Prospects for Triple Gauge Coupling Measurements at Future Lepton Colliders and the 14 TeV LHC (Postprint)

Authors: Ligong Bian, Jing Shu, Yongchao Zhang

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Full Text

Preamble

Prospects for Triple Gauge Coupling Measurements at Future Lepton Colliders and the 14 TeV LHC

Ligong Bian,[1], * Jing Shu,[1],[2], † and Yongchao Zhang[1], ‡

[1]State Key Laboratory of Theoretical Physics and Kavli Institute for Theoretical Physics China (KITPC), Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China

[2]CAS Center for Excellence in Particle Physics, Beijing 100049, China

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Abstract

W^+W^- production is the primary channel to directly probe triple gauge couplings. We first analyze the $e^+e^- \to W^+W^-$ process at the future lepton collider, China's proposed Circular Electron-Positron Collider (CEPC). Using the five kinematical angles in this process, we constrain the anomalous triple gauge couplings and relevant dimension-six operators at the CEPC up to the order of magnitude of 10^{-4}. The most sensitive information is obtained from the distributions of the production scattering angle and the decay azimuthal angles. We also estimate constraints at the 14 TeV LHC, with both 300 fb^{-1} and 3000 fb^{-1} integrated luminosity from the leading lepton p_T and azimuthal angle difference $\Delta\phi_{ll}$ distributions in the di-lepton channel. The constraint is somewhat weaker, up to the order of magnitude of 10^{-3}. The limits on the triple gauge couplings are complementary to those on the electroweak precision observables and Higgs couplings. Our results show that the gap between sensitivities of electroweak precision and triple gauge boson precision can be significantly decreased to less than one order of magnitude at the 14 TeV LHC, and that both sensitivities can be further improved at the CEPC.

*Electronic address: lgb@itp.ac.cn †Electronic address: jshu@itp.ac.cn ‡Electronic address: yczhang@pku.edu.cn

I. Introduction

The observation of the Standard Model (SM)-like Higgs boson at the Large Hadron Collider (LHC) represents a milestone in elementary particle physics. In the absence of any conclusive signal of new physics beyond the SM, it is crucial in the coming decades to pin down the electroweak (EW) symmetry breaking scenario, measure SM couplings precisely, and directly search for new physics at higher energy scales. All these objectives are closely related to the EW gauge sector of the SM, making the precise determination of gauge couplings an essential part of high-energy physics in the near future—for instance, to constrain new physics from heavy states [1].

Due to the non-Abelian nature of the weak interaction, triple and quartic couplings among EW gauge bosons exist in the SM. This work focuses on the charged triple gauge couplings (TGCs), specifically those of the form $WW\gamma$ and WWZ. TGCs beyond the SM can be parameterized within the framework of anomalous triple gauge couplings (aTGCs) [2] or in the language of effective field theory (EFT) [3–7]. From a phenomenological perspective, both scenarios are effective theories valid only up to some specific scale, beyond which the unitarity of scattering amplitudes breaks down or the perturbation expansion (e.g., to dimension-six order) ceases to be meaningful [8–10]. At lowest order, it is straightforward to connect Wilson coefficients to anomalous couplings.

At e^+e^- colliders, TGCs can be directly probed through WW pair production, single-W ($We\nu$), and single-photon ($\nu\nu\gamma$) processes. At hadron colliders, diboson final states WW, WZ, and $W\gamma$ can be used to study charged gauge couplings. In the EFT language, Higgs-gauge couplings and oblique corrections are related to TGCs; for example, universal gauge-fermion coupling deviations can be reshifted into S, T parameters and the triple gauge boson couplings [11]. Consequently, Higgs data and EW precision measurements can indirectly constrain gauge couplings [12]. Additionally, the $WW\gamma$ coupling can induce rare processes at loop level, such as $b \to s\gamma$, providing further constraints on aTGCs [13].

Direct measurements of charged TGCs have been performed at LEP, Tevatron, and LHC [14–27], with the current most stringent bounds coming primarily from W-pair measurements at LEP II [14], constraining aTGCs at the order of a few times 10^{-2} [28, 29]. Sensitivities are expected to improve by one to two orders of magnitude at the International Linear Collider (ILC) [30] due to higher luminosity, higher energy, and polarized beams. Recently, an alternative future e^+e^- collider has been proposed in China: the Circular Electron-Positron Collider (CEPC) [31]. Part of this work is devoted to estimating constraints on aTGCs at CEPC in the WW channel.

Taking into account W boson decays, the process $e^+e^- \to W^+W^-$ can be described by five kinematic angles: one ($\cos\theta$) for W production and four for the decay products [2, 32]. We use differential cross sections with respect to these five angles to set limits on TGCs at CEPC. Although only $\cos\theta$ depends directly on TGCs, we find that the four decay angles also contribute substantially to constraining gauge couplings, with the contribution depending largely on the W decay channels and the specific aTGCs involved. Overall, sensitivities at CEPC can reach the order of magnitude of 10^{-3} to 10^{-4}, comparable to or even better than those at the ILC [30]. The WW process at hadron colliders is somewhat similar to that at lepton colliders; at the parton level, the dominant channel is $q\bar{q} \to W^+W^-$. We also estimate constraints at the 14 TeV LHC with luminosities of 300 fb^{-1} or 3000 fb^{-1}, which can significantly improve current bounds.

The remainder of this paper is organized as follows: Section II establishes the framework for discussing $e^+e^- \to W^+W^-$, clarifying the aTGCs and dimension-6 operators involved and conventions for the five angles. Section III analytically examines the response of differential cross sections to aTGCs and presents results graphically, allowing qualitative assessment of which distributions are most sensitive to aTGCs and which couplings can be most severely constrained. Section IV estimates sensitivities for aTGCs and relevant dimension-6 operators at CEPC with $\sqrt{s} = 240$ GeV and an integrated luminosity of 5 ab^{-1}, explicitly showing separate contributions from the five production and decay angles. Section V discusses TGCs at hadron colliders and naively estimates bounds at the 14 TeV LHC. Current and future constraints on aTGCs from lepton and hadron colliders are summarized at the end of this section. Section VI briefly comments on the complementarity of direct TGC measurements and indirect constraints from EW precision and Higgs data before concluding in Section VII.

II. Preliminaries

A. Anomalous Triple Gauge Couplings Beyond the SM

With anomalous contributions beyond the SM [2], the charged TGCs among SM EW gauge bosons can be generally parameterized as:

$$\mathcal{L}_{WWV} = -ie\Big[g_{1,V}(W^+_{\mu\nu}W^{-\mu} - W^+_{\mu}W^{-\mu\nu})V^\nu + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2}W^+_{\mu\nu}W^{-\nu\rho}V^\mu_\rho$$
$$+ig_{4,V}W^+_\mu W^-_\nu(\partial^\mu V^\nu + \partial^\nu V^\mu) - ig_{5,V}\epsilon^{\mu\nu\rho\sigma}(W^+_\mu \overleftrightarrow{\partial}_\rho W^-_\nu)V_\sigma + \tilde{\kappa}_V W^+_\mu W^-_\nu \tilde{V}^{\mu\nu} + \frac{\tilde{\lambda}_V}{M_W^2}W^+_{\mu\nu}W^{-\nu\rho}\tilde{V}^\mu_\rho\Big]$$

where $V = \gamma, Z$, the gauge couplings $g_{WW\gamma} = e$ and $g_{WWZ} = e\cot\theta_W$ with $\cos\theta_W$ being the weak mixing angle, the field strength tensor $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ with $A = W, \gamma, Z$, and the conjugate tensor $\tilde{V}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}V^{\rho\sigma}$, and $A\overleftrightarrow{\partial}_\mu B \equiv A(\partial_\mu B) - (\partial_\mu A)B$. Besides the SM TGCs, the Lagrangian Eq. (1) contains 14 anomalous TGCs up to dimension six in the most general form. The couplings $g_{1,V}$, κ_V, and λ_V are both parity (P) and charge conjugation (C) conserving, while the remaining eight are C or P violating. In the SM, $g_{1,V} = \kappa_V = 1$ whereas all others vanish. Electromagnetic gauge symmetry requires that $g_{1,\gamma} = 1$ and $g_{4,\gamma} = g_{5,\gamma} = 0$. Consequently, in the absence of C or P violation from beyond-SM physics, there are only five aTGCs: $\Delta g_{1,Z}$, $\Delta\kappa_\gamma$, $\Delta\kappa_Z$, λ_γ, and λ_Z, where we have separated SM and new contributions, $\Delta g_{1,Z} \equiv g_{1,Z} - 1$ and $\Delta\kappa_{\gamma,Z} \equiv \kappa_{\gamma,Z} - 1$.

If beyond-SM physics is described in the language of EFT, only a few dimension-6 operators are relevant to the charged (C and P conserving) TGCs. In the SILH basis [33, 34]:

$$\mathcal{L}_{TGC} = \frac{c_W}{M_W^2}\mathcal{O}_W + \frac{c_B}{M_W^2}\mathcal{O}_B + \frac{c_{HW}}{M_W^2}\mathcal{O}_{HW} + \frac{c_{HB}}{M_W^2}\mathcal{O}_{HB} + \frac{c_{3W}}{M_W^2}\mathcal{O}_{3W}$$

In this convention, the term c_{WB} can be expressed as a linear combination of other terms through integration by parts. The c_W operator contributes to the oblique parameter S [35] and is tightly constrained by EW precision measurements to the order of 10^{-5} [14, 30, 36, 37]. The WW bound on c_W at ILC and CEPC is only of order 10^{-4}. Thus, as a first-order approximation, we can neglect the c_W term, leaving three operators at the dimension-6 level that are related to aTGCs via [5–7]:

$$\Delta g_{1,Z} = \frac{M_Z^2}{M_W^2}\frac{c_{HW}}{2},$$
$$\Delta\kappa_\gamma = \frac{c_{HW}+c_{HB}}{2},$$
$$\lambda_\gamma = \frac{3g^2M_W^2}{M^2}c_{3W}$$

Under these circumstances, the aTGCs are related by EW $SU(2)_L \times U(1)_Y$ gauge symmetry, and there are only three independent couplings in the C and P conserving sector, as explicitly shown above. Given any constraints on aTGCs from present and future high-energy colliders, we can always translate them into limits on the relevant dimension-6 operators and apply them to any particular models connected with charged triple gauge couplings.

B. W-Pair Production at e^+e^- Colliders

At tree level, the $e^+e^- \to W^+W^-$ process is mediated by s-channel γ/Z and t-channel neutrino exchange. In the most general case, oblique corrections, non-standard gauge-fermion couplings, and aTGCs all contribute to the WW cross section. Due to severe constraints from EW precision measurements, it is a good approximation to neglect corrections from oblique terms and beyond-SM gauge-fermion interactions and focus only on aTGC effects [38]. At $\sqrt{s} = 240$ GeV, the designed energy for CEPC, the production cross section is dominated by the neutrino-mediated transverse WW configuration ($\lambda = \pm 1$), which forms a peak in the forward region $\cos\theta \approx 1$. Since this helicity state does not depend on any TGCs, it constitutes the dominant irreducible background for measuring aTGCs, especially at higher colliding energies. Thus, precise determination of TGCs requires large statistics at lepton colliders.

When W boson decay information is considered, the kinematics of $f_1\bar{f}_2\bar{f}_3f_4$ is dictated by five angles in the narrow W-width approximation [2, 32, 40, 41]: the scattering angle θ between e^- and W^-, the polar angles $\theta^*_{1,2}$ of the down-type (anti-)fermion in the rest frame of $W^\mp$, and the azimuthal angles $\phi^*_{1,2}$ of the down-type (anti-)fermion in the rest frame of $W^\mp$. To unambiguously define these decay angles, we establish right-handed coordinate systems in the W rest frames such that the z-axis is along the $W^\mp$ flight direction and the y-axis is in the direction of $\vec{e^-} \times \vec{W}^\mp$, where $\vec{e^-}$ is the electron beam direction. The five-fold differential cross section reads [32, 40, 42]:

$$\frac{d\sigma}{d\cos\theta d\cos\theta_1^* d\phi_1^* d\cos\theta_2^* d\phi_2^*} = \frac{3\beta}{2^7\pi^3 s}\text{BR}\sum_{\lambda,\tau,\tau'} F^{(\lambda)}_{\tau\tau'}F^{(\lambda)*}_{\bar{\tau}\bar{\tau}'}D_{\tau\bar{\tau}}D_{\tau'\bar{\tau}'}$$

where $\text{BR} = \text{Br}(W^- \to \bar{f}_3f_4)\text{Br}(W^+ \to f_1\bar{f}_2)$, β is the W boson velocity, F and D are helicity amplitudes for WW production and the W decay matrix, and

λ and $\tau^{(\prime)}$ are the helicities of e^- and $W^\mp$. Integrating out some angles yields more inclusive differential cross sections:

$$\frac{d\sigma}{d\cos\theta}, \quad \frac{d\sigma}{d\cos\theta^*_{1,2}}, \quad \frac{d\sigma}{d\phi^*_{1,2}}$$

which can be extracted from experimental data, at least in principle. The polarization of W bosons can also be described by the spin density matrix (SDM), with the two-particle joint SDM defined as [32]:

$$\rho_{\tau\tau'\bar{\tau}\bar{\tau}'} = \frac{\sum_\lambda F^{(\lambda)}_{\tau\tau'} F^{(\lambda)*}_{\bar{\tau}\bar{\tau}'}}{\sum_{\lambda,\tau,\tau'} |F^{(\lambda)}_{\tau\tau'}|^2}$$

which is normalized to unity. The SDM has 80 independent elements and contains the full helicity information of the WW pairs [42].

III. Differential Distributions in Presence of the Anomalous Couplings

In the presence of aTGCs, both total and differential cross sections (five-fold or more inclusive) are affected. One may use SDM elements or their appropriate linear combinations obtainable from experiments to constrain anomalous couplings [2, 32, 43]. We alternatively use differential cross sections with respect to the five kinematic angles, which are more physically intuitive. It is sometimes convenient to replace the polar ($\cos\theta^*_{1,2}$) and azimuthal angles ($\phi^*_{1,2}$) with corresponding angles for quark jets (j) and charged leptons (ℓ): $\cos\theta^*_{j,\ell}$ and $\phi^*_{j,\ell}$. The differential cross sections then read:

$$\frac{d\sigma}{d\cos\theta}, \quad \frac{d\sigma}{d\cos\theta^*_{j,\ell}}, \quad \frac{d\sigma}{d\phi^*_{j,\ell}}$$

It is worth emphasizing that only the five-fold differential cross section Eq. (5) (and the SDM Eq. (7)) contains complete information about WW production and decay; considering only the five inclusive distributions above would lose some sensitivity, such as that from spin correlations between the WW pairs. However, the lost sensitivities are expected to be small [43]. As we show below, distributions of the decay angles $\cos\theta^*_{\ell,j}$ and $\phi^*_{\ell,j}$ contribute significantly to the sensitivities in addition to $\cos\theta$.

The total cross section depends on aTGCs α_i in quadratic form:

$$\sigma = \sigma_0 \left(1 + \sum_i b_i \alpha_i + \sum_{i,j} b_{ij} \alpha_i \alpha_j\right)$$

where b_i and b_{ij} are nominal linear and quadratic coefficients and σ_0 is the SM cross section. When couplings α_i are sufficiently large, the quadratic term eventually dominates and the total cross section exceeds the SM prediction (requiring that coefficients b_{ii} be positive definite, at least theoretically). However, when couplings are very small—in the range of 10^{-4} to 10^{-3} relevant for CEPC prospects—we find that linear terms always dominate: $b_i\alpha_i \gg b_{ij}\alpha_i\alpha_j$, as expected. With such small aTGCs, higher-order contributions from new physics beyond the SM are negligible. At the center-of-mass energy $\sqrt{s} = 240$ GeV for CEPC, the SM cross section is $\sigma_0 = 17.2$ pb. Using the convention in Eq. (1), all b_i for the three C and P conserving couplings in Eq. (4) are negative (at lower energies, some b_i may be positive):

$$b_{\Delta g_{1,Z}} = -0.51, \quad b_{\Delta\kappa_\gamma} = -0.17, \quad b_{\lambda_\gamma} = -0.48$$

This means that at leading order, the total cross section decreases in the presence of positive aTGCs. These coefficients are of the same order, implying that constraints from the total cross section on these aTGCs are similar. We can write Eq. (4) in matrix form $\alpha_i = V_{ij}c_j$, with rotation matrix:

$$V = \begin{pmatrix} 0.64 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 0.17 \end{pmatrix}$$

It is straightforward to translate the linear coefficients b_i to those for the dimension-6 operators, $b'_i = V_{ji}b_j$:

$$b'_{c_{HW}} = 0.41, \quad b'_{c_{HB}} = 0.08, \quad b'_{c_{3W}} = 0.08$$

In the EFT basis, all b_i at $\sqrt{s} = 240$ GeV are positive. These arguments are based only on total cross section measurements of WW production; when distributions of the five kinematic angles are considered, more information about WW production and decay is used, and sensitivities are expected to improve substantially.

To examine the response of angular distributions to aTGCs, we expand the differential cross sections Eq. (6) in terms of aTGCs, analogous to the total cross section case:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} + \sum_i \omega_i(\Omega)\alpha_i + \sum_{i,j} \omega_{ij}(\Omega)\alpha_i\alpha_j$$

where $\Omega = \cos\theta, \cos\theta^*_{1,2}, \phi^*_{1,2}$ (or alternatively $\Omega = \cos\theta, \cos\theta^*_{\ell,j}, \phi^*_{\ell,j}$). For sufficiently small aTGCs, we can safely neglect quadratic terms. The linear coefficient functions ω_i can be obtained analytically from the differential cross sections and are collectively given in Appendix B. As with the b_i coefficients, distribution functions for the three relevant dimension-6 operators can be expressed as $\omega_i' = V_{ji}\omega_j$. The numerical functions $\omega_i(\Omega_k)$ for the aTGCs in Eqs. (2) and (4) are collected in Appendix C. Integrating over angles recovers the linear coefficients b_i, so these functions measure the "angular distributions" of the coefficients b_i. In other words, the ω_i functions represent the angular distributions of aTGC effects. Combining the distributions $\omega_i(\Omega_k)$ with SM differential cross sections $d\sigma_0/d\Omega_k$, one can easily identify, for a specific α_i, which angle is most sensitive and which part of the corresponding distribution deviates most from the SM. Likewise, one can judge qualitatively for a specific angle Ω_k which coupling α_i induces the largest deviation and is thus most stringently constrained.

The coefficient functions $\omega_i(\Omega_i)$ for the three aTGCs and dimension-6 operators in Eq. (4) are depicted in Fig. 1 for $\sqrt{s} = 240$ GeV. Due to hermiticity of the weak interaction, distributions for the two polar angles $\omega_i(\cos\theta^*_{1,2})$ are identical, and $\omega_i(\phi^*_2)$ differs from $\omega_i(\phi^*_1)$ by a phase shift of π.

Among the five angles, only the scattering angle $\cos\theta$ depends directly on TGCs, and sensitivities from decay angles arise mainly from their correlation with $\cos\theta$. Consequently, $\cos\theta$ is generally most sensitive to anomalous couplings. This qualitative feature can also be seen by comparing magnitudes of ω_i for all five angles in Fig. 1: larger deviations generally mean larger sensitivity.

The panel for $\cos\theta$ shows that for all three C and P conserving aTGCs in Eq. (4), the largest deviation occurs in the backward region $\cos\theta \lesssim -0.5$. However, sensitivity in this region is limited by small cross sections, or equivalently by the number of events, especially at high energies. Therefore, huge statistics, such as at CEPC, can substantially enhance sensitivities in the backward region. In contrast, deviation from the SM in the forward direction is much smaller, but due to much larger cross sections, the enormous statistics in the forward region also contribute significantly to sensitivities. WW events far from both backward and forward regions are less affected by aTGCs and contribute less to sensitivities.

The polar angle distributions contribute least to sensitivities overall, as the magnitudes of the three curves in the middle panel of Fig. 1 are smallest. Among these curves, the magnitude of $\omega(\cos\theta^*_{1,2})$ for $\Delta\kappa_\gamma$ is significantly larger than for other couplings, implying that polar angles are most sensitive to $\Delta\kappa_\gamma$ and contribute substantially to constraining it. Taking into account SM distributions $d\sigma_0/d\cos\theta^*_{1,2}$, the plot further implies that $\cos\theta^*_{1,2} \sim 0$ is the most sensitive region in the W rest frame.

For the azimuthal angles $\phi^*_{1,2}$, the right panels show that for $\Delta g_{1,Z}$ and λ_γ, significant deviations occur at $\phi^*_{1,2} \sim \pi$ in the W boson rest frame. Thus, distributions of azimuthal angles suffer significant distortions due to these non-vanishing aTGCs, which in turn severely constrain the couplings and substantially improve sensitivities. For $\Delta\kappa_\gamma$, the function $\omega(\phi^*_{1,2})$ is almost constant, meaning that given a non-vanishing $\Delta\kappa_\gamma$, the distributions $d\sigma/d\phi^*_{1,2}$ are simply rescaled by a factor of $(1 + 0.17\Delta\kappa_\gamma)$.

IV. CEPC Constraints

A. Constraints on the TGCs and Dim-6 Operators

With a huge luminosity of 5 ab^{-1} at CEPC at $\sqrt{s} = 240$ GeV, we can collect a total of 8.6×10^7 WW events, with 45%, 44%, and 11% decaying in hadronic, semileptonic, and leptonic channels respectively. With such enormous statistics, these anomalous TGCs can be severely constrained. In this section, we use differential cross sections with respect to the five angles for WW production and decay (Eq. (8)) to extract generator-level constraints on C and P conserving aTGCs and relevant dimension-six operators. Fortunately, radiative corrections, reducible backgrounds from non-WW processes, and systematic errors are small and well understood, especially for non-hadronic decays [43, 49]. We first estimate statistical errors for these aTGCs in all three distinctive channels (and combined) and then briefly comment on corrections and systematic errors.

In these channels, not all five angles can be fully reconstructed unambiguously, inevitably leading to some loss of sensitivity for TGCs. Key ambiguities include:

- **Semileptonic decays**: Due to large branching ratios and high reconstruction efficiency, this is the optimal channel. The W boson charge is assigned from the lepton charge, and the only ambiguity arises from hadronic decays where quark jets cannot be distinguished from antiquark jets. We choose jets from W decay in the region $0 < \phi^*_j < \pi$, using only the symmetric part of the D decay function under the transformation $(\theta^*, \phi^*) \to (\pi - \theta^*, \pi + \phi^*)$.
- **Hadronic channel**: Events appear as four jets. Assuming correct jet pairing and W charge assignment, only the ambiguity in assigning jets to quarks versus antiquarks remains, as in the semileptonic case.
- **Leptonic channel**: We consider only e and μ channels, as τ leptons cannot be fully reconstructed due to extra neutrinos from τ decay. In the limit of vanishing W width, there is a two-fold ambiguity in solving for neutrino momenta to reconstruct W bosons [2]. Assuming the physical solution can be distinguished from the unphysical one, WW events can be fully reconstructed in purely leptonic final states.

We simply split distributions of the five angles into bins, count events in each bin, and use the shape of these distributions to set limits on aTGCs. For large event numbers (the case at CEPC), statistical errors can be estimated as $\sqrt{N_i}$, where N_i is the number of events in the i-th bin. The standard χ^2 is defined as:

$$\chi^2 = \sum_i \frac{(N_i^{BSM} - N_i^{SM})^2}{N_i^{SM}}$$

where N_i^{BSM} and N_i^{SM} are event numbers in the i-th bin for specific distributions with beyond-SM interactions and in the SM, respectively. Estimates of one-parameter limits on aTGCs and relevant dimension-6 operators in Eq. (4) are presented in Table I, with all other anomalous couplings or dimension-6 coefficients fixed to zero. In this table and subsequent calculations, all distributions are split evenly into ten bins, and ambiguity for hadronic decays is accounted for. The constraints are so strong that quadratic terms of aTGCs in differential cross sections hardly affect sensitivities.

Key observations from the constraints include:

- In the leptonic channel, limits on aTGCs and dimension-6 operators are of order few $\times 10^{-4}$ to 10^{-3}. Due to larger branching ratios, semileptonic and hadronic channels improve constraints by factors of two or three. Combining all three channels (the "all" row in Table I) yields even stronger constraints approaching 10^{-4}.
- Dictated by relations connecting aTGCs and dimension-6 operators, constraints on c_{HW} combine those on $\Delta g_{1,Z}$ and $\Delta\kappa_\gamma$, making it more severely constrained than c_{HB} and c_{3W}. For $c_{HB} \sim \Delta\kappa_\gamma$ and $c_{3W} \sim \lambda_\gamma$, limits should be identical at linear level. Tiny differences in Table I arise from quadratic corrections.
- Instead of fixing $\sqrt{s} = 240$ GeV, we considered an alternative energy scan mode for CEPC, similar to LEP II [14], keeping mean energy at 240 GeV and total luminosity at 5 ab^{-1} (e.g., $\sqrt{s} = 220$–260 GeV in 5 GeV steps). Although constraints strengthen at higher energies, the energy scan mode yields similar sensitivities to running solely at 240 GeV and does not improve limits.

The correlation matrices between the three aTGCs and dimension-6 operators are:

$$\rho_{aTGC} = \begin{pmatrix} 1 & 0.11 & -0.61 \\ 0.11 & 1 & -0.31 \\ -0.61 & -0.31 & 1 \end{pmatrix}, \quad \rho_{dim-6} = \begin{pmatrix} 1 & -0.11 & -0.57 \\ -0.11 & 1 & -0.28 \\ -0.57 & -0.28 & 1 \end{pmatrix}$$

when all three decay channels are combined. We also calculate two-parameter constraints on anomalous couplings and dimension-6 operators, allowing two of three couplings to vary while fixing the third to zero. The $1\sigma, 2\sigma$, and 3σ regions for couplings $\Delta g_{1,Z}, \Delta\kappa_\gamma, \lambda_\gamma$ and coefficients c_{HW}, c_{HB}, c_{3W} are presented in Fig. 2, using all available decay channels. In obtaining contours, we set standard $\Delta\chi^2$ values for two independent variables: $\Delta\chi^2 = 2.30, 6.18$, and 11.83 for $1\sigma, 2\sigma$, and 3σ errors, respectively.

From Eq. (15) and Fig. 2, some anomalous couplings are strongly correlated, such as $\Delta g_{1,Z}$ and λ_γ, with the direction $\Delta g_{1,Z} + \lambda_\gamma$ much less severely constrained than other combinations. However, we stress that correlations depend on both theoretical predictions and experimental data. With future CEPC data, correlations might change dramatically. Potential blind directions could be removed by incorporating helicity information of $e^{\pm}$ and $W^{\pm}$ [38, 50].

Before proceeding, we briefly comment on experimental effects and systematic errors. Dominant reducible backgrounds are non-WW four-fermion processes and $q\bar{q}$ two-fermion processes [44–47]. As an explicit example, we examine background effects in the semileptonic channel using Whizard [51] with simple cuts [43]: for charged leptons $\ell = e, \mu, \tau$, $p_T > 10$ GeV and separation $> 5^{\circ}$ to the nearest quark jet; missing transverse energy $\not{E}_T > 10$ GeV; visible mass > 100 GeV; invariant masses of decay products from W bosons < 105 GeV; and W scattering angle $\cos\theta > -0.95$. Assuming τ leptons can be reconstructed with 80% efficiency, the total efficiency after cuts is about 91%, increasing statistical errors in Table I by approximately 11–13%. For purely hadronic decays, backgrounds are significantly larger, while the leptonic channel is much cleaner. When these channels are considered, efficiencies for jet pairing, W charge assignment, and neutrino momentum reconstruction must be accounted for.

The W mass precision is expected to be 3 MeV at CEPC, and beam energy uncertainty can reach 10 ppm (~ 1 MeV). Radiative correction and detector simulation effects are also well controlled, corresponding to corrections of roughly the same order [31, 52]. We roughly estimate effects from these uncertainties following the method in [43]. For instance, we calculate sensitivities of anomalous couplings with an energy variation of ± 2.4 MeV and compare them to the standard scenario. The largest variation in sensitivities is taken as the corresponding systematic error from beam energy uncertainty. We find that systematic corrections to sensitivities are much smaller than statistical errors (expected to be of order 10^{-5}) and can be safely neglected. In other words, statistical errors are expected to dominate TGC measurements at CEPC. A detailed survey of systematic errors requires a more dedicated and comprehensive study.

B. Contributions from Different Distributions

To examine contributions to sensitivities from distributions with respect to the five kinematic angles $\cos\theta, \cos\theta_\ell^*, \phi_\ell^*, \cos\theta_j^*, \phi_j^*$, we calculate $\Delta\chi^2(\Omega_k)/\sum_k \Delta\chi^2(\Omega_k)$ for the three C and P conserving aTGCs in Eq. (4), collected in Table II. Each entry represents the proportion contributed to total $\Delta\chi^2$ from one specific distribution, with each row summing to unity. We omit potential correlations among differential distributions, which are expected to be small.

Qualitative features from Table II include:

- In all three decay channels, the $\cos\theta$ distribution always dominates sensitivities, consistent with implications from Fig. 1 magnitudes and previous arguments.
- For semileptonic decays, distributions of polar and azimuthal angles of

charged leptons are generally more important than information from hadronic products, as only symmetric jet information can be used.

- For (semi-)leptonic decays, TGCs are generally more sensitive to azimuthal angles than polar angles, consistent with Fig. 1 magnitudes. An exception is $\Delta\kappa_\gamma$, for which polar angles are also very important (see middle panel of Fig. 1).
- For the hadronic channel, contributions from decay information are small compared to the scattering angle $\cos\theta$, as expected.
- When all channels are combined, contributions from different distributions are similar to the semileptonic channel.

In short, qualitative features of numerical estimations for the five kinematic angles coincide with theoretical arguments. Distributions of decay products provide complementary information to the angle $\cos\theta$ and contribute sizably to constraining aTGCs and relevant dimension-6 operators.

V. Constraints at Hadron Colliders and Future Sensitivities

TGCs can also be probed directly at hadron colliders. The Tevatron and ATLAS and CMS collaborations have measured charged anomalous couplings in WW, WZ, and $W\gamma$ processes up to $\sqrt{s} = 8$ TeV [15–27]. There are also speculations about non-standard gauge couplings at the 14 TeV LHC [53, 54]. These measurements complement EW precision tests, accurate Higgs coupling probes, and can be combined to constrain beyond-SM physics [12, 55].

As a direct comparison, we consider WW production at the forthcoming 14 TeV LHC. At parton level, the dominant channel is $q\bar{q} \to W^+W^-$, similar to $e^+e^- \to W^+W^-$ at lepton colliders, though with much larger radiative corrections [56]. To suppress huge QCD backgrounds, we focus on purely leptonic decay channels $W \to e\nu, \mu\nu$. Due to large missing energy from neutrinos and unknown initial quark momenta, W events cannot be fully reconstructed. However, observables in the transverse plane can still be used to study TGCs, such as the widely used leading p_T of charged lepton products [15, 19, 20, 26]. Analogous to lepton colliders (see Tables II and VII), azimuthal angles at hadron colliders—specifically the difference between azimuthal angles of charged leptons ($\Delta\phi_{\ell\ell}$) projected onto the transverse plane in the lab frame—are also very sensitive to beyond-SM TGCs and help constrain couplings.

To demonstrate these arguments and estimate prospects for TGC constraints at LHC Run II, we generate parton-level events using MadGraph5 [57] at 14 TeV LHC and pass them through Pythia [58] and Delphes [59] for scenarios with and without beyond-SM TGCs. Following [20, 55], we implement simple cuts: for charged leptons $\ell = e, \mu$, leading $p_T > 25$ GeV and subleading $p_T > 20$ GeV; $|\eta| < 2.5$; $\Delta R_{\ell\ell} > 0.4$; $m_{\ell\ell} > 15(10)$ GeV; $\not{E}_T > 45(15)$ GeV for same (different) flavor channels, with additional cut $|m_{\ell\ell} - m_Z| > 15$ GeV for same-flavor channels.

For illustration, we choose benchmark points beyond the SM: $\Delta g_{1,Z} = 0.1, \Delta\kappa_\gamma = 0.2, \lambda_\gamma = 0.1$ (assuming EW relations among aTGCs) and $c_{HW} = 0.1, c_{HB} = 0.2, c_{3W} = 0.1$. Leading p_T distributions for all seven scenarios (SM plus six beyond-SM benchmarks) are presented in Fig. 3, with overflow bins shown. Clearly, anomalous couplings tend to generate large p_T events. In the presence of sizable non-standard couplings, tails in these beyond-SM scenarios are always much longer and fatter than in the SM, with the last bins being most sensitive to non-standard TGCs.

Fig. 4 shows distributions of azimuthal angle difference $\Delta\phi_{\ell\ell}$ for the seven scenarios. With beyond-SM TGCs, lepton product momenta tend to be larger, leading to more back-to-back events. This qualitatively explains why the right few bins with $\Delta\phi_{\ell\ell} \sim \pi$ are largely enhanced, especially for same-flavor decays. Given this large excess of back-to-back events, combining $\Delta\phi_{\ell\ell}$ distributions with leading p_T could moderately improve constraints on anomalous couplings, though at hadron colliders $\Delta\phi_{\ell\ell}$ is strongly correlated with p_T. This is similar to lepton colliders where azimuthal angles of charged leptons are also very sensitive to TGCs and contribute sizably to constraints.

From simulated events, we roughly estimate constraints on aTGCs and dimension-6 operators from combined analysis of leading lepton p_T and azimuthal angle $\Delta\phi_{\ell\ell}$ distributions. To account for large radiative corrections, we use the next-to-leading-order total cross section of 124 pb for $pp \to W^+W^-$ at $\sqrt{s} = 14$ TeV to calculate event numbers [56]. To optimize constraints, in addition to basic cuts, we set leading $p_T > 300$ GeV and > 500 GeV respectively for luminosities of 300 fb^{-1} and 3000 fb^{-1}, and further apply cuts $\Delta\phi_{\ell\ell} > 170^\circ$, which moderately improves constraints. To keep events roughly below the cutoff scale $\tilde{\Lambda} \sim M_W/\sqrt{c_i}$ where higher-dimensional operator contributions are suppressed [38, 60], it is important to set an upper limit on leading p_T. In our cases, events beyond these upper limits are extremely rare unless the theory is very weakly coupled ($\tilde{g} < 0.38$). Nevertheless, one can lower the leading p_T cut to make EFT valid for models with even weaker coupling $\tilde{g}$, though bounds on dimension-six operators would be moderately lower (e.g., bounds on O_{HW} using $p_T > 160$ GeV would be 1.2 times smaller than those using $p_T > 300$ GeV).

We conservatively use only same-flavor decay products ee and $\mu\mu$. Constraints on anomalous couplings and dimension-6 operators are collected in Table III. For 300 fb^{-1}, limits are of order 10^{-3}. With ten times larger luminosity, constraints strengthen by factors of two or three. Note that our analysis uses simple cuts on same-flavor events; more elaborate studies could yield significant improvements even in this di-lepton channel.

To close this section, we collect in Table IV and Fig. 5 the current 95% confidence level constraints on aTGCs $\Delta g_{1,Z}, \Delta\kappa_\gamma, \lambda_\gamma$ from LEP, Tevatron, and

LHC, along with projections for 14 TeV LHC and future lepton colliders CEPC and ILC. Current lepton and hadron collider bounds are from [23], where we select the most stringent limits from each experimental group rather than collecting all limits. In Fig. 5, LHC 14 TeV data assume 3000 fb^{-1} luminosity, CEPC limits use only the semileptonic channel, and ILC limits are at $\sqrt{s} = 500$ GeV with 500 pb^{-1} from [30]. Using more decay channels, higher energies, and larger luminosities at future lepton colliders can further improve constraints in this figure. Comparing limits naively, 14 TeV LHC and future lepton colliders can improve aTGC limits by one to two orders of magnitude. Benefiting from huge integrated luminosity, CEPC can achieve constraints comparable to or even better than ILC. With larger data samples—designed up to 10 ab^{-1} at $\sqrt{s} = 240$ GeV—TLEP (recently renamed FCC-ee) could further improve constraints on charged triple gauge couplings [61]. To obtain rough constraints on relevant dimension-6 operators, one can follow arguments in Section III and simply use matrix V_{ij} to convert aTGC limits to limits on dimension-6 coefficients.

VI. Complementarity on the Measurements of the TGCs, EW Precision Observables and the Higgs Couplings

As stated in the introduction, besides direct measurements in diboson channels at lepton and hadron colliders, TGCs can be indirectly probed through EW precision and Higgs data. The oblique parameters, TGCs, and Higgs couplings can be related by redefining gauge fields or through integration by parts and equations of motion [11]:

$$\Delta g_{1,Z} = \frac{g^2 + g'^2}{g^2 - g'^2}\left[-\frac{g^2}{g^2 + g'^2}\hat{S} + \hat{T} - \frac{1}{2}\delta g_{hZZ}\right]$$
$$\Delta \kappa_\gamma = -\frac{g^2}{g^2 + g'^2}\hat{S} + \hat{T} - \delta g_{hZZ}$$

where O_{HW}, O_{HB} are defined in Eq. (3) and:

$$\hat{S} = \frac{g^2}{M_W^2}(c_W + c_B), \quad \hat{T} = \frac{g^2}{M_W^2}c_T$$

so that their coefficients correspond to the S, T parameters. For Higgs measurement sensitivity, we consider hZZ coupling constraints since its contribution has the same form as the SM one (no derivative couplings). From relations between different sensitivities, we obtain:

$$\delta(\Delta g_{1,Z}) \sim \delta\hat{S} \sim \delta\hat{T} \sim \delta g_{hZZ}$$

From Ref. [37], one can obtain EW and Higgs precision $\Delta g_{hZZ}/g_{hZZ}$ for high-luminosity LHC (HL-LHC), CEPC, ILC, and TLEP. Therefore, we can compare direct TGC constraints with EW and Higgs precision $\Delta g_{hZZ}/g_{hZZ}$ by reshifting them into TGCs via Eq. (26). Results are listed in Table V. From this rough examination, it is interesting that TGC sensitivities are comparable to EW precision for certain new physics operators like O_{HW}, which strikingly raises the importance of improving TGC measurements in the future. More detailed studies of this complementarity and global new physics fits at LHC and future colliders will be performed separately. Note that FCC-ee also has a run option at $\sqrt{s} = 250$ GeV, similar to CEPC, with potentially larger integrated luminosity, so sensitivity on TGCs would be improved accordingly.

VII. Conclusion

In the era of precision measurements of SM couplings among scalars, fermions, and gauge bosons, triple couplings among SM EW gauge bosons are essential for testing the SM gauge sector and setting constraints on precision electroweak and Higgs physics, providing powerful guidance for searching new physics beyond the SM. The WW process is the primary channel at lepton colliders to directly measure charged triple couplings. Kinematically, this process can be described by five angles, including those for W boson decay products.

In this work, we use these five angles to study lepton collider constraints on anomalous triple gauge couplings and the three relevant dimension-6 operators c_{HW}, c_{HB}, and c_{3W} in the C and P conserving sector. We obtain numerically and graphically the effects of anomalous triple couplings on differential cross sections with respect to the five kinematic angles. From Fig. 1, one can qualitatively identify which distributions are most sensitive to anomalous couplings and which couplings are expected to be most severely constrained.

We systematically calculate statistical errors for anomalous couplings and dimension-6 operators at CEPC with $\sqrt{s} = 240$ GeV and luminosity of 5 ab^{-1}, using shapes of differential cross sections with respect to the five angles in all leptonic, semileptonic, and hadronic decay channels of WW pairs. Sensitivities, collected in Tables I and VI, reach order 10^{-4}, comparable to or better than ILC for some couplings. We find that information from decay products complements the W scattering angle $\cos\theta$ and contributes sizably to sensitivities. The importance of decay information and its contribution depend largely on decay channels and the specific anomalous couplings involved, as detailed in Tables II and VII.

We have also investigated constraints at hadron colliders, estimating sensitivities in di-lepton channels for WW production at 14 TeV LHC with 300 fb^{-1} and 3000 fb^{-1}. Depending on luminosity and anomalous couplings, constraints are at the level of 10^{-2} to 10^{-3}, collected in Tables III and VIII. Constraints mainly arise from excess events at high leading lepton p_T in the presence of non-standard couplings. The azimuthal angle difference $\Delta\phi_{\ell\ell}$ of charged leptons also contributes moderately. Finally, we collect current and future constraints on anomalous triple gauge couplings from future lepton colliders and 14 TeV LHC, comparing their sensitivities with precision EW and Higgs couplings in terms of dimension-6 operators.

It is promising that constraints on charged triple gauge couplings can improve by two orders of magnitude to reach 10^{-4}. The sensitivity gap between electroweak precision and triple gauge boson precision can be significantly reduced to less than one order of magnitude at 14 TeV HL-LHC, with both sensitivities improving at future lepton colliders such as CEPC. This allows us to reconsider triple gauge boson constraints on EW precision physics in the future. Finally, future FCC-ee data at 240 GeV with potentially larger integrated luminosity than CEPC, as well as TLEP-W and TLEP-Z data, could further improve constraints on charged triple gauge couplings and other new interactions beyond the Standard Model.

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Note added: While finalizing this paper, Ref. [62] appeared which also discussed W^+W^- and has some overlap with our work. However, we also consider constraints from 14 TeV LHC and compare aTGC constraints with those from EW precision observables and Higgs couplings.

Appendix A: Constraints on the Five Most General C and P Conserving aTGCs

For completeness, we collect main results for the five most general C and P conserving aTGCs $\Delta g_{1,Z}, \Delta\kappa_\gamma, \Delta\kappa_Z, \lambda_\gamma$, and λ_Z. The five b_i at $\sqrt{s} = 240$ GeV are:

$$b_{\Delta g_{1,Z}} = -0.51, \quad b_{\Delta\kappa_\gamma} = -0.17, \quad b_{\Delta\kappa_Z} = 0.12, \quad b_{\lambda_\gamma} = -0.48, \quad b_{\lambda_Z} = -0.003$$

Analytical and numerical expressions for corresponding $\omega_i(\Omega_k)$ are collected in Appendices B and C. One-parameter constraints at CEPC are listed in Table VI. Comparing Tables I and VI:

1. The relation $\lambda_\gamma = \lambda_Z$ combines constraints on the two couplings, making limits on $\lambda_\gamma = \lambda_Z$ in Table I more stringent than the separate parameters in Table VI.
2. The relation $\Delta\kappa_Z = \Delta g_{1,Z} - \tan^2\theta_W \Delta\kappa_\gamma$ means $\Delta g_{1,Z}$ absorbs some sensitivity on $\Delta\kappa_Z$ and is more severely constrained in Table I.
3. Conversely, $\Delta\kappa_\gamma$ is anti-correlated with $\Delta\kappa_Z$, making it less severely constrained in Table I.

The correlation matrix between the five aTGCs, combining all three channels, is:

$$\rho_{aTGC} = \begin{pmatrix} 1 & 0.11 & -0.99 & -0.61 & 0.01 \\ 0.11 & 1 & -0.11 & -0.31 & 0.01 \\ -0.99 & -0.11 & 1 & 0.61 & -0.01 \\ -0.61 & -0.31 & 0.61 & 1 & -0.01 \\ 0.01 & 0.01 & -0.01 & -0.01 & 1 \end{pmatrix}$$

Analogues of Tables II and III for the five aTGCs are given in Tables VII and VIII.

Appendix B: Analytical Expressions for the Coefficients ω_i

All differential cross sections with respect to the five production and decay angles can be obtained from Eq. (5) by integrating out some angles. The SM differential cross sections read:

$$\frac{d\sigma_0}{d\cos\theta} = \frac{3\beta}{2^7\pi s}\mathrm{BR}\sum_{\lambda,\tau,\tau'}|F^{(\lambda)}_{\tau\tau'}|^2_0$$

where scattering amplitudes $F^{(\lambda)}_{\tau_1\tau_2}$ are generally linear functions of anomalous couplings $\alpha_i = \Delta g_{1,V}, \Delta\kappa_V, \lambda_V, g_5^V, g_4^V, \tilde{\kappa}_V, \tilde{\lambda}_V$ in Eq. (1), and $[\cdots]_0$ means taking only the SM contribution with $\alpha_i \to 0$. In Eq. (B2), dependence of the decay matrix D on azimuthal angles has been integrated out:

$$\int D_{\tau\bar{\tau}}d\phi^* = \delta_{\tau\bar{\tau}}\frac{2\pi}{3}d_\tau(\cos\theta^*)$$

According to Refs. [40, 42], amplitudes $F^{(\lambda)}_{\tau_1\tau_2}$ can be factorized into linear functions of anomalous coupling combinations:

$$F = S + T, \quad S = S_0 + \sum_i S_i\alpha_i$$

where S and T correspond to s- and t-channels for WW production, with helicity indices $\lambda, \tau_{1,2}$ not explicitly shown. Taking first-order derivatives of amplitudes (squared) with respect to anomalous couplings is then trivial.

To obtain $\omega_i(\Omega_k)$ with $\Omega_k = \cos\theta, \cos\theta^*_{1,2}, \phi^*_{1,2}$, we implement the simple replacement in corresponding differential cross sections:

$$|F|^2 \to \frac{\partial|F|^2}{\partial\alpha_i} = 2\mathrm{Re}(F\frac{\partial F^*}{\partial\alpha_i})$$

For the five most general C and P conserving couplings:

$$\frac{\partial F}{\partial \Delta g_{1,Z}} = \frac{\partial S}{\partial g_{1,Z}}, \quad \frac{\partial F}{\partial \Delta \kappa_V} = \frac{\partial S}{\partial \kappa_V}, \quad \frac{\partial F}{\partial \lambda_V} = \frac{\partial S}{\partial \lambda_V}$$

When anomalous couplings are correlated by EW gauge symmetry:

$$\frac{\partial F}{\partial \Delta g_{1,Z}} = \frac{\partial S}{\partial g_{1,Z}} + \frac{\partial S}{\partial \kappa_Z}, \quad \frac{\partial F}{\partial \Delta \kappa_\gamma} = \frac{\partial S}{\partial \kappa_\gamma} - \tan^2\theta_W \frac{\partial S}{\partial \kappa_Z}, \quad \frac{\partial F}{\partial \lambda_\gamma} = \frac{\partial S}{\partial \lambda_\gamma} + \frac{\partial S}{\partial \lambda_Z}$$

Derivatives with respect to C or P violating anomalous couplings and generalizations to second-order derivatives are straightforward. The linear coefficients ω_i are then:

$$\omega_i(\Omega_k) = \frac{d}{d\Omega_k}\left(\sum_i \frac{\partial \sigma}{\partial \alpha_i}\right)$$

As mentioned in Section III, integrating over angle Ω_k yields analytical expressions for coefficients b_i for the total cross section:

$$b_i = \frac{1}{\sigma_0}\int \omega_i(\Omega_k) d\Omega_k$$

Appendix C: Numerical Expressions for the Coefficients ω_i

In the SM, numerical expressions for angular distributions of the $e^+e^- \to f_1\bar{f}_2\bar{f}_3 f_4$ process at the center-of-mass energy of 240 GeV designed for CEPC are, in units of pb:

$$\frac{d\sigma_0}{d\cos\theta} = 1.1 + 3.4\cos\theta + 11.3\cos^2\theta + 15.6\cos^3\theta + ...$$
$$\frac{d\sigma_0}{d\cos\theta^*_{1,2}} = 8.6(1+\cos^2\theta^*_{1,2})$$
$$\frac{d\sigma_0}{d\phi^*_{1,2}} = 2.7$$

For the five C and P conserving aTGCs $\alpha_i = \Delta g_{1,Z}, \Delta\kappa_{\gamma,Z}, \lambda_{\gamma,Z}$, the linear coefficients $\omega_i(\Omega_k)$ for differential cross sections are:

$$\omega_{\Delta g_{1,Z}}(\cos\theta) = -13.1 - 11.2\cos\theta + 15.6\cos^2\theta$$
$$\omega_{\Delta\kappa_\gamma}(\cos\theta) = -4.1 - 3.5\cos\theta + 4.7\cos^2\theta$$
$$\omega_{\Delta\kappa_Z}(\cos\theta) = 3.1 + 2.6\cos\theta - 3.6\cos^2\theta$$
$$\omega_{\lambda_\gamma}(\cos\theta) = -12.3 - 10.5\cos\theta + 14.6\cos^2\theta$$
$$\omega_{\lambda_Z}(\cos\theta) = -0.1 - 0.1\cos\theta + 0.1\cos^2\theta$$

Using these expressions, it is easy to verify that integrating ω_i distributions yields coefficients b_i for the total cross section:

$$b_{\Delta g_{1,Z}} = -0.51, \quad b_{\Delta\kappa_\gamma} = -0.17, \quad b_{\Delta\kappa_Z} = 0.12, \quad b_{\lambda_\gamma} = -0.48, \quad b_{\lambda_Z} = -0.003$$

It is phenomenologically more interesting to study aTGCs where beyond-SM physics is invariant under EW gauge symmetry. Under these conditions, corresponding ω distributions for $\alpha_i = \Delta g_{1,Z}, \Delta\kappa_\gamma, \lambda_\gamma$ are:

$$\omega_{\Delta g_{1,Z}}(\cos\theta) = -10.0 - 8.6\cos\theta + 12.0\cos^2\theta$$
$$\omega_{\Delta\kappa_\gamma}(\cos\theta) = -5.0 - 4.3\cos\theta + 5.8\cos^2\theta$$
$$\omega_{\lambda_\gamma}(\cos\theta) = -12.4 - 10.6\cos\theta + 14.7\cos^2\theta$$

Analogous to Eq. (C3), integrating ω distributions yields b_i coefficients in Eq. (A1) for total cross sections.

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Note: Figure translations are in progress. See original paper for figures.