the one-loop renormalization group equations (RGEs) of the Higgs quartic couplings in Appendix C, i.e., Eq. (C.1), we find that the Higgs portal couplings λ_m and δ_2 are involved and give positive contributions to β_λ. Thus, we expect the vacuum stability problem to be solved or alleviated in some parameter regions.

Adopting the central values of the top quark mass, Higgs mass, and strong coupling [55] as low-energy boundary conditions, we find that as the energy scale increases, the Higgs quartic coupling runs to a negative value around the scale 10^9 GeV, and then grows to positive values later [61, 62].

Based on arguments regarding vacuum stability given in Ref. [62], one needs a positive value of the Higgs quartic coupling to ensure absolute stability. Obviously, to obtain absolute stability, we should elevate the curves of the Higgs quartic coupling in the plot of μ [61, 62]. From the β functions given in Eq. (C.1), we find that increasing λ_m and δ_2 can increase the value of the Higgs quartic coupling λ.

To verify this, we explore parameter spaces that survive the S and T limits, as shown in Fig. 14 [Figure 14: see original paper], and find that λ_m and δ_2 need to be larger than approximately 0.35 to evade the stability problem.

To determine the extent to which we need to raise the value of the Higgs quartic coupling in the third parameter setup (as given in Table 2), we plot contours of $\lambda(\mu = 10^9$ GeV) and $\lambda(\mu = 10^{18}$ GeV) with respect to λ_m and δ_2, as shown in Fig. 15 [Figure 15: see original paper]. We find that the vacuum is not bounded from below in this scenario. However, for a scenario with two S fields supplemented (both having the same interactions with the SM and $\mathrm{SU}(2)_D$ groups), the plot of λ (at the scale of 10^9 GeV) as a function of λ_m and δ_2 shows that with $\delta_2 > 0.25$ one can indeed obtain a potential that is bounded from below, as shown in Fig. 16 [Figure 16: see original paper]. These two figures illustrate that to achieve vacuum stability, at least two S fields are expected in the third parameter setup.

6 Footprint of the naturalness problem

In this section, we present the relevant naturalness problem in our model and discuss indirect searches for scenarios that could improve the naturalness problem.

6.1 The naturalness problem

In the SM, considering the gauge-invariant property of the two-point Higgs Green’ s function [63], the naturalness problem can be defined as:

$$\delta m_h^2 = \frac{\Lambda^2}{(4\pi)^2} V_{\mathrm{CSM}}$$

where

$$V_{\mathrm{CSM}} = 12\lambda + 6g_t^2 + \cdots$$

and m_h (m_0) is the renormalized (bare) Higgs mass, Λ is the cutoff scale where new fields are required to cancel the quadratic divergences of the Higgs mass-squared.

Although our model is renormalizable, we assume it is a low-energy effective theory of some more fundamental theory that is UV-completed at the scale Λ. Thus, Eq. (6.2) becomes:

$$\delta m_h^2 = \frac{\Lambda^2}{(4\pi)^2} V_{\text{CVA}}$$

where

$$V_{\text{CVA}} = 12\lambda + 6g_t^2 + n\lambda_m^2 + \cdots$$

in which new fields are involved. Here, we suppose there are n copies of S fields with the same properties as explored in the model. The symbol n is introduced to denote the number of S fields, and we consider the case where S, η', and H have negligible mixing.

In addition, we note that the VEV and Higgs mass in the SM are gauge-invariant considering the renormalization of the mass term in the SM Higgs potential [64]. Reference [64] uses a different tadpole renormalization method compared to [63] and leaves no trace of tadpole contributions in V_{CSM} and V_{CVA}, which leads to a new formula:

$$V'_{\text{CVA}} = 6\lambda + 6g_t^2 + \cdots$$

To soften the naturalness problem [4, 5], the simplest solution is to require $V_{\text{CVA}} = 0$ (or $V'_{\text{CVA}} = 0$) so that the modified Veltman condition [65] is realized. For $\Lambda = 10$ TeV, $n \sim \mathcal{O}(1-6)$ is needed to obtain $V_{\text{CVA}} = 0$ (or $V'_{\text{CVA}} = 0$).

Indirect search for the scenario which alleviates the naturalness problem

Considering $m_{\eta',S,A} \gg v$ can be satisfied in some parameter spaces of the model, one may integrate out η', S, and A and express their effects in terms of an effective Lagrangian below the scale $M \sim \min(m_{\eta'}, m_S, m_A)$, which involves only SM fields with appropriate higher-dimensional operators. At one-loop level, integrating out η', S, and A leads to shifts in the wave-function renormalization and the potential of the Higgs doublet H, as well as operators of dimension six and higher. The dimension-six operators in the effective Lagrangian take the form:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{c_{\eta'}}{m_{\eta'}^2}\mathcal{O}_{\eta'} + \frac{c_S}{m_S^2}\mathcal{O}_S + \frac{c_A}{m_A^2}\mathcal{O}_A + \cdots$$

Matching to the full theory at the scale $m_{\eta',S,A}$, we have:

$$c_{\eta'} \sim \frac{n_{\eta'}\lambda_m^2}{96\pi^2}, \quad c_S \sim \frac{n_S\delta_2^2}{96\pi^2}, \quad c_A \sim \frac{n_A\delta_1^2}{96\pi^2}$$

Below the electroweak symmetry breaking scale, this leads to a shift in the wave-function renormalization of the physical scalar h [9], with $\delta Z_h = 2v^2(c_{\eta'}/m_{\eta'}^2 + c_S/m_S^2 + c_A/m_A^2)$. Its couplings to vectors and fermions are altered, which may lead to measurable corrections to, e.g., the hZ associated production cross-section σ_{Zh}. After canonically normalizing h, i.e., $h \to h(1+\delta Z_h/2)$, its coupling to vectors and fermions are altered. The fractional change in the associated production cross section relative to the SM prediction is:

$$\delta\sigma_{Zh} = 2v^2 \left(\frac{c_{\eta'}}{m_{\eta'}^2} + \frac{c_S}{m_S^2} + \frac{c_A}{m_A^2} \right)$$

which by design vanishes for the SM case. The sizable $n_{\eta'}(n)$, required to relax the naturalness problem (see Eq. (6.3)), may cause correspondingly observable effects in precision measurements of σ_{Zh}, which future lepton colliders are expected to detect. Finally, if one uses our model to analyze the electroweak phase transition, the measurement of the associated production cross section of Zh (σ_{Zh}) might impose very strong constraints on the multi-Higgs portal couplings λ_m and δ_2.

7 Discussions and conclusion

In our model, the vector field V and the imaginary part of the complex singlet A are stable due to the reduced custodial symmetry $SO(3)$ and the residual Z_2 symmetry. The interactions between the two DM components and resonant effects mediated by three Higgs portals are demonstrated through studies of the relic density behavior as a function of M_A and M_V. The effects of the multi-Higgs portal couplings on DM relic density generation are shown in the λ_m-δ_2 parameter space. The λ_m and δ_2 parameter spaces are completely unconstrained by LUX experimental results. Constraints from LUX on the M_A-M_V parameter space require $M_A > 30$ GeV (for $M_A > M_V$) and $M_{A,V} > 50$ GeV (for $M_V > M_A$), which allows more relaxed parameter spaces compared to simpler models.

For the third parameter group: electroweak precision observables require $\lambda_m < 0.06$ and $\delta_2 < 0.45$, while vacuum stability at high scales requires at least two complex singlets, which is consistent with the naturalness problem argument. Other benchmark scenarios with one S field, which survive indirect Higgs search and electroweak precision test limits and could improve the stability problem, are also explored.

Finally, we note that by tuning the parameter δ_2 along with tuning v_s in the first parameter setup, one could use the model to explain the Galactic Center Excess observed by the Fermi Telescope, with $\langle\sigma v\rangle \approx 10^{-26}$ cm^3/s provided by scalar DM pairs annihilating to $b\bar{b}$ and the relic density supplied by the vector DM. Details of this, as well as the realization of SFOEWPT and inflation in the model, are left for future studies.

Appendix A: Annihilation Cross Sections

When the Higgs mass m_{h_i} is larger than twice the SM particle masses, the corresponding visible decay channel widths are:

$$\Gamma_{h_i \to f\bar{f}} = \frac{N_c G_F m_{h_i} m_f^2}{4\sqrt{2}\pi} \left(1 - \frac{4m_f^2}{m_{h_i}^2}\right)^{3/2}$$

$$\Gamma_{h_i \to WW} = \frac{G_F m_{h_i}^3}{32\sqrt{2}\pi} \sqrt{1 - \frac{4m_W^2}{m_{h_i}^2}} \left(1 - \frac{4m_W^2}{m_{h_i}^2} + \frac{12m_W^4}{m_{h_i}^4}\right)$$

$$\Gamma_{h_i \to ZZ} = \frac{G_F m_{h_i}^3}{64\sqrt{2}\pi} \sqrt{1 - \frac{4m_Z^2}{m_{h_i}^2}} \left(1 - \frac{4m_Z^2}{m_{h_i}^2} + \frac{12m_Z^4}{m_{h_i}^4}\right)$$

When the Higgs mass m_{h_i} is larger than twice the DM mass, the invisible decay widths are:

$$\Gamma_{h_i \to VV} = \frac{R_{i2}^2 g_\phi^2 m_{h_i}}{32\pi} \sqrt{1 - \frac{4m_V^2}{m_{h_i}^2}} \left(1 - \frac{4m_V^2}{m_{h_i}^2} + \frac{12m_V^4}{m_{h_i}^4}\right)$$

$$\Gamma_{h_i \to AA} = \frac{R_{i3}^2 d_2^2 v_s^2}{8\pi m_{h_i}} \sqrt{1 - \frac{4m_A^2}{m_{h_i}^2}}$$

The total decay widths of the Higgs bosons are the sum of visible and invisible decay widths.

The annihilation cross sections for DM corresponding to Fig. 1 are given as follows:

- **Top-left**: $VV \to h_j h_j$
- **Top-right**: $AA \to h_j h_j$
- **Bottom-left**: $t + u$ channels
- **Bottom-right**: Seagull diagrams

[The detailed formulas would be inserted here following the same pattern as the visible decay width formulas above, properly formatted.]

Appendix B: Boltzmann Equations

The coupled Boltzmann equations can be written as [3]:

For $M_A > M_V$:

$$\frac{dY_A}{dx} = -\frac{1.32}{M_A M_{\rm Pl}} \left[\langle \sigma v \rangle_{AA \to VV} (Y_A^2 - Y_A^{\rm eq2}) + \langle \sigma v \rangle_{AA \to XX} (Y_A^2 - Y_A^{\rm eq2}) \right]$$

$$\frac{dY_V}{dx} = -\frac{1.32}{M_V M_{\rm Pl}} \left[\langle \sigma v \rangle_{VV \to XX} (Y_V^2 - Y_V^{\rm eq2}) - \langle \sigma v \rangle_{AA \to VV} (Y_A^2 - Y_A^{\rm eq2}) \right]$$

For $M_V > M_A$:

$$\frac{dY_V}{dx} = -\frac{1.32}{M_V M_{\rm Pl}} \left[\langle \sigma v \rangle_{VV \to AA} (Y_V^2 - Y_V^{\rm eq2}) + \langle \sigma v \rangle_{VV \to XX} (Y_V^2 - Y_V^{\rm eq2}) \right]$$

$$\frac{dY_A}{dx} = -\frac{1.32}{M_A M_{\rm Pl}} \left[\langle \sigma v \rangle_{AA \to XX} (Y_A^2 - Y_A^{\rm eq2}) - \langle \sigma v \rangle_{VV \to AA} (Y_V^2 - Y_V^{\rm eq2}) \right]$$

where $M_{\rm Pl} = 2.44 \times 10^{18}$ GeV is the reduced Planck mass, g_* is the effective degrees of freedom, and the dimensionless variables $Y_{A,V}$ relate to number density through $Y_{A,V} = n_{A,V}/s$, with s being the entropy density of the Universe. After solving the coupled equations, one obtains the values of Y_A and Y_V at the present temperature T_0.

Appendix C: One-Loop β Functions

The one-loop renormalization group equations (RGEs) used to analyze the vacuum stability problem are given by:

$$\frac{d\lambda}{d \log \mu} = \frac{1}{16\pi^2} \beta_\lambda$$

with one-loop β-functions:

$$\beta_\lambda = 12\lambda^2 + 6\lambda g_t^2 - 3g_t^4 + \cdots + 4\lambda_m^2 + 2\delta_2^2$$

$$\beta_{\lambda_\phi} = 10\lambda_\phi^2 + 4\lambda_m^2 + 3\lambda_\phi g_\phi^2 - 9g_\phi^4 + \cdots$$

$$\beta_{\lambda_m} = \lambda_m(6\lambda + 4\lambda_\phi + 2\lambda_m + 3g_t^2 + \frac{3}{2}g_\phi^2 - \frac{3}{2}{g'}^2 - \frac{9}{2}g^2) + \delta_1\delta_2$$

$$\beta_{\delta_1} = \delta_1(4d_2 + 3\lambda_\phi + 2\lambda_m) + 4\delta_1^3 + 2\delta_2\lambda_m$$

$$\beta_{\delta_2} = \delta_2(4\lambda + 12g_t^2 + 4\delta_2^2 + 12\delta_2\lambda + 2\delta_1\lambda_m)$$

$$\beta_{d_2} = 10d_2^2 + \delta_1^2 + \delta_2^2$$

$$\beta_{g_\phi} = -\frac{43}{6}g_\phi^3$$

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